

The Science and Engineering of Materials

THIRD S.I. EDITION

Solutions manual

Donald R. Askeland

Prepared by Gregory Lea

Adaptation by Paul Porgess,
Steve Horne and
Howard Taylor



SPRINGER-SCIENCE+BUSINESS MEDIA, B.V.

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Chapter 1 Introduction to Materials

1.1 Iron is often coated with a thin layer of zinc if it is to be used outside. What characteristics do you think the zinc provides to this coated, or galvanized, steel? What precautions should be considered in producing this product? How will the recyclability of the product be affected?

The zinc provides corrosion resistance to the iron in two ways. If the iron is completely coated with zinc, the zinc provides a barrier between the iron and the surrounding environment, therefore protecting the underlying iron. If the zinc coating is scratched to expose the iron, the zinc continues to protect the iron because the zinc corrodes preferentially to the iron (see Chapter 22). To be effective, the zinc should bond well to the iron so that it does not permit reactions to occur at the interface with the iron and so that the zinc remains intact during any forming of the galvanized material. When the material is recycled, the zinc will be lost by oxidation and vaporization, often producing a "zinc dust" that may pose an environmental hazard. Special equipment may be required to collect and either recycle or dispose of the zinc dust.

1.2 We would like to produce a transparent canopy for an aircraft. If we were to use a ceramic (that is, traditional window glass) canopy, rocks or birds might cause it to shatter. Design a material that would minimize damage or at least keep the canopy from breaking into pieces.

We might sandwich a thin sheet of a transparent polymer between two layers of the glass. This approach, used for windshields of automobiles, will prevent the "safety" glass from completely disintegrating when it fails, with the polymer holding the broken pieces of glass together until the canopy can be replaced.

Another approach might be to use a transparent, "glassy" polymer material such as polycarbonate. Some polymers have reasonably good impact properties and may resist failure. The polymers can also be toughened to resist impact by introducing tiny globules of a rubber, or elastomer, into the polymer; these globules improve the energy-absorbing ability of the composite polymer, while being too small to interfere with the optical properties of the material.

1.3 Coiled springs ought to be very strong and stiff. Si_3N_4 is a strong, stiff material. Would you select this material for a spring? Explain.

Springs are intended to resist high elastic forces, where only the atomic bonds are stretched when the force is applied. The silicon nitride would satisfy this requirement. However, we would like to also have good resistance to impact and at least some ductility (in case the spring is overloaded) to assure that the spring will not fail catastrophically. We also would like to be sure that all springs will perform satisfactorily. Ceramic materials such as silicon nitride have virtually no ductility, poor impact properties, and often are difficult to manufacture without introducing at least some small flaws that cause to fail even for relatively low forces. The silicon nitride is NOT recommended.

1.4 Temperature indicators are sometimes produced from a coiled metal strip that uncoils a specific amount when the temperature increases. How does this work; from what kind of material would the indicator be made; and what are the important properties that the material in the indicator must possess?

Bimetallic materials are produced by bonding two materials having different coefficients of thermal expansion to one another, forming a laminar composite. When the temperature changes, one of the materials will expand or contract more than the other material. This difference in expansion or contraction causes the bimetallic material to change shape; if the original shape is that of a coil, then the device will coil or uncoil, depending on the direction of the temperature change. In order for the material to perform well, the two materials must have very different coefficients of thermal expansion and should have high enough modulus of elasticity so that no permanent deformation of the material occurs.

1.5 You would like to design an aircraft that can be flown by human power nonstop for a distance of 30 km. What types of material properties would you recommend? What materials might be appropriate?

Such an aircraft must possess enough strength and stiffness to resist its own weight, the weight of the human "power source", and any aerodynamic forces imposed on it. On the other hand, it must be as light as possible to assure that the human can generate enough work to operate the aircraft. Composite materials, particularly those based on a polymer matrix, might comprise the bulk of the aircraft. The polymers have a light weight (with densities of less than half that of aluminium) and can be strengthened by introducing strong, stiff fibres made of glass, carbon, or other polymers. Composites having the strength and stiffness of steel, but with only a fraction of the weight, can be produced in this manner.

1.6 You would like to place a three-foot diameter microsatellite into orbit. The satellite will contain delicate electronic equipment that will send and receive radio signals from earth. Design the outer shell within which the electronic equipment is contained. What properties will be required and what kind of materials might be considered?

The shell of the microsatellite must satisfy several criteria. The material should have a low density, minimizing the satellite weight so that it can be lifted economically into its orbit; the material must be strong, hard, and impact resistant in order to assure that any "space dust" that might strike the satellite does not penetrate and damage the electronic equipment; the material must be transparent to the radio signals that provide communication between the satellite and earth; and the material must provide some thermal insulation to assure that solar heating does not damage the electronics.

One approach might be to use a composite shell of several materials. The outside surface might be a very thin reflective metal coating that would help reflect solar heat. The main body of the shell might be a light weight fibre-reinforced composite that would provide impact resistance (preventing penetration by dust particles) but would be transparent to radio signals.

1.7 What properties should the head of a carpenter's hammer possess? How would you manufacture a hammer head?

The head for a carpenter's hammer is produced by forging, a metal-working process; a simple steel shape is heated and formed in several steps while hot into the required shape. The head is then heat treated to produce the required mechanical and physical properties.

The striking face and claws of the hammer should be hard -- the metal should not dent or deform when driving or removing nails. Yet these portions must also possess some impact resistance, particularly so that chips do not flake off the striking face and cause injuries.

1.8 The hull of the space shuttle consists of ceramic tiles bonded to an aluminium skin. Discuss the design requirements of the shuttle hull that led to the use of this combination of materials. What problems in producing the hull might the designers and manufacturers have faced?

The space shuttle experiences extreme temperatures during re-entry into earth's atmosphere; consequently a thermal protection system must be used to prevent damage to the structure of the shuttle (not to mention its contents!). The skin must therefore be composed of a material that has an exceptionally low thermal conductivity. The material must be capable of being firmly attached to the skin of the shuttle and to be easily repaired when damage occurs.

The tiles used on the space shuttle are composed of silica fibres bonded together to produce a very low density ceramic. The thermal conductivity is so low that a person can hold on to one side of the tile while the opposite surface is red hot. The tiles are attached to the shuttle skin using a rubbery polymer that helps assure that the forces do not break the tile loose, which would then expose the underlying skin to high temperatures.

1.9 You would like to select a material for the electrical contacts in an electrical switching device which opens and closes frequently and forcefully. What properties should the contact material possess? What type of material might you recommend? Would Al_2O_3 be a good choice? Explain.

The material must have a high electrical conductivity to assure that no electrical heating or arcing occurs when the switch is closed. High purity (and therefore very soft) metals such as copper, aluminium, silver or gold provide the high conductivity. However the device must also have good wear resistance, requiring that the material be hard. Most hard, wear resistant materials have poor electrical conductivity.

One solution to this problem is to produce a particulate composite material composed of hard ceramic particles embedded in a continuous matrix of the electrical conductor. For example, silicon carbide particles could be introduced into pure aluminium; the silicon carbide particles provide wear resistance while aluminium provides conductivity. Other examples of these materials are described in Chapter 16.

Al_2O_3 by itself would not be a good choice -- alumina is a ceramic material and is an electrical insulator. However alumina particles dispersed into a copper matrix might provide wear resistance to the composite.

1.10 Aluminium has a density of $2.7 \text{ Mg}\cdot\text{m}^{-3}$. Suppose you would like to produce a composite material based on aluminium having a density of $1.5 \text{ Mg}\cdot\text{m}^{-3}$. Design a material that would have this density. Would introducing beads of polyethylene, with a density of $0.95 \text{ Mg}\cdot\text{m}^{-3}$, into the aluminium be a likely possibility? Explain.

In order to produce an aluminium-matrix composite material with a density of 1.5 Mg.m^{-3} , we would need to select a material having a density considerably less than 1.5 Mg.m^{-3} . While polyethylene's density would make it a possibility, the polyethylene has a very low melting point compared to aluminium; this would make it very difficult to introduce the polyethylene into a solid aluminium matrix -- processes such as casting or powder metallurgy would destroy the polyethylene. Therefore polyethylene would NOT be a likely possibility.

One approach, however, might be to introduce hollow glass beads. Although ceramic glasses have densities comparable to that of aluminium, a hollow bead will have a very low density. The glass also has a high melting temperature and could be introduced into liquid aluminium for processing as a casting.

1.11 You would like to be able to identify different materials without resorting to chemical analysis or lengthy testing procedures. Describe some possible testing and sorting techniques you might be able to use based on the physical properties of materials.

Some typical methods might include: measuring the density of the material (may help in separating metal groups such as aluminium, copper, steel, magnesium, etc.), determining the electrical conductivity of the material (may help in separating ceramics and polymers from metallic alloys), measuring the hardness of the material (perhaps even just using a file), and determining whether the material is magnetic or nonmagnetic (may help separate iron from other metallic alloys).

1.12 You would like to be able to physically separate different materials in a scrap recycling plant. Describe some possible methods that might be used to separate materials such as polymers, aluminium alloys, and steels from one another.

Steels can be magnetically separated from the other materials; steel (or carbon-containing iron alloys) are ferromagnetic and will be attracted by magnets. Density differences could be used -- polymers have a density near that of water; the specific gravity of aluminium alloys is around 2.7; that of steels is between 7.5 and 8. Electrical conductivity measurements could be used -- polymers are insulators, aluminium has a particularly high electrical conductivity.

1.13 Some pistons for automobile engines might be produced from a composite material containing small, hard silicon carbide particles in an aluminium alloy matrix. Explain what benefits each material in the composite may provide to the overall part. What problems might the different properties of the two materials cause in producing the part?

Aluminium provides good heat transfer due to its high thermal conductivity. It has good ductility and toughness, reasonably good strength, and is easy to cast and process. The silicon carbide, a ceramic, is hard and strong, providing good wear resistance, and also has a high melting temperature. It provides good strength to the aluminium, even at elevated temperatures. However there may be problems producing the material -- for example, the silicon carbide may not be uniformly distributed in the aluminium matrix if the pistons are produced by casting. We need to assure good bonding between the particles and the aluminium -- the surface chemistry must therefore be understood.

Differences in expansion and contraction with temperature changes may cause debonding and even cracking in the composite.

Chapter 2 Atomic Structure

2.1 Aluminium foil used for storing food weighs about 0.3 g per square inch. How many atoms of aluminium are contained in this sample of foil?

In a one square inch sample:

$$\text{number} = \frac{(0.3 \text{ g})(6.02 \times 10^{23} \text{ atoms.mol}^{-1})}{26.981 \text{ g.mol}^{-1}} = 6.69 \times 10^{21} \text{ atoms}$$

2.2 Using the densities and atomic weights given in Appendix A, calculate and compare the number of atoms per cubic centimeter in (a) lead and (b) lithium.

$$1 \text{ Mg.m}^{-3} = 1 \times 10^{-6} \text{ g m}^{-3} = 1 \text{ g cm}^{-3}.$$

(a) *In lead:*

$$\frac{(11.36 \text{ Mg.m}^{-3})(1 \text{ cm}^3)(6.02 \times 10^{23} \text{ atoms.mol}^{-1})}{207.19 \text{ g.mol}^{-1}}$$

$$= 3.3 \times 10^{22} \text{ atoms/cm}^3$$

(b) *In lithium:*

$$\frac{(0.534 \text{ Mg.m}^{-3})(1 \text{ cm}^3)(6.02 \times 10^{23} \text{ atoms.mol}^{-1})}{6.94 \text{ g.mol}^{-1}}$$

$$= 4.63 \times 10^{22} \text{ atoms/cm}^3$$

2.3 Using data in Appendix A, calculate the number of iron atoms in one tonne (A metric tonne = 1Mg.)

$$\frac{(1 \text{ Mg})(6.02 \times 10^{23} \text{ atoms.mol}^{-1})}{55.847 \times 10^{-6} \text{ Mg.mol}^{-1}} = 1.08 \times 10^{28} \text{ atoms/tonne}$$

2.4 Using data in Appendix A, calculate the volume in cubic centimeters occupied by one mole of boron.

$$\frac{(1 \text{ mol})(10.81 \text{ g.mol}^{-1})}{2.3 \text{ Mg.m}^{-3}} = 4.7 \text{ cm}^3$$

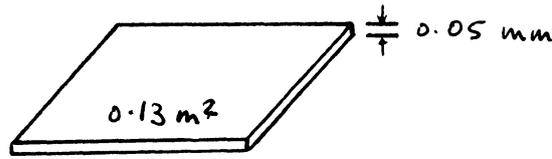
2.5 In order to plate a steel part having a surface area of 0.13 m² with a 0.05 mm. thick layer of nickel, (a) how many atoms of nickel are required and (b) how many moles of nickel are required?

$$\text{Volume} = (0.13 \text{ m}^2)(0.05 \times 10^{-3} \text{ m}) = 6.50 \times 10^{-6} \text{ m}^3$$

$$(a) \frac{(6.50 \times 10^{-6} \text{ m}^3)(8.902 \text{ Mg.m}^{-3})(6.02 \times 10^{23} \text{ atoms.mol}^{-1})}{58.71 \times 10^{-6} \text{ Mg.mol}^{-1}}$$

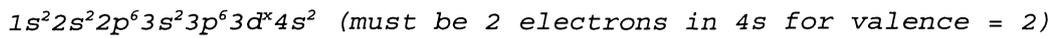
$$= 5.98 \times 10^{23} \text{ atoms}$$

$$(b) \frac{(6.555 \text{ cm}^3)(8.902 \text{ Mg}\cdot\text{m}^{-3})}{58.71 \text{ g}\cdot\text{mol}^{-1}} = 0.994 \text{ mol Ni required}$$



2.6 Suppose an element has a valence of 2 and an atomic number of 27. Based only on the quantum numbers, how many electrons must be present in the 3d energy level?

We can let x be the number of electrons in the 3d energy level. Then:



Since

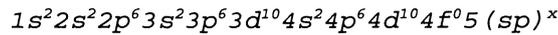
$$27 - (2+2+6+2+6+2) = 7 = x$$

there must be 7 electrons in the 3d level.

2.7 Indium, which has an atomic number of 49, contains no electrons in its 4f energy level. Based only on this information, what must be the valence of indium?

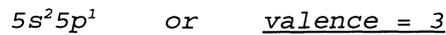
We can let x be the number of electrons in the outer sp energy level.

Then:



$$49 - (2+2+6+2+6+10+2+6+10+0) = 3$$

Therefore the outer 5sp level must be:



2.8 Without consulting Appendix C, describe the quantum numbers for each of the 18 electrons in the M shell of copper, using a format similar to that in Figure 2.2.

For the M shell: $n = 3$; $l = 0, 1, 2$; $m_l = 2l + 1$

$n = 3$	$l = 0$	$m_l = 0$	$m_s = +\frac{1}{2}$	} $3s^2$
			$m_s = -\frac{1}{2}$	
	$l = 1$	$m_l = -1$	$m_s = +\frac{1}{2}$	} $3p^6$
		$m_l = 0$	$m_s = -\frac{1}{2}$	
		$m_l = +1$	$m_s = +\frac{1}{2}$	
			$m_s = -\frac{1}{2}$	
		$m_l = +1$	$m_s = +\frac{1}{2}$	
	$l = 2$	$m_l = -2$	$m_s = +\frac{1}{2}$	} $3d^{10}$
		$m_l = -1$	$m_s = -\frac{1}{2}$	
		$m_l = 0$	$m_s = +\frac{1}{2}$	
		$m_l = +1$	$m_s = -\frac{1}{2}$	
		$m_l = +1$	$m_s = +\frac{1}{2}$	
		$m_l = +2$	$m_s = -\frac{1}{2}$	
		$m_l = +2$	$m_s = +\frac{1}{2}$	

2.9 Electrical charge is transferred in metals by movement of valence electrons. How many potential charge carriers are there in an aluminium wire 1 mm in diameter and 100 m in length?

Aluminium has 3 valence electrons per atom; the volume of the wire is:

$$\text{Volume} = (\pi/4)d^2l = (\pi/4)(1 \times 10^{-3} \text{ m})^2(1000 \text{ m}) = 78.54 \times 10^{-6} \text{ m}^3$$

$$n = \frac{(78.54 \times 10^{-6} \text{ m}^3)(2.699 \text{ Mg.m}^{-3})(6.02 \times 10^{23} \text{ atoms.mol}^{-1})(3 \text{ electrons/atom})}{26.981 \text{ g.mol}^{-1}}$$

$$n = 1.42 \times 10^{25} \text{ carriers}$$

2.10 Increasing the temperature of a semiconductor breaks covalent bonds. For each broken bond, two electrons become free to move and transfer electrical charge. What fraction of valence electrons are free to move and what fraction of the covalent bonds must be broken in order that 5×10^{15} electrons conduct electrical charge in 50 g of silicon?

$$\frac{(50 \text{ g})(6.02 \times 10^{23} \text{ atoms.mol}^{-1})}{28.08 \text{ g.mol}^{-1}} = 1.072 \times 10^{24} \text{ atoms of Si}$$

$$\begin{aligned} \text{Total valence electrons} &= (4 \text{ electrons/atom})(1.072 \times 10^{24} \text{ atoms}) \\ &= 4.288 \times 10^{24} \text{ electrons} \end{aligned}$$

$$\text{Fraction free electrons} = 5 \times 10^{15} / 4.288 \times 10^{24} = 1.17 \times 10^{-9}$$

On average, there is one covalent bond per Si atom (each Si atom is bonded to four others). Thus, there are a total of 1.072×10^{24} bonds. Each bond has 2 electrons, so the number of broken bonds needed to give 5×10^{15} electrons is 2.5×10^{15} . The fraction of broken bonds is:

$$f = \frac{2.5 \times 10^{15}}{1.072 \times 10^{24}} = 2.33 \times 10^{-9}$$

2.11 What fraction of the total silicon atoms must be replaced by arsenic atoms to obtain one million electrons that are free to move in 450g of silicon?

$$\frac{(450 \text{ g})(6.02 \times 10^{23} \text{ atoms.mol}^{-1})}{28.08 \text{ g.mol}^{-1}} = 9.647 \times 10^{24} \text{ Si atoms}$$

As has a valence of 5; therefore, to get 10^6 electrons, we need to replace 10^6 Si atoms. In one pound of Si, the fraction of As must be:

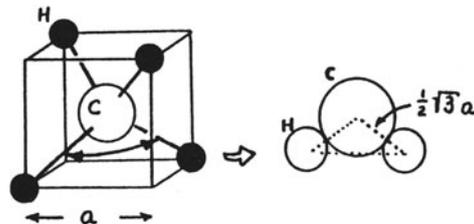
$$f = \frac{1 \times 10^6 \text{ replaced atoms}}{9.647 \times 10^{24} \text{ total Si atoms}} = 1.037 \times 10^{-19}$$

2.12 Methane (CH_4) has a tetrahedral structure similar to that of SiO_2 (Figure 2.8), with a carbon atom of radius 0.77×10^{-8} cm at the center and hydrogen atoms of radius 0.46×10^{-8} cm at four of the eight corners. Calculate the size of the tetrahedral cube for methane.

$$(1/2)\sqrt{3}a = r_c + r_H$$

$$(1/2)\sqrt{3}a = 0.77 \times 10^{-8} + 0.46 \times 10^{-8}$$

$$a = 1.42 \times 10^{-8} \text{ cm}$$



2.13 The compound AlP is a compound semiconducting material having mixed ionic and covalent bonding. Estimate the fraction of the bonding that is ionic.

$$E_{Al} = 1.5 \qquad E_p = 2.1$$

$$f_{\text{covalent}} = \exp(-0.25 \Delta E^2)$$

$$f_{\text{covalent}} = \exp[(-0.25)(2.1 - 1.5)^2] = \exp[-0.09] = 0.914$$

$$f_{\text{ionic}} = 1 - 0.914 = 0.086 \therefore \text{bonding is mostly covalent}$$

2.14 Estimate the fraction of bonding of MgO that is ionic.

$$E_{Mg} = 1.2 \qquad E_o = 3.5$$

$$f_{\text{covalent}} = \exp[(-0.25)(3.5 - 1.2)^2] = \exp(-1.3225) = 0.266$$

$$f_{\text{ionic}} = 1 - 0.266 = 0.734 \therefore \text{bonding is mostly ionic}$$

2.15 Bonding in the intermetallic compound Ni₃Al is predominantly metallic. Explain why there will be little, if any, ionic bonding component. (The electronegativity of nickel is about 1.8.)

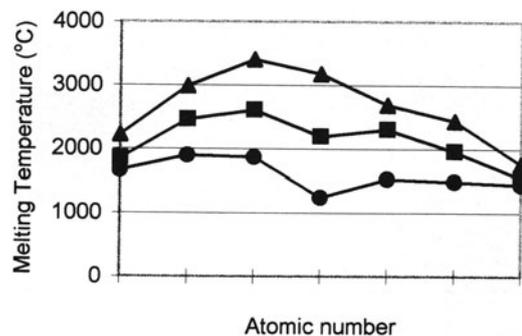
The electronegativity of Al is 1.5, while that of Ni is 1.8. These values are relatively close, so we wouldn't expect much ionic bonding. Also, both are metals and prefer to give up their electrons rather than share or donate them.

2.16 Plot the melting temperatures of elements in the IVB to VIII B columns of the periodic table versus atomic number (that is, plot melting temperatures of Ti through Ni, Zr through Pd, and Hf through Pt). Discuss these relationships, based on atomic bonding and binding energy, (a) as the atomic number increases in each row of the periodic table and (b) as the atomic number increases in each column of the periodic table.

Ti-1668	Zr-1852	Hf-2227
V-1900	Nb-2468	Ta-2996
Cr-1875	Mo-2610	W-3410
Mn-1244	Tc-2200	Re-3180
Fe-1538	Ru-2310	Os-2700
Co-1495	Rh-1963	Ir-2447
Ni-1453	Pd-1552	Pt-1769

For each row, the melting temperature is highest when the outer "d" energy level is partly full. In Cr, there are 5 electrons in the 3d shell; in Mo, there are 5 electrons in the 4d shell; in W there are 4 electrons in the 5d shell.

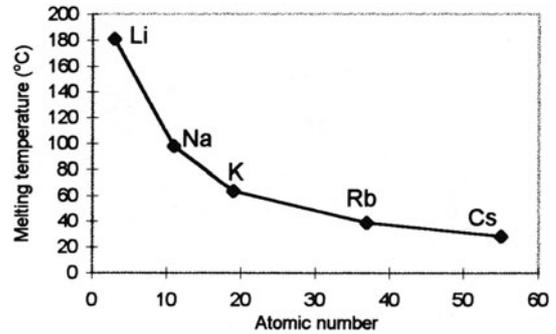
In each column, the melting temperature increases as the atomic number increases -- the atom cores contain a larger number of tightly held electrons, making the metals more stable.



2.17 Plot the melting temperature of the elements in the IA column of the periodic table versus atomic number (that is plot melting temperatures of Li to Cs). Discuss this relationship, based on atomic bonding and binding energy.

$T (^{\circ}\text{C})$
Li-180.7
Na- 97.8
K - 63.2
Rb- 38.9
Cs- 28.6

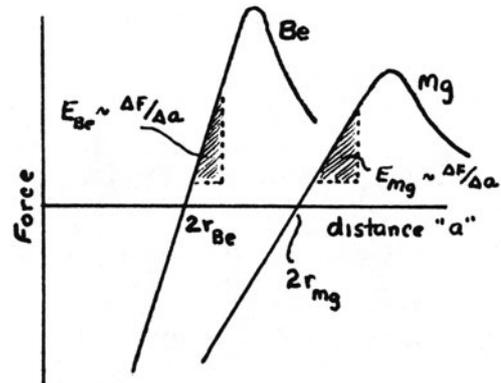
As the atomic number increases, the melting temperature decreases, opposite that found in problem 16.



2.18 Beryllium and magnesium, both in the IIA column of the periodic table, are lightweight metals. Which would you expect to have the higher modulus of elasticity? Explain, considering binding energy and atom radii and using appropriate sketches of force versus interatomic spacing.

4 Be	$1s^2 2s^2$	$E = 300 \text{ GPa}$	$r_{\text{Be}} = 0.1143 \text{ nm}$
12 Mg	$1s^2 2s^2 2p^6 3s^2$	$E = 44 \text{ GPa}$	$r_{\text{Mg}} = 0.1604 \text{ nm}$

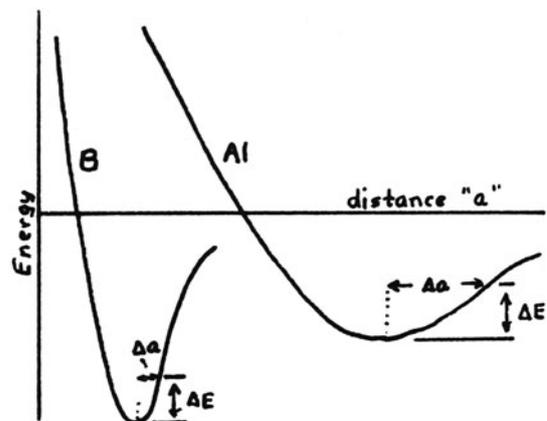
The smaller Be electrons are held closer to the core \therefore held more tightly, giving a higher binding energy.



2.19 Boron has a much lower coefficient of thermal expansion than aluminium, even though both are in the IIIA column of the periodic table. Explain, based on binding energy, atomic size, and the energy well, why this difference is expected.

5 B	$1s^2 2s^2 2p^1$	$r_{\text{B}} = 0.046 \text{ nm}$
13 Al	$1s^2 2s^2 2p^6 3s^2 3p^1$	$r_{\text{Al}} = 0.1432 \text{ nm}$

Electrons in Al are not as tightly bonded as those in B due to the smaller size of the boron atom and the lower binding energy associated with its size.



2.20 Would you expect MgO or magnesium to have the higher modulus of elasticity? Explain.

MgO has ionic bonds, which are strong compared to the metallic bonds in Mg. A higher force will be required to cause the same separation between the ions in MgO compared to the atoms in Mg. Therefore, MgO should have the higher modulus of elasticity. In Mg, $E \approx 44$ GPa; in MgO, $E \approx 207$ GPa.

2.21 Would you expect Al_2O_3 or aluminium to have the higher coefficient of thermal expansion? Explain.

Al_2O_3 has stronger bonds than Al; therefore, Al_2O_3 should have a lower thermal expansion coefficient than Al. In Al, $\alpha = 25 \times 10^{-6} \text{ m.m}^{-1}.\text{K}^{-1}$; in Al_2O_3 , $\alpha = 6.7 \times 10^{-6} \text{ m.m}^{-1}.\text{K}^{-1}$.

2.22 Aluminium and silicon are side-by-side in the periodic table. Which would you expect to have the higher modulus of elasticity? Explain.

Silicon has covalent bonds; aluminium has metallic bonds. Therefore, Si should have a higher modulus of elasticity.

2.23 Explain why the modulus of elasticity of simple thermoplastic polymers, such as polyethylene and polystyrene, is expected to be very low compared with that of metals and ceramics.

The chains in polymers are held to other chains by Van der Waals bonds, which are much weaker than metallic, ionic, and covalent bonds. For this reason, much less force is required to shear these weak bonds and to unkink and straighten the chains.

2.24 Steel is coated with a thin layer of ceramic to help protect against corrosion. What do you expect to happen to the coating when the temperature of the steel is increased significantly? Explain.

Ceramics are expected to have a low coefficient of thermal expansion due to strong ionic/covalent bonds; steel has a high thermal expansion coefficient. When the structure heats, steel expands more than the coating, which may crack and expose the underlying steel to corrosion.

Chapter 3 Atomic Arrangement

3.1 Calculate the atomic radius in cm for the following: (a) BCC metal with $a_0 = 0.3294 \text{ nm}$ and one atom per lattice point and (b) FCC metal with $a_0 = 0.40862 \text{ nm}$ and one atom per lattice point.

(a) For BCC metals,

$$r = \frac{(\sqrt{3})a_0}{4} = \frac{(\sqrt{3})(0.3294 \text{ nm})}{4} = 0.1426 \text{ nm}$$

(b) For FCC metals,

$$r = \frac{(\sqrt{2})a_0}{4} = \frac{(\sqrt{2})(0.40862 \text{ nm})}{4} = 0.14447 \text{ nm}$$

3.2 Determine the crystal structure for the following: (a) a metal with $a_0 = .49489 \text{ nm}$, $r = 0.175 \text{ nm}$ and one atom per lattice point and (b) a metal with $a_0 = 0.42906 \text{ nm}$, $r = 0.1858 \text{ nm}$ and one atom per lattice point.

We want to determine if "x" in the calculations below equals $\sqrt{2}$ (for FCC) or $\sqrt{3}$ (for BCC):

$$(a) (x)(0.49489 \text{ nm}) = (4)(0.175 \text{ nm})$$

$$x = \sqrt{2} \quad \text{or} \quad \text{FCC}$$

$$(b) (x)(0.42906 \text{ nm}) = (4)(0.1858 \text{ nm})$$

$$x = \sqrt{3} \quad \text{or} \quad \text{BCC}$$

3.3 The density of potassium, which has the BCC structure and one atom per lattice point, is 0.855 Mg.m^{-3} . The atomic weight of potassium is 39.09 g.mol^{-1} . Calculate (a) the lattice parameter and (b) the atomic radius of potassium.

(a) Using Equation 3.5:

$$0.855 \text{ Mg.m}^{-3} = \frac{(2 \text{ atoms/cell})(39.09 \text{ g.mol}^{-1})}{(a_0)^3 (6.02 \times 10^{23} \text{ atoms.mol}^{-1})}$$

$$a_0^3 = 1.5189 \times 10^{-28} \text{ m}^3 \quad \text{or} \quad a_0 = 5.3355 \times 10^{-10} \text{ m} = 0.53355 \text{ nm}$$

(b) From the relationship between atomic radius and lattice parameter:

$$r = \frac{(\sqrt{3})(0.53355 \text{ nm})}{4} = 0.23103 \text{ nm}$$

3.4 The density of thorium, which has the FCC structure and one atom per lattice point, is 11.72 Mg.m^{-3} . The atomic weight of thorium is 232 g.mol^{-1} . Calculate (a) the lattice parameter and (b) the atomic radius of thorium.

(a) From Equation 3.5:

$$11.72 \text{ Mg.m}^{-3} = \frac{(4 \text{ atoms/cell})(232 \text{ g.mol}^{-1})}{(a_0)^3 (6.02 \times 10^{23} \text{ atoms.mol}^{-1})}$$

$$a_0^3 = 1.31529 \times 10^{-28} \text{ m}^3 \quad \text{or} \quad a_0 = 0.50856 \text{ nm}$$

(b) From the relationship between atomic radius and lattice parameter:

$$r = \frac{\sqrt{2} (0.50856 \text{ nm})}{4} = 0.1798 \text{ nm}$$

3.5 A metal having a cubic structure has a density of 2.6 Mg.m⁻³, an atomic weight of 87.62 g.mol⁻¹, and a lattice parameter of 0.60849 nm. One atom is associated with each lattice point. Determine the crystal structure of the metal.

$$2.6 \text{ Mg.m}^{-3} = \frac{(x \text{ atoms/cell}) (87.62 \text{ g.mol}^{-1})}{(0.60849 \text{ nm})^3 (6.02 \times 10^{23} \text{ atoms.mol}^{-1})}$$

$$x = 4 \quad \text{or} \quad \text{FCC}$$

3.6 A metal having a cubic structure has a density of 1.892 Mg.m⁻³, an atomic weight of 132.91 g.mol⁻¹, and a lattice parameter of 0.613 nm. One atom is associated with each lattice point. Determine the crystal structure of the metal.

$$1.892 \text{ Mg.m}^{-3} = \frac{(x \text{ atoms/cell}) (132.91 \text{ g.mol}^{-1})}{(0.613 \text{ nm})^3 (6.02 \times 10^{23} \text{ atoms.mol}^{-1})}$$

$$x = 2 \quad \text{or} \quad \text{BCC}$$

3.7 Indium has a tetragonal structure with a₀ = 0.32517 nm and c₀ = 0.49459 nm. The density is 7.286 Mg.m⁻³ and the atomic weight is 114.82 g.mol⁻¹. Does indium have the simple tetragonal or body-centered tetragonal structure?

$$7.286 \text{ Mg.m}^{-3} = \frac{(x \text{ atoms/cell}) (114.82 \text{ g.mol}^{-1})}{(0.32517 \text{ nm})^2 (0.49459 \text{ nm}) (6.02 \times 10^{23} \text{ atoms.mol}^{-1})}$$

$$x = 2 \quad \text{or} \quad \text{BCT (body-centered tetragonal)}$$

3.8 Bismuth has a hexagonal structure, with a₀ = 0.4546 nm and c₀ = 1.186 nm. The density is 9.808 Mg.m⁻³ and the atomic weight is 208.98 g.mol⁻¹. Determine (a) the volume of the unit cell and (b) how many atoms are in each unit cell.

(a) The volume of the unit cell is $V = a_0^2 c_0 \cos 30^\circ$.

$$V = (0.4546 \text{ nm})^2 (1.186 \text{ nm}) (\cos 30^\circ) = 0.21226 \text{ nm}^3$$

(b) If "x" is the number of atoms per unit cell, then:

$$9.808 \text{ Mg.m}^{-3} = \frac{(x \text{ atoms/cell}) (208.98 \text{ g.mol}^{-1})}{(0.21226 \times 10^{-22} \text{ cm}^3) (6.02 \times 10^{23} \text{ atoms.mol}^{-1})}$$

$$x = 6 \text{ atoms/cell}$$

3.9 Gallium has an orthorhombic structure, with a₀ = 0.45258 nm, b₀ = 0.45186 nm, and c₀ = 0.76570 nm. The atomic radius is 0.1218 nm. The density is 5.904 Mg.m⁻³ and the atomic weight is 69.72 g.mol⁻¹. Determine (a) the number of atoms in each unit cell and (b) the packing factor in the unit cell.

The volume of the unit cell is $V = a_0 b_0 c_0$ or

$$V = (0.45258 \text{ nm})(0.45186 \text{ nm})(0.76570 \text{ nm}) = 0.1566 \text{ nm}^3$$

(a) From the density equation:

$$5.904 \text{ Mg.m}^{-3} = \frac{(x \text{ atoms/cell})(69.72 \text{ g.mol}^{-1})}{(0.1566 \text{ nm}^3)(6.02 \times 10^{23} \text{ atoms.mol}^{-1})}$$

$$x = 8 \text{ atoms/cell}$$

(b) From the packing factor equation:

$$PF = \frac{(8 \text{ atoms/cell})(4\pi/3)(0.1218 \text{ nm})^3}{0.1566 \text{ nm}^3} = 0.387$$

3.10 Beryllium has a hexagonal crystal structure, with $a_0 = 0.22858 \text{ nm}$ and $c_0 = 0.35842 \text{ nm}$. The atomic radius is 0.1143 nm , the density is 1.848 Mg.m^{-3} , and the atomic weight is 9.01 g.mol^{-1} . Determine (a) the number of atoms in each unit cell and (b) the packing factor in the unit cell.

$$V = (0.22858 \text{ nm})^2(0.35842 \text{ nm})\cos 30 = 0.01622 \text{ nm}^3$$

(a) From the density equation:

$$1.848 \text{ Mg.m}^{-3} = \frac{(x \text{ atoms/cell})(9.01 \text{ g.mol}^{-1})}{(0.01622 \text{ nm}^3)(6.02 \times 10^{23} \text{ atoms.mol}^{-1})}$$

$$x = 2 \text{ atoms/cell}$$

(b) The packing factor is:

$$PF = \frac{(2 \text{ atoms/cell})(4\pi/3)(0.1143 \text{ nm})^3}{0.01622 \text{ nm}^3} = 0.77$$

3.11 Above 882°C , titanium has a BCC crystal structure, with $a = 0.332 \text{ nm}$. Below this temperature, titanium has a HCP structure, with $a = 0.2978 \text{ nm}$ and $c = 0.4735 \text{ nm}$. Determine the percent volume change when BCC titanium transforms to HCP titanium. Is this a contraction or expansion?

We can find the volume of each unit cell. Two atoms are present in both BCC and HCP titanium unit cells, so the volumes of the unit cells can be directly compared.

$$V_{\text{BCC}} = (0.332 \text{ nm})^3 = 0.03659 \text{ nm}^3$$

$$V_{\text{HCP}} = (0.2978 \text{ nm})^2(0.4735 \text{ nm})\cos 30 = 0.03637 \text{ nm}^3$$

$$\Delta V = \frac{V_{\text{HCP}} - V_{\text{BCC}}}{V_{\text{BCC}}} \times 100 = \frac{0.03637 \text{ nm}^3 - 0.03659 \text{ nm}^3}{0.03659 \text{ nm}^3} \times 100$$

$$= -0.6\%$$

Therefore titanium contracts 0.6% during cooling.

3.12 α -Mn has a cubic structure with $a_0 = 0.8931 \text{ nm}$ and a density of 7.47 Mg.m^{-3} . β -Mn has a different cubic structure, with $a_0 = 0.6326 \text{ nm}$ and a density of 7.26 Mg.m^{-3} . The atomic weight of manganese is $54.938 \text{ g.mol}^{-1}$ and the atomic radius is 0.112 nm . Determine the percent volume change that would occur if α -Mn transforms to β -Mn.

First we need to find the number of atoms in each unit cell so we can determine the volume change based on equal numbers of atoms. From the density equation, we find for the α -Mn:

$$7.47 \text{ Mg.m}^{-3} = \frac{(x \text{ atoms/cell}) (54.938 \text{ g.mol}^{-1})}{(0.8931 \text{ nm})^3 (6.02 \times 10^{23} \text{ atoms.mol}^{-1})}$$

$$x = 58 \text{ atoms/cell} \quad V_{\alpha\text{-Mn}} = (0.8931 \text{ nm})^3 = 7.12 \times 10^{-28} \text{ m}^3$$

For β -Mn:

$$7.26 \text{ Mg.m}^{-3} = \frac{(x \text{ atoms/cell}) (54.938 \text{ g.mol}^{-1})}{(0.6326 \times 10^{-9} \text{ m})^3 (6.02 \times 10^{23} \text{ atoms.mol}^{-1})}$$

$$x = 20 \text{ atoms/cell} \quad V_{\beta\text{-Mn}} = (0.6326 \times 10^{-9} \text{ m})^3 = 2.53 \times 10^{-28} \text{ m}^3$$

The volume of the β -Mn can be adjusted by a factor of 58/20, to account for the different number of atoms per cell. The volume change is then:

$$\begin{aligned} \Delta V &= \frac{(58/20)V_{\beta\text{-Mn}} - V_{\alpha\text{-Mn}}}{V_{\alpha\text{-Mn}}} \times 100 = \frac{(58/20)(2.53) - 7.12}{7.12} \times 100 \\ &= + 3.05\% \end{aligned}$$

The manganese expands by 3.05% during the transformation.

3.13 A typical paper clip weighs 0.59 g and consists of BCC iron. Calculate (a) the number of unit cells and (b) the number of iron atoms in the paper clip. (See Appendix A for required data)

The lattice parameter for BCC iron is 0.2866 nm. Therefore

$$V_{\text{unit cell}} = (0.2866 \text{ nm})^3 = 2.354 \times 10^{-29} \text{ m}^3$$

(a) The density is 7.87 Mg.m⁻³. The number of unit cells is:

$$\begin{aligned} \text{number} &= \frac{0.59 \text{ g}}{(7.87 \text{ Mg.m}^{-3}) (2.354 \times 10^{-29} \text{ m}^3/\text{cell})} \\ &= 3.185 \times 10^{21} \text{ cells} \end{aligned}$$

(b) There are 2 atoms/cell in BCC iron. The number of atoms is:

$$\begin{aligned} \text{number} &= (3.185 \times 10^{21} \text{ cells}) (2 \text{ atoms.cell}^{-1}) \\ &= 6.37 \times 10^{21} \text{ atoms} \end{aligned}$$

3.14 Aluminium foil used to package food is approximately 25 μm thick. Assume that all of the unit cells of the aluminium are arranged so that a_0 is perpendicular to the foil surface. For a 100mm x 100mm square of the foil, determine (a) the total number of unit cells in the foil and (b) the thickness of the foil in number of unit cells. (See Appendix A)

The lattice parameter for aluminium is 0.404958 nm. Therefore:

$$V_{\text{unit cell}} = (0.404958 \times 10^{-9})^3 = 6.6409 \times 10^{-29} \text{ m}^3$$

The volume of the foil is:

$$V_{\text{foil}} = (0.1 \text{ m}) (0.1 \text{ m}) (25 \times 10^{-6} \text{ m}) = 2.5 \times 10^{-7} \text{ m}^3$$

(a) The number of unit cells in the foil is:

$$\text{number} = \frac{2.5 \times 10^{-7} \text{ m}^3}{6.6409 \times 10^{-29} \text{ m}^3/\text{cell}} = 3.765 \times 10^{21} \text{ cells}$$

(b) The thickness of the foil, in number of unit cells, is:

$$\text{number} = \frac{(25 \times 10^{-6} \text{ m})}{0.404958 \text{ nm}} = 6.27 \times 10^4 \text{ cells}$$

3.15 Determine the Miller indices for the directions in the cubic unit cell shown in Figure 3.33.

$$A: 0, 1, 0 - 0, 1, 1 = 0, 0, -1 = [001]$$

$$B: \frac{1}{2}, 0, 0 - 0, 1, 0 = \frac{1}{2}, -1, 0 = [120]$$

$$C: 0, 1, 1 - 1, 0, 0 = -1, 1, 1 = [111]$$

$$D: 1, 0, \frac{1}{2} - 0, \frac{1}{2}, 1 = 1, -\frac{1}{2}, -\frac{1}{2} = [211]$$

3.16 Determine the indices for the directions in the cubic unit cell shown in Figure 3.34.

$$A: 0, 0, 1 - 1, 0, 0 = -1, 0, 1 = [101]$$

$$B: 1, 0, 1 - \frac{1}{2}, 1, 0 = \frac{1}{2}, -1, 1 = [122]$$

$$C: 1, 0, 0 - 0, \frac{3}{4}, 1 = 1, -\frac{3}{4}, -1 = [434]$$

$$D: 0, 1, \frac{1}{2} - 0, 0, 0 = 0, 1, \frac{1}{2} = [021]$$

3.17 Determine the indices for the planes in the cubic unit cell shown in Figure 3.35.

$$A: \begin{array}{l} x = 1 \quad 1/x = 1 \\ y = -1 \quad 1/y = -1 \\ z = 1 \quad 1/z = 1 \end{array} \quad (111)$$

$$B: \begin{array}{l} x = \infty \quad 1/x = 0 \\ y = 1/3 \quad 1/y = 3 \\ z = \infty \quad 1/z = 0 \end{array} \quad (030)$$

$$C: \begin{array}{l} x = 1 \quad 1/x = 1 \\ y = \infty \quad 1/y = 0 \\ z = -\frac{1}{2} \quad 1/z = -2 \end{array} \quad (102) \quad (\text{origin at } 0, 0, 1)$$

3.18 Determine the indices for the planes in the cubic unit cell shown in Figure 3.36.

$$A: \begin{array}{l} x = -1 \quad 1/x = -1 \quad x \cdot 3 = -3 \\ y = \frac{1}{2} \quad 1/y = 2 \quad x \cdot 3 = 6 \\ z = \frac{1}{4} \quad 1/z = 4 \quad x \cdot 3 = 4 \end{array} \quad (364) \quad (\text{origin at } 1, 0, 0)$$

$$B: \begin{array}{l} x = 1 \quad 1/x = 1 \quad x \cdot 3 = 3 \\ y = -3/4 \quad 1/y = -4/3 \quad x \cdot 3 = -4 \\ z = \infty \quad 1/z = 0 \quad x \cdot 3 = 0 \end{array} \quad (340) \quad (\text{origin at } 0, 1, 0)$$

$$C: \begin{array}{l} x = 2 \quad 1/x = \frac{1}{2} \quad x \cdot 6 = 3 \\ y = 3/2 \quad 1/y = 2/3 \quad x \cdot 6 = 4 \\ z = 1 \quad 1/z = 1 \quad x \cdot 6 = 6 \end{array} \quad (346)$$

3.19 Determine the indices for the directions in the hexagonal lattice shown in Figure 3.37, using both the three-digit and four-digit systems.

$$\begin{aligned}
 \text{A: } & 1, -1, 0 - 0, 0, 0 = 1, -1, 0 = [110] \\
 & h = (1/3)(2 + 1) = 1 \\
 & k = (1/3)(-2 - 1) = -1 = [1100] \\
 & i = -(1/3)(1 - 1) = 0 \\
 & l = 0 \\
 \\
 \text{B: } & 1, 1, 0 - 0, 0, 1 = 1, 1, -1 = [111] \\
 & h = (1/3)(2 - 1) = (1/3) \\
 & k = (1/3)(2 - 1) = (1/3) = [1123] \\
 & i = -(1/3)(1 + 1) = -(2/3) \\
 & l = -1 \\
 \\
 \text{C: } & 0, 1, 1 - 0, 0, 0 = 0, 1, 1 = [011] \\
 & h = (1/3)(0 - 1) = -(1/3) \\
 & k = (1/3)(2 - 0) = (2/3) = [1213] \\
 & i = -(1/3)(0 + 1) = -(1/3) \\
 & l = 1
 \end{aligned}$$

3.20 Determine the indices for the directions in the hexagonal lattice shown in Figure 3.38, using both the three-digit and four-digit systems.

$$\begin{aligned}
 \text{A: } & 0, 1, 1 - \frac{1}{2}, 1, 0 = -\frac{1}{2}, 0, 1 = [102] \\
 & h = (1/3)(-2 - 0) = -(2/3) \\
 & k = (1/3)(0 + 1) = (1/3) = [2116] \\
 & i = -(1/3)(-1 + 0) = (1/3) \\
 & l = 2 \\
 \\
 \text{B: } & 1, 0, 0 - 1, 1, 1 = 0, -1, -1 = [011] \\
 & h = (1/3)(0 + 1) = (1/3) \\
 & k = (1/3)(-2 + 0) = -(2/3) = [1213] \\
 & i = -(1/3)(0 - 1) = (1/3) \\
 & l = -1 \\
 \\
 \text{C: } & 0, 0, 0 - 1, 0, 1 = -1, 0, -1 = [101] \\
 & h = (1/3)(-2 + 0) = -(2/3) \\
 & k = (1/3)(0 + 1) = (1/3) = [2113] \\
 & i = -(1/3)(-1 + 0) = (1/3) \\
 & l = -1
 \end{aligned}$$

3.21 Determine the indices for the planes in the hexagonal lattice shown in Figure 3.39.

$$\begin{aligned}
 \text{A : } & \begin{array}{ll} a_1 = 1 & 1/a_1 = 1 \\ a_2 = -1 & 1/a_2 = -1 \\ a_3 = \infty & 1/a_3 = 0 \\ c = 1 & 1/c = 1 \end{array} & (1101) & \text{(origin at } a_2 = 1) \\
 \\
 \text{B : } & \begin{array}{ll} a_1 = \infty & 1/a_1 = 0 \\ a_2 = \infty & 1/a_2 = 0 \\ a_3 = \infty & 1/a_3 = 0 \\ c = 2/3 & 1/c = 3/2 \end{array} & (0003) \\
 \\
 \text{C : } & \begin{array}{ll} a_1 = 1 & 1/a_1 = 1 \\ a_2 = -1 & 1/a_2 = -1 \\ a_3 = \infty & 1/a_3 = 0 \\ c = \infty & 1/c = 0 \end{array} & (1100)
 \end{aligned}$$

3.22 Determine the indices for the planes in the hexagonal lattice shown in Figure 3.40.

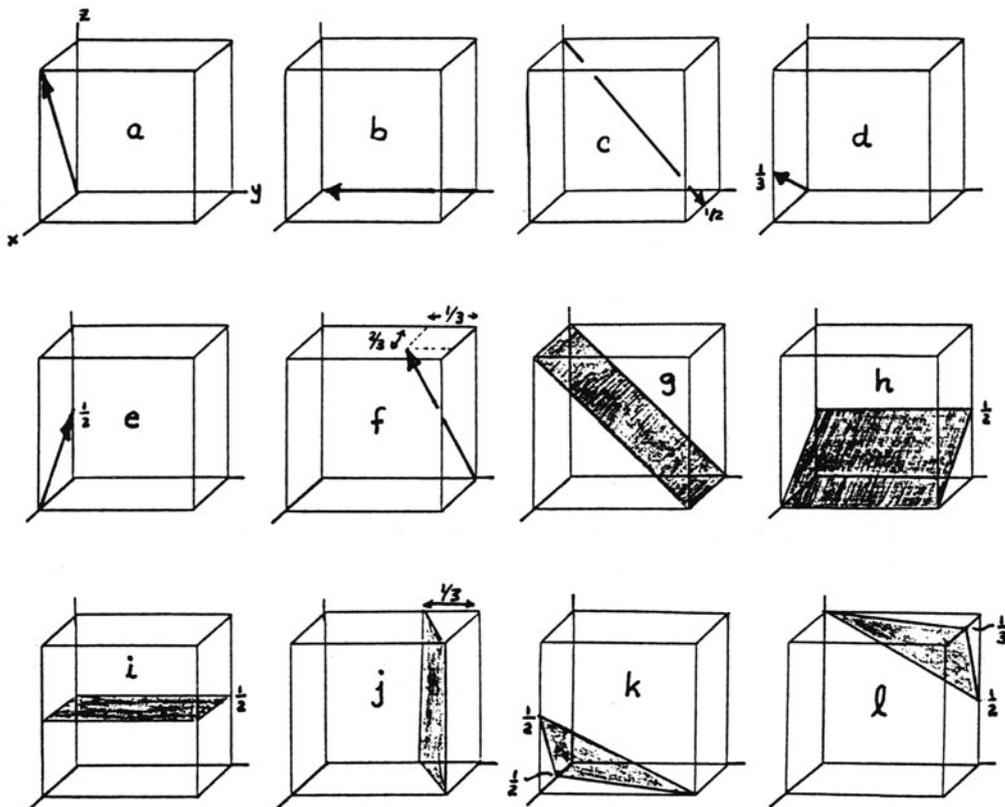
A: $a_1 = 1$ $1/a_1 = 1$
 $a_2 = -1$ $1/a_2 = -1$ (1102)
 $a_3 = \infty$ $1/a_3 = 0$
 $c = \frac{1}{2}$ $1/c = 2$

B: $a_1 = \infty$ $1/a_1 = 0$
 $a_2 = 1$ $1/a_2 = 1$ (0111)
 $a_3 = -1$ $1/a_3 = -1$
 $c = 1$ $1/c = 1$

C: $a_1 = -1$ $1/a_1 = -1$
 $a_2 = \frac{1}{2}$ $1/a_2 = 2$ (1210)
 $a_3 = -1$ $1/a_3 = -1$
 $c = \infty$ $1/c = 0$

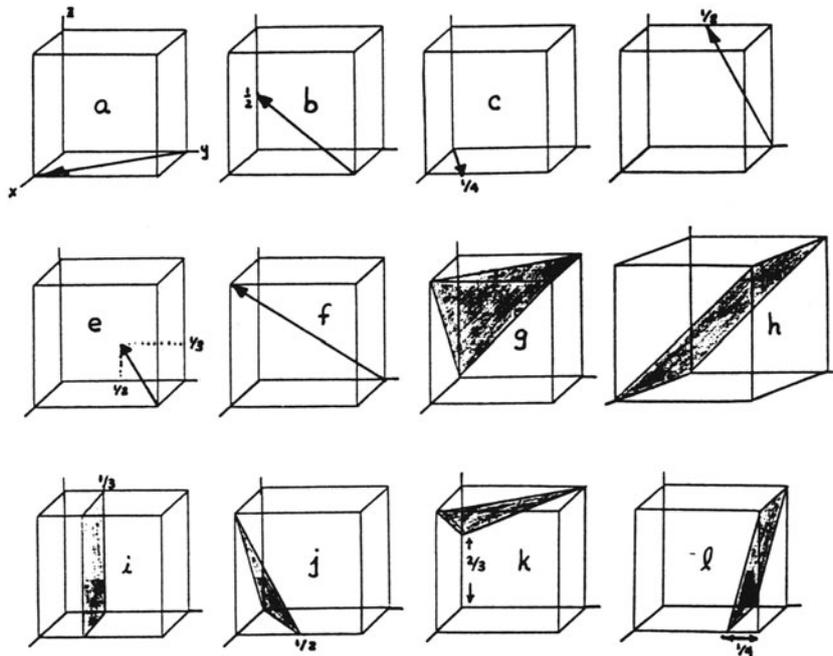
3.23 Sketch the following planes and directions within a cubic unit cell.

- a. [101] b. [010] c. [122] d. [301] e. [201]
 f. [213] g. (011) h. (102) i. (002) j. (130)
 k. (212) l. (312)



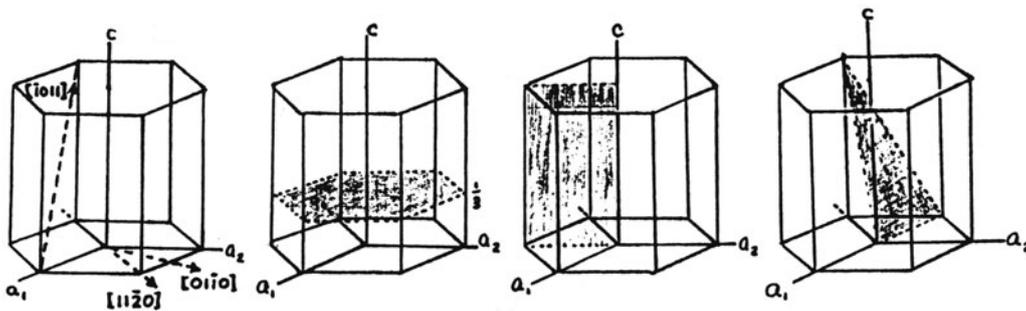
3.24 Sketch the following planes and directions within a cubic unit cell.

- a. [110] b. [221] c. [410] d. [012] e. [321]
 f. [111] g. (111) h. (011) i. (030) j. (121)
 k. (113) l. (041)



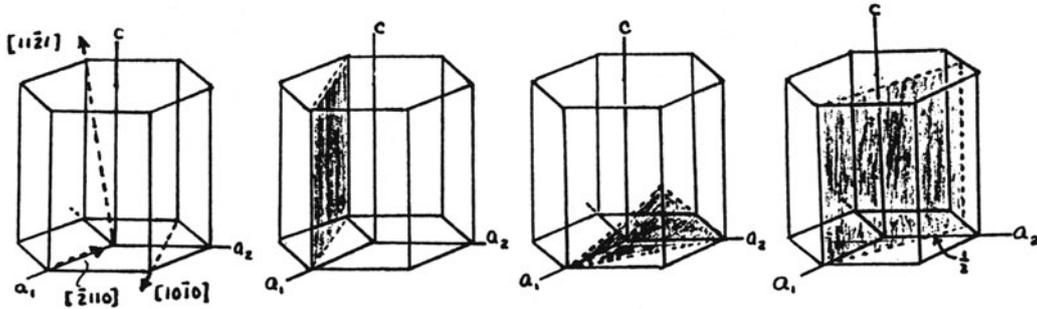
3.25 Sketch the following planes and directions within a hexagonal unit cell.

- a. [0110] b. [1120] c. [1011] d. (0003) e. (1010)
 f. (0111)



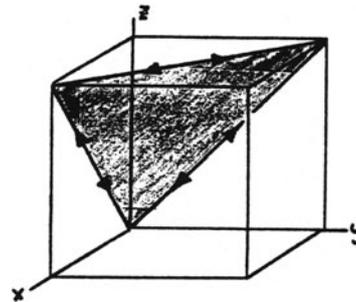
3.26 Sketch the following planes and directions within a hexagonal unit cell.

- a. $[2110]$ b. $[1121]$ c. $[1010]$ d. (1210) e. (1122)
 f. (1230)



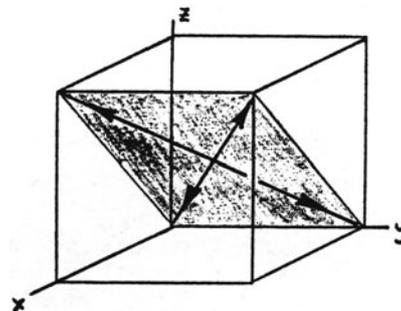
3.27 What are the indices of the six directions of the form $\langle 110 \rangle$ that lie in the (111) plane of a cubic cell?

- $[110]$ $[101]$ $[011]$
 $[\bar{1}10]$ $[\bar{1}01]$ $[0\bar{1}1]$



3.28 What are the indices of the four directions of the form $\langle 111 \rangle$ that lie in the (011) plane of a cubic cell?

- $[111]$ $[\bar{1}\bar{1}1]$
 $[1\bar{1}\bar{1}]$ $[\bar{1}11]$



3.29 Determine the number of directions of the form $\langle 110 \rangle$ in a tetragonal unit cell and compare to the number of directions of the form $\langle 110 \rangle$ in an orthorhombic unit cell.

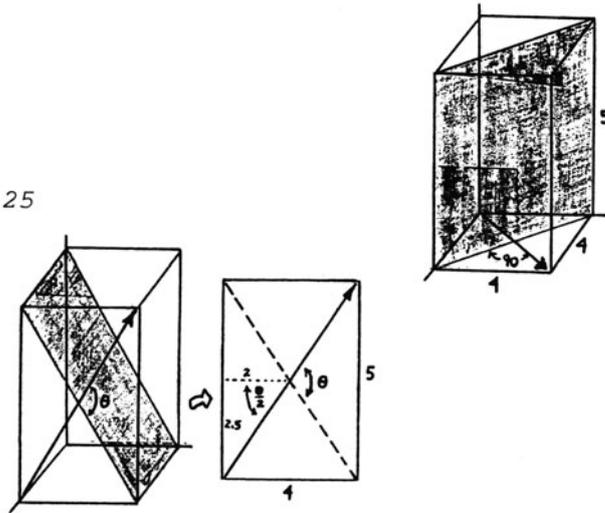
- Tetragonal: $[110], [1\bar{1}0], [110], [1\bar{1}0] = 4$
 Orthorhombic: $[110], [1\bar{1}0] = 2$

Note that in cubic systems, there are 12 directions of the form $\langle 110 \rangle$.

3.30 Determine the angle between the $[110]$ direction and the (110) plane in a tetragonal unit cell; then determine the angle between the $[011]$ direction and the (011) plane in a tetragonal cell. The lattice parameters are $a_0 = 4 \text{ \AA}$ and $c_0 = 5 \text{ \AA}$. What is responsible for the difference?

$$[110] \perp (110)$$

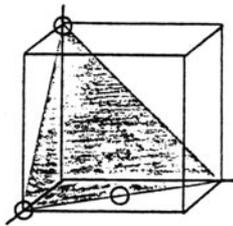
$$\begin{aligned} \tan(\theta/2) &= 2.5 / 2 = 1.25 \\ \theta/2 &= 51.34^\circ \\ \theta &= 102.68^\circ \end{aligned}$$



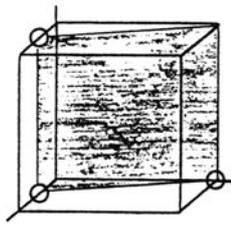
The lattice parameters in the x and y directions are the same; this allows the angle between $[110]$ and (110) to be 90° . But the lattice parameters in the y and z directions are different!

3.31 Determine the Miller indices of the plane that passes through three points having the following coordinates.

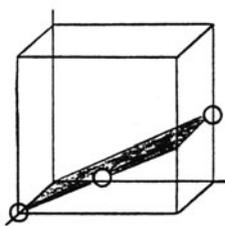
- $0, 0, 1$; $1, 0, 0$; and $\frac{1}{2}, \frac{1}{2}, 0$
- $\frac{1}{2}, 0, 1$; $\frac{1}{2}, 0, 0$; and $0, 1, 0$
- $1, 0, 0$; $0, 1, \frac{1}{2}$; and $1, \frac{1}{2}, \frac{1}{4}$
- $1, 0, 0$; $0, 0, \frac{1}{4}$; and $\frac{1}{2}, 1, 0$



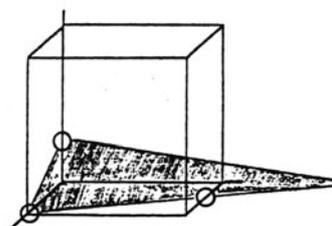
(a) (111)



(b) (210)



(c) (012)

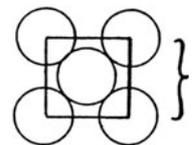


(d) (218)

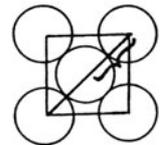
3.32 Determine the repeat distance, linear density, and packing fraction for FCC nickel, which has a lattice parameter of 0.35167 nm , in the $[100]$, $[110]$, and $[111]$ directions. Which of these directions is close-packed?

$$r = (\sqrt{2}) (0.35167) / 4 = 0.1243 \text{ nm}$$

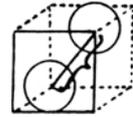
For $[100]$: repeat distance = $a_0 = 0.35167 \text{ nm}$
 linear density = $1/a_0 = 2.84 \text{ points.nm}^{-1}$
 linear packing fraction = $(2)(0.1243)(2.84) = 0.707$



For $[110]$: repeat distance = $\sqrt{2}a_0/2 = 0.2487 \text{ nm}$
 linear density = $2/\sqrt{2}a_0 = 4.02 \text{ points.nm}^{-1}$
 linear packing fraction = $(2)(0.1243)(4.02) = 1.0$



For [111]: repeat distance = $\sqrt{3}a_0 = 0.6091 \text{ nm}$
 linear density = $1/\sqrt{3}a_0 = 1.642 \text{ points.nm}^{-1}$
 linear packing fraction = $(2)(0.1243)(1.642) = 0.408$

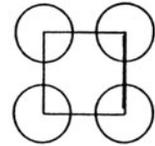


Only the [110] is close packed; it has a linear packing fraction of 1.

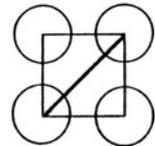
3.33 Determine the repeat distance, linear density, and packing fraction for BCC lithium, which has a lattice parameter of 0.35089 nm, in the [100], [110], and [111] directions. Which of these directions is close-packed?

$$r = \sqrt{3} (0.35089)/4 = 0.1519 \text{ nm}$$

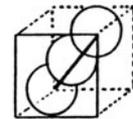
For [100]: repeat distance = $a_0 = 0.35089 \text{ nm}$
 linear density = $1/a_0 = 2.85 \text{ points.nm}^{-1}$
 linear packing fraction = $(2)(0.1519)(2.85) = 0.866$



For [110]: repeat distance = $\sqrt{2}a_0 = 0.496 \text{ nm}$
 linear density = $1/\sqrt{2}a_0 = 2.015 \text{ points.nm}^{-1}$
 linear packing fraction = $(2)(0.1519)(2.015) = 0.612$



For [111]: repeat distance = $\sqrt{3}a_0/2 = 0.3039 \text{ nm}$
 linear density = $2/\sqrt{3}a_0 = 3.291 \text{ points.nm}^{-1}$
 linear packing fraction = $(2)(0.1519)(3.291) = 1$



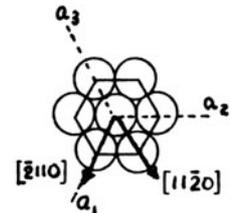
The [111] directions are close packed;
 the linear packing factor is 1.

3.34 Determine the repeat distance, linear density, and packing fraction for HCP magnesium in the [2110] direction and the [1120] direction. The lattice parameters for HCP magnesium are given in Appendix A.

$$a_0 = 0.32087 \text{ nm} \quad r = 0.1604 \text{ nm}$$

For [2110]:
 repeat distance = $a_0 = 0.32087 \text{ nm}$
 linear density = $1/a_0 = 3.116 \text{ points.nm}^{-1}$
 linear packing fraction = $(2)(0.1604)(3.116) = 1$

(Same for [1120])



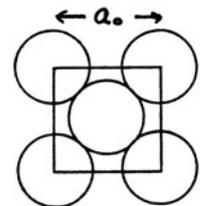
3.35 Determine the planar density and packing fraction for FCC nickel in the (100), (110), and (111) planes. Which, if any, of these planes is close-packed?

$$a_0 = 0.35167 \text{ nm}$$

For (100):

$$\text{planar density} = \frac{2}{(0.35167 \text{ nm})^2} = 1.617 \times 10^{19} \text{ points.m}^{-3}$$

$$\text{packing fraction} = \frac{2(\pi r^2)}{(4r/\sqrt{2})^2} = 0.7854$$

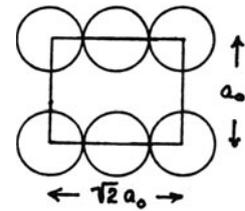


For (110):

$$\text{planar density} = \frac{2 \text{ points}}{(0.35167 \text{ nm}) (\sqrt{2}) (0.35167 \text{ nm})}$$

$$= 1.144 \times 10^{-19} \text{ points.m}^{-2}$$

$$\text{packing fraction} = \frac{2(\pi r^2)}{\sqrt{2}(4r/\sqrt{2})^2} = 0.555$$



For (111):

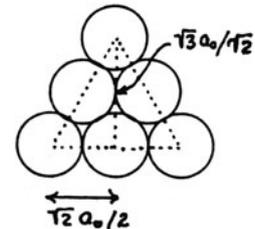
From the sketch, we can determine that the area of the (111) plane is $(\sqrt{2}a_0/2)(\sqrt{3}a_0/\sqrt{2}) = 0.866a_0^2$. There are $(3)(1/2) + (3)(1/6) = 2$ atoms in this area.

$$\text{planar density} = \frac{2 \text{ points}}{0.866(0.35167 \text{ nm})^2}$$

$$= 1.867 \times 10^{-19} \text{ points.m}^{-2}$$

$$\text{packing fraction} = \frac{2\pi(\sqrt{2}a_0/4)^2}{0.866a_0^2} = 0.907$$

The (111) is close packed.



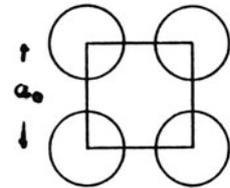
3.36 Determine the planar density and packing fraction for BCC lithium in the (100), (110), and (111) planes. Which, if any, of these planes is close-packed?

$$a_0 = 0.35089 \text{ nm}$$

For (100):

$$\text{planar density} = \frac{1}{(0.35089 \text{ nm})^2} = 8.12 \times 10^{18} \text{ points.m}^{-2}$$

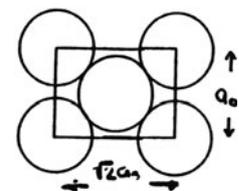
$$\text{packing fraction} = \frac{\pi[(\sqrt{3}a_0/4)]^2}{a_0^2} = 0.589$$



For (110):

$$\text{planar density} = \frac{2}{\sqrt{2}(0.35089 \text{ nm})^2} = 1.149 \times 10^{-19} \text{ points.m}^{-2}$$

$$\text{packing fraction} = \frac{2\pi(\sqrt{3}a_0/4)^2}{\sqrt{2}a_0^2} = 0.833$$



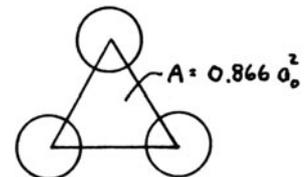
For (111):

There are only $(3)(1/6) = 1/2$ points in the plane, which has an area of $0.866a_0^2$.

$$\text{planar density} = \frac{1/2}{0.866(0.35089 \text{ nm})^2} = 4.69 \times 10^{18} \text{ points.m}^{-2}$$

$$\text{packing fraction} = \frac{1/2 \pi (\sqrt{3}a_0/4)^2}{0.866a_0^2} = 0.34$$

There is no close-packed plane in BCC structures.



3.37 Suppose that FCC rhodium is produced as a 1-mm-thick sheet, with the (111) plane parallel to the surface of the sheet. How many (111) interplanar spacings d_{111} thick is the sheet? See Appendix A for necessary data.

$$d_{111} = \frac{a_o}{\sqrt{1^2+1^2+1^2}} = \frac{0.3796 \text{ nm}}{\sqrt{3}} = 2.1916 \times 10^{-10} \text{ m}$$

$$\text{thickness} = \frac{1 \times 10^{-3} \text{ m}}{2.1916 \times 10^{-10} \text{ m}} = 4.563 \times 10^6 \text{ } d_{111} \text{ spacings}$$

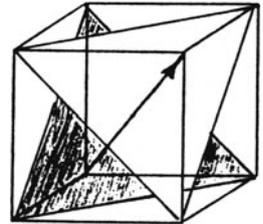
3.38 In a FCC unit cell, how many d_{111} are present between the 0,0,0 point and the 1,1,1 point?

The distance between the 0,0,0 and 1,1,1 points is $\sqrt{3}a_o$.
The interplanar spacing is

$$d_{111} = a_o / \sqrt{1^2 + 1^2 + 1^2} = a_o / \sqrt{3}$$

Therefore the number of interplanar spacings is

$$\text{number of } d_{111} \text{ spacings} = \sqrt{3}a_o / (a_o/\sqrt{3}) = 3$$



3.39 Determine the minimum radius of an atom that will just fit into (a) the tetrahedral interstitial site in FCC nickel and (b) the octahedral interstitial site in BCC lithium.

(a) For the tetrahedral site in FCC nickel ($a_o = 0.35167 \text{ nm}$):

$$r_{Ni} = \frac{\sqrt{2}(0.35167 \text{ nm})}{4} = 0.1243 \text{ nm}$$

$r/r_{Ni} = 0.225$ for a tetrahedral site. Therefore:

$$r = (0.1243 \text{ nm})(0.225) = 0.02797 \text{ nm}$$

(b) For the octahedral site in BCC lithium ($a_o = 0.35089 \text{ nm}$):

$$r_{Li} = \frac{\sqrt{3}(0.35089)}{4} = 0.1519 \text{ nm}$$

$r/r_{Li} = 0.414$ for an octahedral site. Therefore:

$$r = (0.1519 \text{ nm})(0.414) = 0.0629 \text{ nm}$$

3.40 What is the radius of an atom that will just fit into the octahedral site in FCC copper without disturbing the lattice?

$$r_{Cu} = 0.1278 \text{ nm}$$

$r/r_{Cu} = 0.414$ for an octahedral site. Therefore:

$$r = (0.1278 \text{ nm})(0.414) = 0.0529 \text{ nm}$$

3.41 Using the ionic radii given in Appendix B, determine the coordination number expected for the following compounds.

- | | | | |
|-------------|-----------|--------|--------------|
| a. Y_2O_3 | b. UO_2 | c. BaO | d. Si_3N_4 |
| e. GeO_2 | f. MnO | g. MgS | h. KBr |

(a) $r_Y/r_O = 0.089 = 0.67$ CN=6 (e) $r_{Ge}/r_O = 0.053 = 0.40$ CN=4

$$\begin{array}{ll}
 0.132 & 0.132 \\
 (b) \ r_u/r_o = \frac{0.097}{0.132} = 0.73 \quad \text{CN}=6 & (f) \ r_{Mn}/r_o = \frac{0.080}{0.132} = 0.61 \quad \text{CN}=6 \\
 (c) \ r_o/r_{Ba} = \frac{0.132}{0.134} = 0.99 \quad \text{CN}=8 & (g) \ r_{Mg}/r_s = \frac{0.066}{0.132} = 0.50 \quad \text{CN}=6 \\
 (d) \ r_N/r_{Si} = \frac{0.015}{0.042} = 0.36 \quad \text{CN}=4 & (h) \ r_K/r_{Br} = \frac{0.133}{0.196} = 0.68 \quad \text{CN}=6
 \end{array}$$

3.42 Would you expect NiO to have the caesium chloride, sodium chloride, or zinc blende structure? Based on your answer, determine (a) the lattice parameter, (b) the density, and (c) the packing factor for NiO.

$$r_{Ni} = 0.069 \text{ nm} \quad r_o = 0.132 \text{ nm} \quad r_{Ni}/r_o = 0.52 \quad \text{CN} = 6$$

A coordination number of 8 is expected for the CsCl structure, and a coordination number of 4 is expected for ZnS. But a coordination number of 6 is consistent with the NaCl structure.

$$(a) \ a_o = 2(0.069) + 2(0.132) = 0.402 \text{ nm}$$

$$\begin{aligned}
 (b) \ \rho &= \frac{(4 \text{ of each ion/cell})(58.71 + 16 \text{ g.mol}^{-1})}{(0.402 \text{ nm})^3 (6.02 \times 10^{23} \text{ atoms.mol}^{-1})} \\
 &= 7.64 \text{ Mg.m}^{-3}
 \end{aligned}$$

$$(c) \ PF = \frac{(4\pi/3)(4 \text{ ions/cell})[(0.069)^3 + (0.132)^3]}{(0.402)^3} = 0.678$$

3.43 Would you expect UO_2 to have the sodium chloride, zinc blende, or fluorite structure? Based on your answer, determine (a) the lattice parameter, (b) the density, and (c) the packing factor for UO_2 .

$$r_u = 0.097 \text{ nm} \quad r_o = 0.132 \text{ nm} \quad r_u/r_o = 0.97/1.32 = 0.735$$

valence of U = +4, valence of O = -2

The radius ratio predicts a coordination number of 8; however there must be twice as many oxygen ions as uranium ions in order to balance the charge. The fluorite structure will satisfy these requirements, with:

$$U = \text{FCC position (4)} \quad O = \text{tetrahedral position (8)}$$

$$(a) \ \sqrt{3} a_o = 4r_u + 4r_o = 4(0.097 + 0.132) = 0.916 \text{ or } a_o = 0.52885 \text{ nm}$$

$$(b) \ \rho = \frac{4(238.03 \text{ g.mol}^{-1}) + 8(16 \text{ g.mol}^{-1})}{(0.52885 \text{ nm})^3 (6.02 \times 10^{23} \text{ atoms.mol}^{-1})} = 12.13 \text{ Mg.m}^{-3}$$

$$(c) \ PF = \frac{(4\pi/3)[4(0.097)^3 + 8(0.132)^3]}{(0.52885)^3} = 0.624$$

3.44 Would you expect BeO to have the sodium chloride, zinc blende, or fluorite structure? Based on your answer, determine (a) the lattice parameter, (b) the density, and (c) the packing factor for BeO.

$$r_{Be} = 0.035 \text{ nm} \quad r_o = 0.132 \text{ nm}$$

$$r_{Be}/r_o = 0.265 \quad \text{CN} = 4 \quad \therefore \text{Zinc Blende}$$

$$(a) \ \sqrt{3}a_o = 4r_{Be} + 4r_o = 4(0.035 + 0.132) = 0.668 \text{ or } a_o = 0.38567 \text{ nm}$$

$$(b) \rho = \frac{4(9.01 + 16 \text{ g.mol}^{-1})}{(0.38567 \text{ nm})^3 (6.02 \times 10^{23} \text{ atoms.mol}^{-1})} = 2.897 \text{ Mg.m}^{-3}$$

$$(c) PF = \frac{(4\pi/3)(4)[(0.035)^3 + (0.132)^3]}{(0.38567)^3} = 0.684$$

3.45 Would you expect CsBr to have the sodium chloride, zinc blende, fluorite, or cesium chloride structure? Based on your answer; determine (a) the lattice parameter, (b) the density, and (c) the packing factor for CsBr.

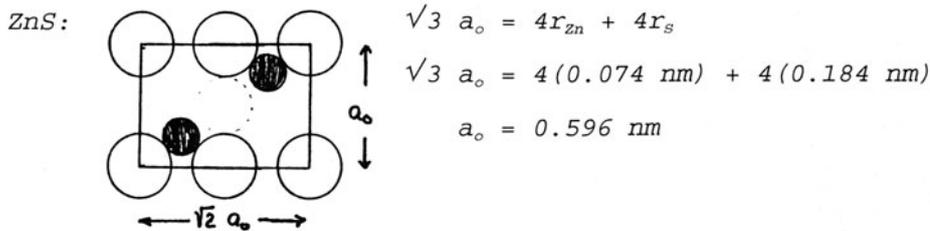
$$\begin{aligned} r_{Cs} &= 0.167 \text{ nm} & r_{Br} &= 0.196 \text{ nm} \\ r_{Cs}/r_{Br} &= 0.852 & CN &= 8 \quad \therefore \text{CsCl} \end{aligned}$$

$$(a) \sqrt{3}a_0 = 2r_{Cs} + 2r_{Br} = 2(0.196 + 0.167) = 0.726 \text{ or } a_0 = 0.41916 \text{ nm}$$

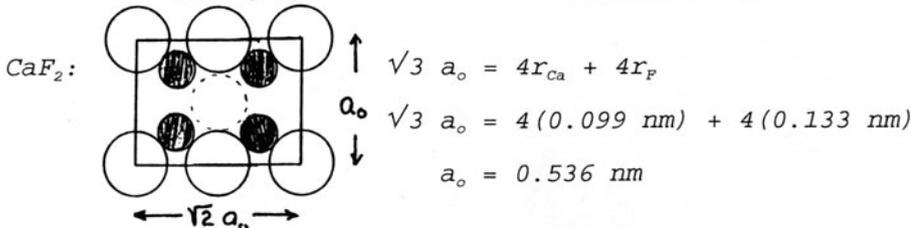
$$(b) \rho = \frac{79.909 + 132.905 \text{ g.mol}^{-1}}{(0.41916 \text{ nm})^3 (6.02 \times 10^{23} \text{ atoms.mol}^{-1})} = 4.8 \text{ Mg.m}^{-3}$$

$$(c) PF = \frac{(4\pi/3)[(0.196)^3 + (0.167)^3]}{(0.41916)^3} = 0.693$$

3.46 Sketch the ion arrangement on the (110) plane of ZnS (with the zinc blende structure) and compare this arrangement to that on the (110) plane of CaF₂ (the fluorite structure). Compare the planar packing fraction on the (110) planes for these two materials.



$$PPF = \frac{(2)(\pi r_{Zn}^2) + (2)(\pi r_S^2)}{(\sqrt{2} a_0) a_0} = \frac{2\pi(0.074)^2 + 2\pi(0.184)^2}{\sqrt{2}(0.596 \text{ nm})^2} = 0.492$$



$$PPF = \frac{(2)(\pi r_{Ca}^2) + (4)(\pi r_F^2)}{(\sqrt{2} a_0) a_0} = \frac{2\pi(0.099)^2 + 4\pi(0.133)^2}{\sqrt{2}(0.536 \text{ nm})^2} = 0.699$$

3.47 MgO, which has the sodium chloride structure, has a lattice parameter of 0.396 nm. Determine the planar density and the planar packing fraction for the (111) and (222) planes of MgO. What ions are present on each plane?

As described in the answer to Problem 3.35, the area of the (111) plane is $0.866a_0^2$.

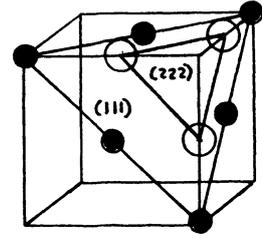
$$a_0 = 2r_{Mg} + 2r_O = 2(0.066 + 0.132) = 0.396 \text{ nm}$$

$$(111): P.D. = \frac{2 \text{ Mg}}{(0.866)(0.396 \text{ nm})^2} = 1.473 \times 10^{-19} \text{ points.m}^{-3}$$

$$PPF = \frac{2\pi(0.066)^2}{(0.866)(0.396)^2} = 0.202$$

$$(222): P.D. = 1.473 \times 10^{-19} \text{ points.m}^{-2}$$

$$PPF = \frac{2\pi(1.32)^2}{(0.866)(0.396)^2} = 0.806$$



3.48 Polypropylene forms an orthorhombic unit cell with lattice parameters of $a_o = 1.450 \text{ nm}$, $b_o = 0.569 \text{ nm}$, and $c_o = 0.740 \text{ nm}$. The chemical formula for the propylene molecule, from which the polymer is produced, is C_3H_6 . The density of the polymer is about 0.90 Mg.m^{-3} . Determine the number of propylene molecules, the number of carbon atoms, and the number of hydrogen atoms in each unit cell.

$$MW_{PP} = 3 C + 6 H = 3(12) + 6 = 42 \text{ g.mol}^{-1}$$

$$0.90 \text{ Mg.m}^{-3} = \frac{(x C_3H_6)(42 \text{ g.mol}^{-1})}{(1.450 \text{ nm})(0.569 \text{ nm})(0.740 \text{ nm})(6.023 \times 10^{23} \text{ molecules.mol}^{-1})}$$

$$x = 8 C_3H_6 \text{ molecules} \quad \text{or} \quad 24 C \text{ atoms and } 48 H \text{ atoms}$$

3.49 The density of cristobalite is about 1.538 Mg.m^{-3} , and it has a lattice parameter of 0.8037 nm . Calculate the number of SiO_2 ions, the number of silicon ions, and the number of oxygen ions in each unit cell.

$$1.538 \text{ Mg.m}^{-3} = \frac{(x SiO_2)[28.08 + 2(16) \text{ g.mol}^{-1}]}{(0.8037 \text{ nm})^3(6.02 \times 10^{23} \text{ ions.mol}^{-1})}$$

$$x = 8 SiO_2 \quad \text{or} \quad 8 Si \text{ ions and } 16 O \text{ ions}$$

3.50 A diffracted X-ray beam is observed from the (220) planes of iron at a 2θ angle of 99.1° when X-rays of 0.15418 nm wavelength are used. Calculate the lattice parameter of the iron.

$$\sin \theta = \lambda / 2d_{220}$$

$$\sin(99.1/2) = \frac{0.15418 \sqrt{2^2 + 2^2 + 0^2}}{2a_o}$$

$$a_o = \frac{(0.15418)\sqrt{8}}{2 \sin(49.55)} = 0.2865 \text{ nm}$$

3.51 A diffracted X-ray beam is observed from the (311) planes of aluminium at a 2θ angle of 78.3° when X-rays of 0.15418 nm wavelength are used. Calculate the lattice parameter of the aluminium.

$$\sin \theta = \lambda / d_{311}$$

$$a_o = \frac{0.15418 \sqrt{3^2 + 1^2 + 1^2}}{2 \sin(78.3/2)} = 0.40497 \text{ nm}$$

3.52 Figure 3.41 shows the results of an X-ray diffraction experiment in the form of the intensity of the diffracted peak versus the 2θ diffraction angle. If X-rays with a wavelength of 0.15418 nm are used, determine (a) the crystal structure of the metal, (b) the indices of the planes that produce each of the peaks, and (c) the lattice parameter of the metal.

The 2θ values can be estimated from the figure:

	2θ	$\sin^2\theta$	$\sin^2\theta/0.0077$	Planar indices	$d = \lambda/2\sin\theta$	$a_o = d\sqrt{h^2+k^2+l^2}$
1	17.5	0.023	3	(111)	0.5068	0.8778
2	20.5	0.032	4	(200)	0.4332	0.8664
3	28.5	0.061	8	(220)	0.3132	0.8859
4	33.5	0.083	11	(311)	0.2675	0.8872
5	35.5	0.093	12	(222)	0.2529	0.8761
6	41	0.123	16	(400)	0.2201	0.8804
7	45	0.146	19	(331)	0.2014	0.8779
8	46.5	0.156	20	(420)	0.1953	0.8734

The $\sin^2\theta$ values must be divided by 0.077 (one third the first $\sin^2\theta$ value) in order to produce a possible sequence of numbers)

(a) The 3,4,8,11,... sequence means that the material is FCC

(c) The average $a_o = 0.8781$ nm

3.53 Figure 3.42 shows the results of an X-ray diffraction experiment in the form of the intensity of the diffracted peak versus the 2θ diffraction angle. If X-rays with a wavelength of 0.0717 nm are used, determine (a) the crystal structure of the metal, (b) the indices of the planes that produce each of the peaks, and (c) the lattice parameter of the metal.

The 2θ values can be estimated from the figure:

	2θ	$\sin^2\theta$	$\sin^2\theta/0.047$	Planar indices	$d = \lambda/2\sin\theta$	$a_o = d\sqrt{h^2+k^2+l^2}$
1	25.5	0.047	1	(110)	0.1610	0.2277
2	36	0.095	2	(200)	0.1150	0.2300
3	44.5	0.143	3	(211)	0.0938	0.2299
4	51.5	0.189	4	(220)	0.0818	0.2313
5	58	0.235	5	(310)	0.0733	0.2318
6	64.5	0.285	6	(222)	0.0666	0.2307
7	70	0.329	7	(321)	0.06195	0.2318
8	75.5	0.375	8	(400)	0.0580	0.2322

(a) The sequence 1,2,3,4,5,6,7,8 (which includes the "7") means that the material is BCC.

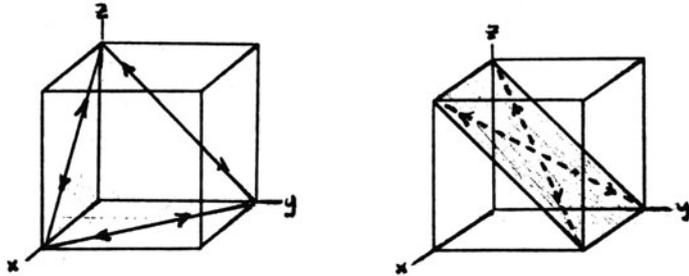
(c) The average $a_o = 0.2307$ nm

Chapter 4 Imperfections in the Atomic Arrangement

- 4.1 What are the Miller indices of the slip directions
 a. on the (111) plane in an FCC unit cell
 b. on the (011) plane in a BCC unit cell?

$[011]$, $[01\bar{1}]$
 $[\bar{1}10]$, $[1\bar{1}0]$
 $[101]$, $[10\bar{1}]$

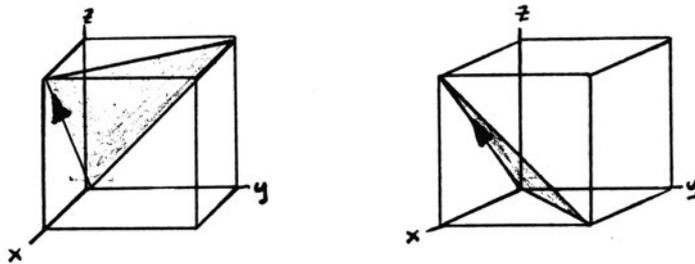
$[\bar{1}11]$, $[1\bar{1}1]$
 $[\bar{1}1\bar{1}]$, $[1\bar{1}\bar{1}]$



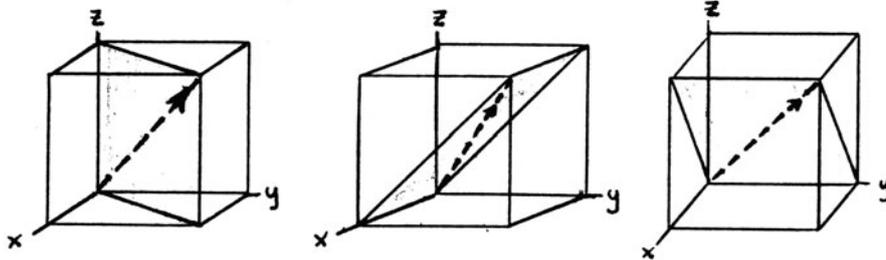
- 4.2 What are the Miller indices of the slip planes in FCC unit cells that include the $[101]$ slip direction?

(111) , $(\bar{1}\bar{1}1)$

$(11\bar{1})$, $(\bar{1}11)$



- 4.3 What are the Miller indices of the $\{110\}$ slip planes in BCC unit cells that include the $[\bar{1}11]$ slip direction?



(110) , $(\bar{1}\bar{1}0)$

(011) , $(0\bar{1}\bar{1})$

(101) , $(10\bar{1})$

4.4 Calculate the length of the Burgers vector in the following materials:
 a. BCC niobium b. FCC silver c. diamond cubic silicon

(a) The repeat distance, or Burgers vector, is half the body diagonal, or:

$$b = \text{repeat distance} = (1/2) (\sqrt{3}) (0.3294 \text{ nm}) = 0.2853 \text{ nm}$$

(b) The repeat distance, or Burgers vector, is half of the face diagonal, or:

$$b = (1/2) (\sqrt{2} a_0) = (1/2) (\sqrt{2}) (0.40862 \text{ nm}) = 0.2889 \text{ nm}$$

(c) The slip direction is [110], where the repeat distance is half of the face diagonal:

$$b = (1/2) (\sqrt{2}) (0.54307 \text{ nm}) = 0.3840 \text{ nm}$$

4.5 Determine the interplanar spacing and the length of the Burgers vector for slip on the expected slip systems in FCC aluminium. Repeat, assuming that the slip system is a (110) plane and a [111] direction. What is the ratio between the shear stress required for slip for the two systems? Assume that $k = 2$ in Equation 4.1.

(a) For (111)/[110],

$$b = (1/2) (\sqrt{2}) (0.404958 \text{ nm}) = 0.2863 \text{ nm} \quad d_{111} = \frac{0.404958 \text{ nm}}{\sqrt{1+1+1}} = 0.2338 \text{ nm}$$

(b) If (110)/[111], then:

$$b = (\sqrt{3}) (0.404958 \text{ nm}) = 0.7014 \text{ nm} \quad d_{110} = \frac{0.404958 \text{ nm}}{\sqrt{1^2+1^2+0^2}} = 0.2863 \text{ nm}$$

(c) If we assume that $k = 2$ in Equation 4.1, then

$$(d/b)_a = \frac{0.2338}{0.2863} = 0.8166 \quad (d/b)_b = \frac{0.2863}{0.7014} = 0.4082$$

$$\therefore \frac{\tau_a}{\tau_b} = \frac{\exp(-2(0.8166))}{\exp(-2(0.4082))} = 0.44$$

4.6 Determine the interplanar spacing and the length of the Burgers vector for slip on the (110)/[111] slip system in BCC tantalum. Repeat, assuming that the slip system is a (111)/[110] system. What is the ratio between the shear stresses required for slip for the two systems? Assume that $k = 2$ in Equation 4.1.

(a) For (110)/[111]:

$$b = (1/2) (\sqrt{3}) (0.33026 \text{ nm}) = 0.2860 \text{ nm} \quad d_{110} = \frac{0.33026 \text{ nm}}{\sqrt{1^2+1^2+0^2}} = 0.2335 \text{ nm}$$

(b) If (111)/[110], then:

$$b = \sqrt{2} (0.33026 \text{ nm}) = 0.4671 \text{ nm} \quad d_{111} = \frac{0.33026 \text{ nm}}{\sqrt{1^2+1^2+1^2}} = 0.1907 \text{ nm}$$

(c) If we assume that $k = 2$ in Equation 4.1, then:

$$(d/b)_a = \frac{0.2335}{0.2860} = 0.8164 \quad (d/b)_b = \frac{0.1907}{0.4671} = 0.4082$$

$$\frac{\tau_a}{\tau_b} = \frac{\exp(-2(0.8164))}{\exp(-2(0.408))} = 0.44$$

4.7 How many grams of aluminium, with a dislocation density of 10^{14} m.m^{-3} , are required to give a total dislocation length that would stretch from New York City to Los Angeles (4800 km)?

$$\frac{(4800 \times 10^3 \text{ m})(2.699 \text{ Mg.m}^{-3})}{(10^{14} \text{ m.m}^{-3})} = 0.13 \text{ g}$$

4.8 The distance from Earth to the Moon is 384,000 km. If this were the total length of dislocation in a 1×10^{-6} cubic metres of material, what would be the dislocation density?

$$\frac{(384000 \times 10^3 \text{ m})}{(1 \times 10^{-6} \text{ m}^3)} = 3.84 \times 10^{14} \text{ m.m}^{-3}$$

4.9 Suppose you would like to introduce an interstitial or large substitutional atom into the lattice near a dislocation. Would the atom fit more easily above or below the dislocation line shown in Figure 4.5(b)? Explain.

The atom would fit more easily into the area just below the dislocation due to the atoms being pulled apart; this allows more space into which the atom can fit.

4.10 Compare the c/a ratios for the following HCP metals, determine the likely slip processes in each, and estimate the approximate critical resolved shear stress. Explain. (See data in Appendix A)

- | | | |
|--------------|--------------|--------------|
| a. zinc | b. magnesium | c. titanium |
| d. zirconium | e. rhenium | f. beryllium |

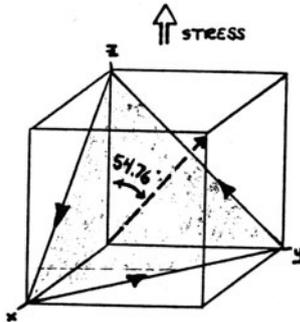
We expect metals with $c/a > 1.633$ to have a low crss:

$$(a) \text{ Zn: } \frac{0.49470}{0.26648} = 1.856 - \text{low crss} \quad (b) \text{ Mg: } \frac{0.5209}{0.32087} = 1.62 - \text{medium crss}$$

$$(c) \text{ Ti: } \frac{0.46831}{0.29503} = 1.587 - \text{high crss} \quad (d) \text{ Zr: } \frac{0.51477}{0.32312} = 1.593 - \text{high crss}$$

$$(e) \text{ Re: } \frac{0.4458}{0.2760} = 1.615 - \text{med crss} \quad (f) \text{ Be: } \frac{0.35842}{0.22858} = 1.568 - \text{high crss}$$

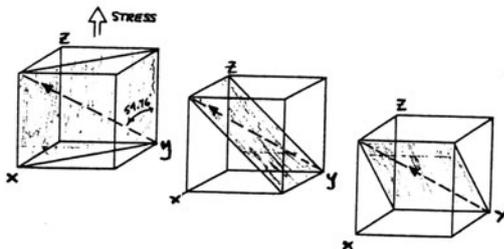
4.11 A single crystal of an FCC metal is oriented so that the [001] direction is parallel to an applied stress of 35 MN.m^{-2} . Calculate the resolved shear stress acting on the (111) slip plane in the [110], [011], and [101] slip directions. Which slip system(s) will become active first?



$$\phi = 54.76^\circ \quad \tau = 35 \cos 54.76 \cos \lambda$$

$\lambda_{[110]} = 90^\circ$	$\tau = 0$	
$\lambda_{[011]} = 45^\circ$	$\tau = 14.29 \text{ MNm}^{-2}$	active
$\lambda_{[101]} = 45^\circ$	$\tau = 14.29 \text{ MNm}^{-2}$	active

4.12 A single crystal of a BCC metal is oriented so that the [001] direction is parallel to the applied stress. If the critical resolved shear stress required for slip is 80 MN.m^{-2} , calculate the magnitude of the applied stress required to cause slip to begin in the [110], [011], and [101] slip planes.



$$\text{CRSS} = 80 \text{ MN.m}^{-2} = \sigma \cos \phi \cos \lambda$$

$$\lambda = 54.76^\circ \quad \frac{80 \text{ MNm}^{-2}}{\cos \phi \cos \lambda} = \sigma$$

$\phi_{[110]} = 90^\circ$	$\sigma = \infty$
$\phi_{[011]} = 45^\circ$	$\sigma = 196 \text{ MN.m}^{-2}$
$\phi_{[101]} = 45^\circ$	$\sigma = 196 \text{ MN.m}^{-2}$

4.13 Calculate the number of vacancies per m^3 expected in copper at 1085°C (just below the melting temperature). The activation energy for vacancy formation is 83700 J.mol^{-1} .

$$n = \frac{(4 \text{ atoms/u.c.})}{(3.6151 \times 10^{-10} \text{ m})^3} = 8.47 \times 10^{28} \text{ atoms/m}^3$$

$$n_v = 8.47 \times 10^{28} \exp[-83700 / (8.31) (1358)]$$

$$= 8.47 \times 10^{28} \exp(-7.41694) = 5.1 \times 10^{25} \text{ vacancies/m}^3$$

4.14 The fraction of lattice points occupied by vacancies in solid aluminium at 660°C is 10^{-3} . What is the activation energy required to create vacancies in aluminium?

$$n_v/n = 10^{-3} = \exp[-Q / (8.31) (933)]$$

$$\ln(10^{-3}) = -6.9078 = -Q / (8.31) (933)$$

$$Q = 53557 \text{ J.mol}^{-1}$$

4.15 The density of a sample of FCC palladium is 11.98 Mg.m^{-3} and its lattice parameter is 0.38902 nm . Calculate (a) the fraction of the lattice points that contain vacancies and (b) the total number of vacancies in a cubic metre of Pd.

(a) Using the density equation,

$$11.98 \text{ Mg.m}^{-3} = \frac{(x \text{ atoms/cell}) (106.4 \text{ g.mol}^{-1})}{(3.8902 \times 10^{-10} \text{ m})^3 (6.02 \times 10^{23} \text{ atoms.mol}^{-1})}$$

$$x = 3.9905 \text{ Pd atoms/cell}$$

$$\text{vacancies/cell} = 4.0 - 3.9905 = 0.0095$$

$$\text{fraction} = \frac{4.0 - 3.9905}{4} = 0.002375$$

$$(b) \text{ number} = \frac{0.0095 \text{ vacancies/u.c.}}{(3.8902 \times 10^{-10} \text{ m})^3} = 1.61 \times 10^{26} \text{ vacs/m}^3$$

4.16 The density of a sample of HCP beryllium is 1.844 Mg.m^{-3} and the lattice parameters are $a_0 = 0.22858 \text{ nm}$ and $c_0 = 0.35842 \text{ nm}$. Calculate (a) the fraction of the lattice points that contain vacancies and (b) the total number of vacancies in a cubic metre.

$$V_{u.c.} = (0.22858 \text{ nm})^2 (0.35842 \text{ nm}) \cos 30 = 0.01622 \text{ nm}^3 = 1.622 \times 10^{-29} \text{ m}^3$$

(a) From the density equation:

$$1.844 \text{ Mg.m}^{-3} = \frac{(x \text{ atoms/cell}) (9.01 \text{ g.mol}^{-1})}{(1.622 \times 10^{-29} \text{ m}^3) (6.02 \times 10^{23} \text{ atoms.mol}^{-1})}$$

$$x = 1.9984 \text{ Be atoms/cell}$$

$$\text{fraction} = \frac{2 - 1.9984}{2} = 0.0008$$

$$(b) \text{ number} = \frac{0.0016 \text{ vacancies/uc}}{1.622 \times 10^{-29} \text{ m}^3} = 9.86 \times 10^{25} \text{ vacancies/m}^3$$

4.17 BCC lithium has a lattice parameter of $3.5089 \times 10^{-10} \text{ m}$ and contains one vacancy per 200 unit cells. Calculate (a) the number of vacancies per cubic metre and (b) the density of Li.

$$(a) \frac{1 \text{ vacancy}}{(200) (3.5089 \times 10^{-10} \text{ m})^3} = 1.157 \times 10^{26} \text{ vacancies/m}^3$$

(b) In 200 unit cells, there are 399 Li atoms. The atoms/cell are $399/200$:

$$\rho = \frac{(399/200) (6.94 \text{ g.mol}^{-1})}{(3.5089 \times 10^{-10} \text{ m})^3 (6.02 \times 10^{23} \text{ atoms.mol}^{-1})} = 0.532 \text{ Mg.m}^{-3}$$

4.18 FCC lead has a lattice parameter of 0.4949 nm and contains one vacancy per 500 Pb atoms. Calculate (a) the density and (b) the number of vacancies per gram of Pb.

(a) The number of atoms/cell = $(499/500) (4 \text{ sites/cell})$

$$\rho = \frac{(499/500) (4) (207.19 \text{ g.mol}^{-1})}{(4.949 \times 10^{-10} \text{ m})^3 (6.02 \times 10^{23} \text{ atoms.mol}^{-1})} = 11.335 \text{ Mg.m}^{-3}$$

(b) The 500 Pb atoms occupy $500 / 4 = 125$ unit cells:

$$\frac{1 \text{ vacancy}}{125 \text{ cells}} \times (1/11.335 \text{ Mg.m}^{-3}) = 5.82 \times 10^{18} \text{ vacancies/g}$$

4.19 A niobium alloy is produced by introducing tungsten substitutional atoms in the BCC structure; eventually an alloy is produced that has a lattice parameter of 0.32554 nm and a density of 11.95 Mg.m⁻³. Calculate the fraction of the atoms in the alloy that are tungsten.

Let x_w = No of W atoms/cell

In bcc metals there are two atoms/cell so

$$x_{Nb} = (2 - x_w)$$

Using the density equation,

$$11.95 \times 10^6 \text{ g.m}^{-3} = \frac{(x_w)(183.85 \text{ g.mol}^{-1}) + (2 - x_w)(92.91 \text{ g.mol}^{-1})}{(3.2554 \times 10^{-10} \text{ m})^3 (6.02 \times 10^{23} \text{ atoms/mol})}$$

$$248.186 = 183.85x_w + 185.82 - 92.91x_w$$

$$x_w = 0.69 \text{ W atoms/cell}$$

$$\text{fraction } f_w = 0.69/2 = 0.345$$

4.20 Tin atoms are introduced into a FCC copper lattice, producing an alloy with a lattice parameter of 3.7589×10^{-10} m and a density of 8.772 Mg.m⁻³. Calculate the atomic percentage tin present in the alloy.

$$8.772 \text{ Mg.m}^{-3} = \frac{(x_{Sn})(118.69 \text{ g.mol}^{-1}) + (4 - x_{Sn})(63.54 \text{ g.mol}^{-1})}{(3.7589 \times 10^{-10} \text{ m})^3 (6.02 \times 10^{23} \text{ atoms.mol}^{-1})}$$

$$280.5 = 55.15x_{Sn} + 254.16 \quad \text{or} \quad x_{Sn} = 0.478 \text{ Sn atoms/cell}$$

There are 4 atoms per cell in FCC metals; therefore the at% Sn is:

$$(0.478/4) = 11.95\%$$

4.21 We replace 7.5 atomic percent of the chromium atoms in its BCC lattice with tantalum. X-ray diffraction shows that the lattice parameter is 0.29158 nm. Calculate the density of the alloy.

$$\rho = \frac{(2)(0.925)(51.996 \text{ g.mol}^{-1}) + (2)(0.075)(180.95 \text{ g.mol}^{-1})}{(2.9158 \times 10^{-10} \text{ m})^3 (6.02 \times 10^{23} \text{ atoms.mol}^{-1})} = 8.265 \text{ Mg.m}^{-3}$$

4.22 Suppose we introduce one carbon atom for every 100 iron atoms in an interstitial position in BCC iron, giving a lattice parameter of 0.2867 nm. For the Fe-C alloy, find (a) the density and (b) the packing factor.

There is one carbon atom per 100 iron atoms, or 1 C / 50 unit cells, or 1/50 C per unit cell. Must add this factor to the usual density and packing factor equations.

$$(a) \quad \rho = \frac{(2)(55.847 \text{ g.mol}^{-1}) + (1/50)(12 \text{ g.mol}^{-1})}{(2.867 \times 10^{-10} \text{ m})^3 (6.02 \times 10^{23} \text{ atoms.mol}^{-1})} = 7.89 \text{ Mg.m}^{-3}$$

$$(b) \quad P.F. = \frac{(2)(4\pi/3)(0.1241 \text{ nm})^3 + (1/50)(4\pi/3)(0.77)^3}{(0.2867 \text{ nm})^3} = 0.681$$

4.23 The density of BCC iron is 7.882 Mg.m⁻³ and the lattice parameter is 0.2866 nm when hydrogen atoms are introduced at interstitial positions. Calculate (a) the atomic fraction of hydrogen atoms and (b) the number of unit cells required on average to contain one hydrogen atom.

$$(a) \quad 7.882 \text{ Mg.m}^{-3} = \frac{2(55.847 \text{ g.mol}^{-1}) + x(1.00797 \text{ g.mol}^{-1})}{(2.866 \times 10^{-10} \text{ m})^3 (6.02 \times 10^{23} \text{ atoms.mol}^{-1})}$$

$$x = 0.0081 \text{ H atoms/cell}$$

The total atoms per cell include 2 Fe atoms and 0.0081 H atoms. Thus:

$$f_H = \frac{0.0081}{2.0081} = 0.004$$

(b) Since there is 0.0081 H/cell, then the number of cells containing H atoms is:

$$\text{cells} = 1/0.0081 = 123.5 \quad \text{or} \quad 1 \text{ H in } 123.5 \text{ cells}$$

4.24 Suppose one Schottky defect is present in every tenth unit cell of MgO. MgO has the sodium chloride crystal structure and a lattice parameter of 0.396 nm. Calculate (a) the number of anion vacancies per m^3 and (b) the density of the ceramic.

In 10 unit cells, we expect 40 Mg + 40 O ions, but due to the defect:

$$\begin{aligned} 40 \text{ Mg} - 1 &= 39 \\ 40 \text{ O} - 1 &= 39 \end{aligned}$$

$$(a) \quad 1 \text{ vacancy}/(10 \text{ cells}) (3.96 \times 10^{-10} \text{ m})^3 = 1.61 \times 10^{27} \text{ vacancies/m}^3$$

$$(b) \quad \rho = \frac{(39/40)(4)(24.312 \text{ g.mol}^{-1}) + (39/40)(4)(16 \text{ g.mol}^{-1})}{(3.96 \times 10^{-10} \text{ m})^3 (6.02 \times 10^{23} \text{ atoms.mol}^{-1})} = 4.205 \text{ Mg.m}^{-3}$$

4.25 ZnS has the zinc blende structure. If the density of the crystal is 3.02 Mg.m^{-3} and the lattice parameter is 0.59583 nm, determine the number of Schottky defects (a) per unit cell and (b) per cubic metre.

Let x be the number of each type of ion in the unit cell. There normally are 4 of each type.

$$(a) \quad 3.02 \text{ Mg.m}^{-3} = \frac{x(65.38 \text{ g.mol}^{-1}) + x(32.064 \text{ g.mol}^{-1})}{(5.9583 \times 10^{-8} \text{ cm})^3 (6.02 \times 10^{23} \text{ ions.mol}^{-1})} \quad x = 3.9465$$

$$4 - 3.9465 = 0.0535 \text{ defects/u.c.}$$

$$(b) \quad \text{No of unit cells/m}^3 = 1/(5.9583 \times 10^{-10} \text{ m})^3 = 4.726 \times 10^{27}$$

$$\text{Schottky defects per m}^3 = (4.726 \times 10^{27})(0.0535) = 2.529 \times 10^{26}$$

4.26 Suppose we introduce the following point defects. What other changes in each structure might be necessary to maintain a charge balance? Explain.

- Mg²⁺ ions substitute for yttrium atoms in Y₂O₃
- Fe³⁺ ions substitute for magnesium ions in MgO
- Li¹⁺ ions substitute for magnesium ions in MgO
- Fe²⁺ ions replace sodium ions in NaCl

(a) Remove 2 Y³⁺ and add 3 Mg²⁺ - create cation interstitial.

(b) Remove 3 Mg²⁺ and add 2 Fe³⁺ - create cation vacancy.

(c) Remove 1 Mg²⁺ and add 2 Li¹⁺ - create cation interstitial.

(d) Remove 2 Na⁺ and add 1 Fe²⁺ - create cation vacancy.

4.27 The strength of titanium is found to be 450 MNm^{-2} when the grain size is $17 \times 10^{-6} \text{ m}$ and 565 MNm^{-2} when the grain size is $0.8 \times 10^{-6} \text{ m}$. Determine (a) the constants in the Hall-Petch equation and (b) the strength of the titanium when the grain size is reduced to $0.2 \times 10^{-6} \text{ m}$.

$$450 = \sigma_0 + K \frac{1}{\sqrt{(17 \times 10^{-6})}} = \sigma_0 + 242.5 K$$

$$565 = \sigma_0 + K \frac{1}{\sqrt{0.8 \times 10^{-6}}} = \sigma_0 + 1118.0 K$$

(a) By solving the two simultaneous equations:

$$K = 0.131 \text{ MN./m}^{-3/2} \quad \sigma_0 = 418 \text{ MN.m}^{-2}$$

(b) $\sigma = 418 + 0.131/\sqrt{(0.2 \times 10^{-6})} = 711 \text{ MN.m}^{-2}$

4.28 A copper-zinc alloy has the following properties:

grain diameter (mm)	strength (MN.m ⁻²)	d ^{-1/2} (m ^{-1/2})
0.015	170	258.2
0.025	158	200.0
0.035	151	169.0
0.050	145	141.4

Determine (a) the constants in the Hall-Petch equation and (b) the grain size required to obtain a strength of 200 MNm^{-2} .

The values of $d^{-1/2}$ are included in the table; the graph shows the relationship. We can determine K and σ_0 either from the graph or by using two of the data points.

(a) $170 = \sigma_0 + K(258.2)$
 $145 = \sigma_0 + K(141.4)$
 $25 = 116.8K$

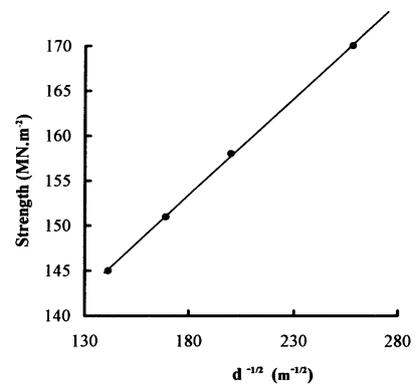
$$K = 0.214 \text{ MNm}^{-3/2} \quad \sigma_0 = 114.7 \text{ MNm}^{-2}$$

(b) To obtain a strength of 200 MNm^{-2} :

$$200 = 114.7 + 0.214/\sqrt{d}$$

$$85.3 = 0.214/\sqrt{d}$$

$$d = 6.29 \times 10^{-6} \text{ m}$$



4.29 For an SI grain size index of 8, calculate the number of grains per square mm.

$$m = 2^{Ge + 3} = 2^{8 + 3} = 2^{11} = 2048 \text{ grains per sq. mm}$$

4.30 Determine the ASTM grain size index if 20 grains/square in are observed at a magnification of 400.

$$(20) (400/100)^2 = 2^{n-1} \quad \log(320) = (n-1)\log(2)$$

$$2.505 = (n-1)(0.301) \quad \text{or} \quad n = 9.3$$

4.31 Determine the SI grain size index if 25 grains/square mm are observed at a magnification of 5.

If 25 grains per square mm are counted at magnification X5, then at X1 magnification

$$m = \frac{(5^2)(25)}{(1)} = 625 \text{ grains/mm}^2$$

$$625 = 2^{G_E + 3}$$

$$\log(625) = (G_E + 3)\log(2)$$

$$2.7959 = (G_E + 3)0.301$$

$$G_E = 6.29$$

- 4.32 Determine the ASTM grain size number for the materials in
 a. Figure 4.16 b. Figure 4.20 c. Figure 1-9(c)

(a) There are about 26 grains in the photomicrograph, which has the dimensions 2.375 in. x 1.875 in. The magnification is 100, thus:

$$\frac{26}{(2.375)(1.875)} = 2^{n-1} \quad \log(5.8386) = 0.7663 = (n-1)\log(2) \quad n = 3.5$$

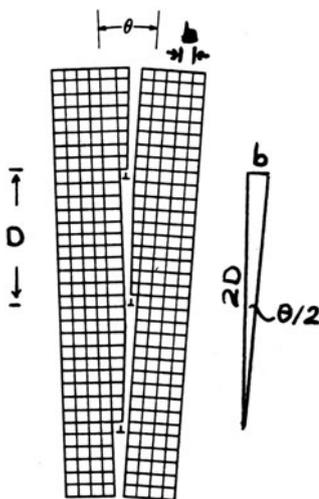
(b) There are about 59 grains in the photomicrograph, which has the dimensions 2.25 in. x 2 in. The magnification is 500, thus:

$$\frac{59(500/100)^2}{(2.25)(2)} = 2^{n-1} \quad \log(328) = 2.516 = (n-1)\log(2) \quad n = 9.4$$

(c) There are about 28 grains in the photomicrograph, which has the dimensions 2 in. x 2.25 in. The magnification is 200, thus:

$$\frac{28(200/100)^2}{(2.25)(2)} = 2^{n-1} \quad \log(24.889) = 1.396 = (n-1)\log(2) \quad n = 5.6$$

- 4.33 The angle θ of a tilt boundary is given by $\sin(\theta/2) = b/2D$ (see Figure 4.17). Verify the correctness of this equation.



From the figure, we note that the grains are offset one Burgers vector, b , only for two spacings D . Then it is apparent that $\sin(\theta/2)$ must be b divided by two D .

4.34 Calculate the angle θ of a small angle grain boundary in FCC aluminium when the dislocations are 500 nm apart. (See Problem 4.33)

Lattice parameter is 0.404958nm. Repeat distance of Burgess vector is half of the face diagonal, or

$$b = (1/2) (\sqrt{2}) (0.404958) = 0.28635 \text{ nm and } D = 500 \text{ nm}$$

$$\sin(\theta/2) = \frac{0.28635}{(2)(500)} = 0.000286$$

$$\theta/2 = 0.0164$$

$$\theta = 0.0328^\circ$$

4.35 For BCC iron, calculate the average distance between dislocations in a small angle grain boundary tilted 0.50°. (See Problem 4.33)

Lattice parameter is 0.2866nm. Repeat distance of Burgers vector is half of the body diagonal, or

$$b = \frac{(\sqrt{3})(0.2866)}{2}$$

$$\sin(0.5/2) = \frac{(1/2)(\sqrt{3})(2.2866)}{2D}$$

$$0.004363 = 0.1241/D$$

$$D = 28.4 \text{ nm}$$

4.36 Our discussion of Schmid's law dealt with single crystals of a metal. Discuss slip and Schmid's law in a polycrystalline material. What might happen as the grain size gets smaller and smaller?

With smaller grains, the movement of the dislocations is impeded by frequent intersections with the grain boundaries. The strength of metals is not nearly as low as might be predicted from the critical resolved shear stress as a consequence of these interactions.

Chapter 5 Atom Movement in Materials

5.1 Atoms are found to move from one lattice position to another at the rate of 5×10^5 jumps per second at 400°C when the activation energy for their movement is $125580 \text{ J.mol}^{-1}$. Calculate the jump rate at 750°C .

$$\text{Rate} = \frac{5 \times 10^5}{x} = \frac{c_o \exp[-125580/(8.314)(673)]}{c_o \exp[-125580/(8.314)(1023)]} = \exp(-22.444 + 14.765)$$

$$\frac{5 \times 10^5}{x} = \exp(-7.679) = 4.62 \times 10^{-4}$$

$$x = \frac{5 \times 10^5}{4.62 \times 10^{-4}} = 1.08 \times 10^9 \text{ jumps/s}$$

5.2 The number of vacancies in a material is related to temperature by an Arrhenius equation. If the fraction of lattice points containing vacancies is 8×10^{-5} at 600°C , determine the fraction at 1000°C .

$$8 \times 10^{-5} = \exp[-Q/(8.314)(873)] \quad Q = 68,470 \text{ J.mol}^{-1}$$

$$f = n_v/n = \exp[-68,470/(8.314)(1273)] = 0.00155$$

5.3 The diffusion coefficient for Cr in Cr_2O_3 is $6 \times 10^{-19} \text{ m}^2.\text{s}^{-1}$ at 727°C and is $1 \times 10^{-13} \text{ m}^2.\text{s}^{-1}$ at 1400°C . Calculate (a) the activation energy and (b) the constant D_o .

$$(a) \frac{6 \times 10^{-19}}{1 \times 10^{-13}} = \frac{D_o \exp[-Q/(8.314)(1000)]}{D_o \exp[-Q/(8.314)(1673)]}$$

$$6 \times 10^{-6} = \exp[-Q(0.0001203 - 0.0000719)] = \exp[-0.0000438 Q]$$

$$-12.024 = -0.0000438 Q \quad \text{or} \quad Q = 248,500 \text{ J.mol}^{-1}$$

$$(b) 1 \times 10^{-13} = D_o \exp[-248,500/(8.314)(1673)] = D_o \exp(-17.87)$$

$$1 \times 10^{-13} = 1.742 \times 10^{-8} D_o \quad \text{or} \quad D_o = 5.74 \times 10^6 \text{ m}^2.\text{s}^{-1}$$

5.4 The diffusion coefficient for O in Cr_2O_3 is $4 \times 10^{-19} \text{ m}^2.\text{s}^{-1}$ at 1150°C and $6 \times 10^{-15} \text{ m}^2.\text{s}^{-1}$ at 1715°C . Calculate (a) the activation energy and (b) the constant D_o .

$$\frac{4 \times 10^{-19}}{6 \times 10^{-15}} = \frac{D_o \exp[-Q/(8.314)(1423)]}{D_o \exp[-Q/(8.314)(1988)]}$$

$$6.67 \times 10^{-5} = \exp[-0.0000240 Q]$$

$$-9.615 = -0.0000240 Q \quad \text{or} \quad Q = 400,000 \text{ J.mol}^{-1}$$

$$4 \times 10^{-19} = D_o \exp[-400,000/(8.314)(1423)] = D_o(2.02 \times 10^{-15})$$

$$D_o = 1.98 \times 10^{-4} \text{ m}^2.\text{s}^{-1}$$

5.5 A 0.2 mm thick wafer of silicon is treated so that a uniform concentration gradient of antimony is produced. One surface contains 1 Sb atom per 10^8 Si atoms and the other surface contains 500 Sb atoms per 10^8 Si atoms. The lattice parameter for Si is given in Appendix A. Calculate the concentration gradient in (a) atomic percent Sb per cm and (b) Sb atoms. $m^{-3}.m^{-1}$.

$$\Delta c/\Delta x = \frac{(1 \times 10^{-8} - 500 \times 10^{-8})}{0.2 \times 10^{-3}m} \times 100\% = -2.495 \text{ at}\% \text{ Sb}.m^{-1}$$

$$a_0 = 0.357 \times 10^{-9} \quad V_{\text{unit cell}} = 1.6016 \times 10^{-28} \text{ m}^3$$

$$c_1 = \frac{(8 \text{ Si atoms/u.c.})(1 \text{ Sb}/10^8 \text{ Si})}{1.6016 \times 10^{-28} \text{ m}^3/\text{u.c.}} = 4.995 \times 10^{20} \text{ Sb}.m^{-3}$$

$$c_2 = \frac{(8 \text{ Si atoms/u.c.})(500 \text{ Sb}/10^8 \text{ Si})}{1.6016 \times 10^{-28} \text{ m}^3/\text{u.c.}} = 2497.5 \times 10^{20} \text{ Sb}.m^{-3}$$

$$\Delta c/\Delta x = \frac{(4.995 - 2497.5) \times 10^{20}}{0.02 \times 10^{-3}m} = -1.246 \times 10^{21} \text{ Sb atoms}.m^3.m^{-1}$$

5.6 When a Cu-Zn alloy solidifies, one portion of the structure contains 25 atomic percent zinc and another portion 0.025 mm away contains 20 atomic percent zinc. If the lattice parameter for the FCC alloy is 3.63×10^{-10} m, determine the concentration gradient in (a) atomic percent Zn per cm, (b) weight percent Zn per cm, and (c) Zn atoms. $m^{-3}.m^{-1}$.

$$(a) \quad \Delta c/\Delta x = \frac{20\% - 25\%}{(0.025 \times 10^{-3}m)} = -200000 \text{ at}\% \text{ Zn}.m^{-1}$$

(b) We now need to determine the wt% of zinc in each portion:

$$\text{wt}\% \text{ Zn} = \frac{(20)(65.38 \text{ g}.mol^{-1})}{(20)(65.38) + (80)(63.54)} \times 100 = 20.46$$

$$\text{wt}\% \text{ Zn} = \frac{(25)(65.38 \text{ g}.mol^{-1})}{(25)(65.38) + (75)(63.54)} \times 100 = 25.54$$

$$\Delta c/\Delta x = \frac{20.46\% - 25.54\%}{0.025 \times 10^{-3}m} = -203200 \text{ wt}\% \text{ Zn}.m^{-1} \\ = -2032 \text{ wt}\% \text{ Zn}.cm^{-1}$$

(c) Now find the number of atoms per m^3 :

$$c_1 = \frac{(4 \text{ atoms/cell})(0.2 \text{ Zn fraction})}{(0.363 \times 10^{-9}m)^3} = 1.67 \times 10^{28} \text{ Zn atoms}.m^{-3}$$

$$c_2 = \frac{(4 \text{ atoms/cell})(0.25 \text{ Zn fraction})}{(0.363 \times 10^{-9}m)^3} = 2.091 \times 10^{28} \text{ Zn atoms}.m^{-3}$$

$$\Delta c/\Delta x = \frac{(1.67 \times 10^{28} - 2.091 \times 10^{28})}{0.025 \times 10^{-3}m} = -1.68 \times 10^{31} \text{ Zn atoms}.cm^{-3} \times (10^{-6}m^3.m^{-1})$$

5.7 A 0.025 mm BCC iron foil is used to separate a high hydrogen gas from a low hydrogen gas at 650°C. 5×10^{14} H atoms. m^{-3} are in equilibrium with the hot side of the foil, while 2×10^9 H atoms. m^{-3} are in equilibrium with the cold side. Determine (a) the concentration gradient of hydrogen and (b) the flux of hydrogen through the foil.

$$(a) \quad \Delta c/\Delta x = \frac{2 \times 10^9 - 5 \times 10^{14}}{(0.025 \text{ mm} \times 10^{-3} \text{ m} \cdot \text{mm}^{-1})} = -2 \times 10^{19} \text{ H atoms} \cdot \text{m}^{-3} \cdot \text{m}^{-1}$$

$$(b) \quad J = -D(\Delta c/\Delta x) = -1.2 \times 10^{-7} \exp[-15050/(8.314)(923)](-2 \times 10^{19})$$

$$J = 3.57 \times 10^{11} \text{ H atoms} \cdot \text{m}^{-2} \cdot \text{s}^{-1}$$

5.8 A 1 mm sheet of FCC iron is used to contain nitrogen in a heat exchanger at 1200°C. The concentration of N at one surface is 0.04 atomic percent and the concentration at the second surface is 0.005 atomic percent. Determine the flux of nitrogen through the foil in atoms.m⁻².s⁻¹.

$$(a) \quad \Delta c/\Delta x = \frac{(0.00005 - 0.0004)(4 \text{ atoms per cell})/(0.3589 \times 10^{-9} \text{ m})^3}{(1 \times 10^{-3} \text{ m})}$$

$$= -3.03 \times 10^{28} \text{ N atoms} \cdot \text{m}^{-3} \cdot \text{m}^{-1}$$

$$(b) \quad J = -D(\Delta c/\Delta x) = -3.4 \times 10^{-7} \exp[-144,900/(8.314)(1473)](-3.03 \times 10^{28})$$

$$= 7.49 \times 10^{16} \text{ N atoms/m}^2 \cdot \text{s}^{-1}$$

5.9 A 4 cm-diameter, 0.5 mm-thick spherical container made of BCC iron holds nitrogen at 700°C. The concentration at the inner surface is 0.05 atomic percent and at the outer surface is 0.002 atomic percent. Calculate the number of grams of nitrogen that are lost from the container per hour.

$$\Delta c/\Delta x = \frac{[0.00002 - 0.0005](2 \text{ atoms/cell})/(2.866 \times 10^{-10} \text{ m})^3}{(0.5 \times 10^{-3} \text{ m/mm})}$$

$$= -8.16 \times 10^{28} \text{ N m}^{-3} \cdot \text{m}^{-1}$$

$$J = -4.7 \times 10^{-7} \exp[-76,600/(8.314)(973)](-8.16 \times 10^{28}) = 2.97 \times 10^{18} \text{ N} \cdot \text{m}^{-2} \cdot \text{s}^{-1}$$

$$A_{\text{sphere}} = 4\pi r^2 = 4\pi(2 \times 10^{-2} \text{ m})^2 = 5.027 \times 10^{-3} \text{ m}^2 \quad t = 3600 \text{ s/h}^{-1}$$

$$N \text{ atoms/h} = (2.97 \times 10^{18})(5.027 \times 10^{-3})(3600) = 5.37 \times 10^{19} \text{ N atoms h}^{-1}$$

$$N \text{ loss} = \frac{(5.37 \times 10^{19} \text{ atoms})(14.007 \text{ g} \cdot \text{mol}^{-1})}{(6.02 \times 10^{23} \text{ atoms} \cdot \text{mol}^{-1})} = 1.245 \times 10^{-3} \text{ g} \cdot \text{h}^{-1}$$

5.10 A BCC iron structure is to be manufactured that will allow no more than 50 g of hydrogen to be lost per year through each square centimeter of the iron at 400°C. If the concentration of hydrogen at one surface is 0.05 H atom per unit cell and is 0.001 H atom per unit cell at the second surface, determine the minimum thickness of the iron.

$$c_1 = 0.05 \text{ H}/(0.2866 \times 10^{-9} \text{ m})^3 = 2.124 \times 10^{27} \text{ H atoms} \cdot \text{m}^{-3}$$

$$c_2 = 0.001 \text{ H}/(0.2866 \times 10^{-9} \text{ m})^3 = 4.25 \times 10^{25} \text{ H atoms} \cdot \text{m}^{-3}$$

$$\Delta c/\Delta x = \frac{[4.25 \times 10^{25} - 2.124 \times 10^{27}]}{\Delta x} = \frac{-2.08 \times 10^{27}}{\Delta x}$$

$$50 \text{ g of H is lost through each cm}^2 = 50 \times 10^4 \text{ g H lost through each m}^2$$

$$J = \frac{(50 \times 10^4 \text{ g} \cdot \text{m}^{-2} \cdot \text{y}^{-1})(6.02 \times 10^{23} \text{ atoms} \cdot \text{mol}^{-1})}{(1.00797 \text{ g} \cdot \text{mol}^{-1})(31.536 \times 10^6 \text{ s} \cdot \text{y}^{-1})}$$

$$= 9.47 \times 10^{21} \text{ H atoms} \cdot \text{m}^{-2} \cdot \text{s}^{-1}$$

$$J = 9.47 \times 10^{21} \text{ H atoms} \cdot \text{m}^{-2} \cdot \text{s}^{-1}$$

$$= (-2.08 \times 10^{27}/\Delta x) (1.7 \times 10^{-7}) \exp[-15,050/(8.314)(673)]$$

$$\Delta x = 1.79 \text{ mm}$$

5.11 Determine the maximum allowable temperature that will produce a flux of less than 2×10^7 H atoms. $m^{-2}.s^{-1}$ through a BCC iron foil when the concentration gradient is -5×10^{24} atoms. $m^{-3}.m^{-1}$.

$$2 \times 10^7 \text{ H atoms}.m^{-2}.s^{-1} = -(4.7 \times 10^{-7} \text{ m}^2.s^{-1}). \exp(-76,600/8.314T) \times (-5 \times 10^{24} \text{ atoms}.m^{-3}.s^{-1})$$

$$\frac{2 \times 10^7}{(4.7 \times 10^{-7})(5 \times 10^{24})} = \exp(-9213/T)$$

$$8.511 \times 10^{-26} = \exp(-9213/T)$$

$$\ln(8.511 \times 10^{-26}) = -57.73 = (-9213/T)$$

$$\therefore T = \frac{9213}{57.73} = 159 \text{ K} = -113^\circ\text{C}$$

5.12 The electrical conductivity of Mn_3O_4 is $8 \times 10^{-16} \text{ ohm}^{-1}.m^{-1}$ at 140°C and is $1 \times 10^{-5} \text{ ohm}^{-1}.m^{-1}$ at 400°C . Determine the activation energy that controls the temperature dependence of conductivity. Explain the process by which the temperature controls conductivity.

$$\frac{8 \times 10^{-16}}{1 \times 10^{-5}} = \frac{C_o \exp[-Q/(8.314)(413)]}{C_o \exp[-Q/(8.314)(673)]}$$

$$8 \times 10^{-11} = \exp(-1.125 \times 10^{-4}Q) \quad \text{or} \quad -23.25 = -1.125 \times 10^{-4}Q$$

$$Q = 206,6000 \text{ J}.mol^{-1}$$

Electrical charge is carried by the diffusion of the atoms; as the temperature increases, more rapid diffusion occurs and consequently the electrical conductivity is higher.

5.13 Compare the rate at which oxygen diffuses in Al_2O_3 with the rate at which aluminium diffuses in Al_2O_3 at 1500°C . Explain the difference.

$$D_o = 0.19 \exp[-636,250/(8.314)(1773)] = 3.42 \times 10^{-20} \text{ m}^2.s^{-1}$$

$$D_{Al} = 2.8 \times 10^{-3} \exp[-477,200/(8.314)(1773)] = 2.44 \times 10^{-17} \text{ m}^2.s^{-1}$$

The ionic radius of the oxygen ion is 0.0132 nm, compared with the aluminium ionic radius of 0.051 nm; consequently it is much easier for the smaller aluminium ion to diffuse in the ceramic.

5.14 Compare the diffusion coefficients of carbon in BCC and FCC iron at the allotropic transformation temperature of 912°C and explain the difference.

$$D_{BCC} = 1.1 \times 10^{-6} \exp[-87,500/(8.314)(1185)] = 1.53 \times 10^{-10} \text{ m}^2.s^{-1}$$

$$D_{FCC} = 2.3 \times 10^{-5} \exp[-137,700/(8.314)(1185)] = 1.96 \times 10^{-11} \text{ m}^2.s^{-1}$$

Packing factor of the BCC lattice (0.68) is less than that of the FCC lattice; consequently atoms are expected to be able to diffuse more rapidly in the BCC iron.

5.15 Compare the diffusion coefficients for hydrogen and nitrogen in FCC iron at 1000°C and explain the difference.

$$D_{H \text{ in FCC}} = 6.3 \times 10^{-7} \exp[-43,100/(8.314)(1273)] = 1.074 \times 10^{-8} \text{ m}^2 \cdot \text{s}^{-1}$$

$$D_{N \text{ in FCC}} = 3.4 \times 10^{-7} \exp[-144,900/(8.314)(1273)] = 3.851 \times 10^{-13} \text{ m}^2 \cdot \text{s}^{-1}$$

Nitrogen atoms have a larger atoms radius (0.0071 nm) compared with that of hydrogen atoms (0.0046 nm); the smaller hydrogen ions are expected to diffuse more rapidly.

5.16 Explain why a polymer balloon filled with helium gas deflates over time.

Helium atoms diffuse through the chains of the polymer material due to the small size of the helium atoms and the ease at which they diffuse between the loosely-packed chains.

5.17 A carburizing process is carried out on a 0.10% C steel by introducing 1.0% C at the surface at 980°C, where the iron is FCC. Calculate the carbon content at 0.1 mm, 0.5 mm, and 1 mm beneath the surface after 1 h.

$$D = 2.3 \times 10^{-5} \exp[-137,700/(8.314)(1253)] = 4.18 \times 10^{-11} \text{ m}^2 \cdot \text{s}^{-1}$$

$$\frac{1 - c_x}{1 - 0.1} = \text{erf}[x/(2\sqrt{(4.18 \times 10^{-11})(3600)})] = \text{erf}[x/7.76 \times 10^{-4}]$$

when $x = 0.1 \text{ mm} = 1 \times 10^{-4} \text{ m}$ $\text{erf}[1 \times 10^{-4}/7.76 \times 10^{-4}] = \text{erf}(0.1289) = 0.144$

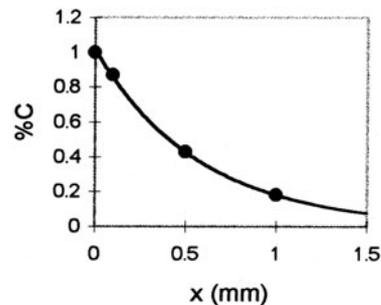
$$\frac{(1 - c_x)}{0.9} = 0.144, \quad c_x = 0.87\% \text{ C}$$

when $x = 0.5 \text{ mm} = 5 \times 10^{-4} \text{ m}$ $\text{erf}[5 \times 10^{-4}/7.76 \times 10^{-4}] = \text{erf}(0.645) = 0.636$

$$\frac{(1 - c_x)}{0.9} = 0.636 \quad c_x = 0.43\% \text{ C}$$

when $x = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$ $\text{erf}[1 \times 10^{-3}/7.76 \times 10^{-4}] = \text{erf}(1.289) = 0.914$

$$\frac{(1 - c_x)}{0.9} = 0.914 \quad c_x = 0.18\% \text{ C}$$



5.18 Iron containing 0.05% C is heated to 912°C in an atmosphere that produces 1.20% C at the surface and is held for 24 h. Calculate the carbon content at 0.5 mm beneath the surface if (a) the iron is BCC and (b) the iron is FCC. Explain the difference.

$$t = (24 \text{ h})(3600 \text{ s/h}) = 86,400 \text{ s}$$

$$D_{\text{BCC}} = 1.1 \times 10^{-6} \exp[-87,500/(8.314)(1185)] = 1.53 \times 10^{-10} \text{ m}^2 \cdot \text{s}^{-1}$$

$$D_{\text{FCC}} = 2.3 \times 10^{-5} \exp[-137,700/(8.314)(1185)] = 1.96 \times 10^{-12} \text{ m}^2 \cdot \text{s}^{-1}$$

$$\text{BCC: } \frac{1.2 - c_x}{1.2 - 0.05} = \text{erf}[0.5 \times 10^{-3} / (2\sqrt{(1.53 \times 10^{-4}) (86,400)})] = \text{erf}[0.0685] \\ = 0.077$$

$$c_x = 1.11\% \text{ C}$$

$$\text{FCC: } \frac{1.2 - c_x}{1.2 - 0.05} = \text{erf}[0.5 \times 10^{-3} / (2\sqrt{(1.96 \times 10^{-12}) (86,400)})] = \text{erf}[0.192] \\ = 0.2139$$

$$c_x = 0.95\% \text{ C}$$

Faster diffusion occurs in the more loosely packed BCC structure, leading to the higher carbon content at depth x.

5.19 What temperature is required to obtain 0.50% C at a distance of 0.5 mm beneath the surface of a 0.20% C steel in 2 h. when 1.10% C is present at the surface? Assume that the iron is FCC.

$$\frac{1.1 - 0.5}{1.1 - 0.2} = 0.667 = \text{erf}[0.5 \times 10^{-3} / 2\sqrt{Dt}]$$

$$0.05 \times 10^{-3} / 2\sqrt{Dt} = 0.685 \quad \text{or} \quad \sqrt{Dt} = 3.65 \times 10^{-4}$$

$$\text{or} \quad Dt = 1.33 \times 10^{-7}$$

$$t = (2 \text{ h}) (3600 \text{ s.h}^{-1}) = 7200 \text{ s}$$

$$D = 1.33 \times 10^{-7} / 7200 = 1.85 \times 10^{-11}$$

$$D = 2.3 \times 10^{-5} \exp[-137,700 / 8.314(T)]$$

$$\exp(-16,560/T) = 8.043 \times 10^{-7}$$

$$T = 1180\text{K} = 907^\circ\text{C}$$

5.20 A 0.15% C steel is to be carburized at 1100°C, giving 0.35% C at a distance of 1 mm beneath the surface. If the surface composition is maintained at 0.90% C, what time is required?

$$\frac{0.9 - 0.35}{0.9 - 0.15} = 0.733 = \text{erf}[1 \times 10^{-3} / 2\sqrt{Dt}]$$

$$1 \times 10^{-3} / 2\sqrt{Dt} = 0.786 \quad \text{or} \quad \sqrt{Dt} = 6.36 \times 10^{-4} \quad \text{or} \quad Dt = 4.05 \times 10^{-7}$$

$$D = 2.3 \times 10^{-5} \exp[-137,700 / (8.314)(1373)] = 1.33 \times 10^{-10} \text{ m}^2.\text{s}^{-1}$$

$$t = 4.05 \times 10^{-7} / 1.33 \times 10^{-10} = 3049 \text{ s} = 51 \text{ min}$$

5.21 A 0.02% C steel is to be carburized at 1200°C in 4 h, with a point 0.6 mm beneath the surface reaching 0.45% C. Calculate the carbon content required at the surface of the steel.

$$\frac{c_s - 0.45}{c_s - 0.02} = \text{erf}[0.6 / 2\sqrt{Dt}]$$

$$D = 2.3 \times 10^{-5} \exp[-137,700 / (8.314)(1473)] = 3.01 \times 10^{-10} \text{ m}^2/\text{s}^{-1}$$

$$t = (4 \text{ h}) (3600) = 14,400 \text{ s}$$

$$\sqrt{Dt} = \sqrt{(3.01 \times 10^{-10})(14,400)} = 2.085 \times 10^{-3}$$

$$\text{erf}[6 \times 10^{-4}/(2)(2.085 \times 10^{-3})] = \text{erf}(0.144) = 0.161$$

$$\frac{c_s - 0.45}{c_s - 0.02} = 0.161 \quad \text{or} \quad c_s = 0.53\% \text{ C}$$

5.22 A 1.2% C tool steel held at 1150°C is exposed to oxygen for 48 h. The carbon content at the steel surface is zero. To what depth will the steel be decarburized to less than 0.20% C?

$$\frac{0 - 0.2}{0 - 1.2} = 0.1667 \quad \therefore x/2\sqrt{Dt} = 0.149$$

$$D = 2.3 \times 10^{-5} \exp[-137,700/(8.314)(1423)] = 2.03 \times 10^{-10} \text{ m}^2 \cdot \text{s}^{-1}$$

$$t = (48 \text{ h})(3600 \text{ s/h}) = 17.28 \times 10^4 \text{ s}$$

$$\sqrt{Dt} = 0.592 \times 10^{-3}$$

$$\text{Then from above, } x = (0.149)(2)(0.592 \times 10^{-3}) = 1.76 \text{ mm}$$

5.23 A 0.80% C steel must operate at 950°C in an oxidizing environment, where the carbon content at the steel surface is zero. Only the outermost 0.02 cm of the steel part can fall below 0.75% C. What is the maximum time that the steel part can operate?

$$\frac{0 - 0.75}{0 - 0.8} = 0.9375 = \text{erf}[x/2\sqrt{Dt}] \quad \therefore x/2\sqrt{Dt} = 1.384$$

$$0.2 \times 10^{-3}/2\sqrt{Dt} = 1.384 \quad \text{or} \quad \sqrt{Dt} = 7.23 \times 10^{-5} \quad \text{or} \quad Dt = 5.22 \times 10^{-9}$$

$$D = 2.3 \times 10^{-5} \exp[-137,700/(8.314)(1223)] = 3.02 \times 10^{-11} \text{ m}^2 \cdot \text{s}^{-1}$$

$$t = 5.22 \times 10^{-9} / 3.02 \times 10^{-11} = 172 \text{ s} = 2.9 \text{ min}$$

5.24 A BCC steel containing 0.001% N is nitrided at 550°C for 5 h. If the nitrogen content at the steel surface is 0.08%, determine the nitrogen content at 0.25 mm from the surface.

$$\frac{0.08 - c_x}{0.08 - 0.001} = \text{erf}[0.25 \times 10^{-3}/2\sqrt{Dt}]$$

$$t = (5 \text{ h})(3600 \text{ s} \cdot \text{h}^{-1}) = 1.8 \times 10^4 \text{ s}$$

$$D = 4.7 \times 10^{-7} \exp[-76,600/(8.314)(823)] = 6.46 \times 10^{-12} \text{ m}^2 \cdot \text{s}^{-1}$$

$$\sqrt{Dt} = 3.41 \times 10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$$

$$\text{erf}[0.25 \times 10^{-3}/(2)(3.41 \times 10^{-4})] = \text{erf}(0.3666) = 0.394$$

$$\frac{0.08 - c_x}{0.079} = 0.394 \quad \text{or} \quad c_x = 0.049\% \text{ N}$$

5.25 What time is required to nitride a 0.002% N steel to obtain 0.12% N at a distance of 0.05 mm beneath the surface at 625°C? The nitrogen content at the surface is 0.15%.

$$\frac{0.15 - 0.12}{0.15 - 0.002} = 0.2027 = \text{erf}[x/2\sqrt{Dt}] \quad \therefore x/2\sqrt{Dt} = 0.2256$$

$$D = 4.7 \times 10^{-7} \exp[-76,600/(8.314)(898)] = 1.65 \times 10^{-11} \text{ m}^2 \cdot \text{s}^{-1}$$

$$x = 0.05 \text{ mm} = 5 \times 10^{-5} \text{ m}$$

$$\frac{5 \times 10^{-5}}{2\sqrt{(1.65 \times 10^{-11})t}} = 0.2256$$

$$t = \frac{(5 \times 10^{-5})^2}{4 \times 1.65 \times 10^{-11} \times (0.2256)^2} = 746 \text{ s} = 12.4 \text{ min}$$

5.26 We currently can successfully perform a carburizing heat treatment at 1200°C in 1 h. In an effort to reduce the cost of the brick lining in our furnace, we propose to reduce the carburizing temperature to 950°C. What time will be required to give us a similar carburizing treatment?

$$D_{1200} = 2.3 \times 10^{-5} \exp[-137,700/(8.314)(1473)] = 3.01 \times 10^{-10} \text{ m}^2 \cdot \text{s}^{-1}$$

$$D_{950} = 2.3 \times 10^{-5} \exp[-137,700/(8.314)(1223)] = 3.02 \times 10^{-11} \text{ m}^2 \cdot \text{s}^{-1}$$

For a similar carburizing process

$$t_{950} D_{950} = t_{1200} D_{1200}$$

or
$$\frac{t_{950}}{D_{950}} = \frac{t_{1200} D_{1200}}{D_{950}}$$

$$= \frac{(1 \text{ hr})(3.01 \times 10^{-10})}{3.02 \times 10^{-11}}$$

$$= 9.97 \text{ h.}$$

5.27 During freezing of a Cu-Zn alloy, we find that the composition is nonuniform. By heating the alloy to 600°C for 3 hours, diffusion of zinc helps to make the composition more uniform. What temperature would be required if we wished to perform this homogenization treatment in 30 minutes?

$$D_{600} = 7.8 \times 10^{-5} \exp[-183,750/(8.314)(873)] = 7.90 \times 10^{-16} \quad \begin{matrix} t_{600} = 3 \text{ h} \\ t_x = 0.5 \text{ h} \end{matrix}$$

$$D_x = D_{600} t_{600}/t_x = (7.90 \times 10^{-16})(3)/0.5$$

$$D_x = 4.74 \times 10^{-15} = 7.8 \times 10^{-5} \exp[-183,750/8.314T]$$

$$\ln(6.07 \times 10^{-11}) = -23.525 = -183,750/8.314 T$$

$$T = 940 \text{ K} = 667^\circ\text{C}$$

5.28 A ceramic part made of MgO is sintered successfully at 1700°C in 90 minutes. To minimize thermal stresses during the process, we plan to reduce the temperature to 1500°C. Which will limit the rate at which sintering can be done -- diffusion of magnesium ions or diffusion of oxygen ions? What time will be required at the lower temperature?

Diffusion of oxygen is the slower of the two, due to the larger ionic radius of the oxygen.

$$D_{1700} = 4.3 \times 10^{-9} \exp[-343,650/(8.314)(1973)] = 3.430 \times 10^{-18} \text{ m}^2 \cdot \text{s}^{-1}$$

$$D_{1500} = 4.3 \times 10^{-9} \exp[-343,650/(8.314)(1773)] = 3.227 \times 10^{-19} \text{ m}^2 \cdot \text{s}^{-1}$$

$$t_{1500} = D_{1700} t_{1700}/D_{1500} = \frac{(3.430 \times 10^{-18})(90)}{3.227 \times 10^{-19}} = 956 \text{ min} = 15.9 \text{ h}$$

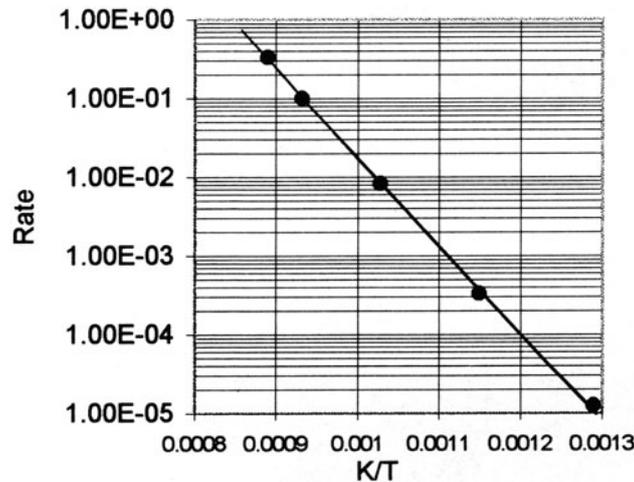
5.29 A Cu-Zn alloy has an initial grain diameter of 0.01 mm. The alloy is then heated to various temperatures, permitting grain growth to occur. The times required for the grains to grow to a diameter of 0.30 mm are

Temperature (°C)	Time (min)
500	80,000
600	3,000
700	120
800	10
850	3

Determine the activation energy for grain growth. Does this correlate with the diffusion of zinc in copper? (Hint - note that rate is the reciprocal of time.)

Temperature (°C)	Temperature (K)	1/T (K ⁻¹)	Time (min)	Rate (min ⁻¹)
500	773	0.00129	80,000	1.25 x 10 ⁻⁵
600	873	0.00115	3,000	3.33 x 10 ⁻⁴
700	973	0.001028	120	8.33 x 10 ⁻³
800	1073	0.000932	10	0.100
850	1123	0.000890	3	0.333

From the graph, we find that $Q = 214,600 \text{ J.mol}^{-1}$ which does correlate with the activation energy for diffusion of zinc in copper.



5.30 A sheet of gold is diffusion-bonded to a sheet of silver in 1 h at 700°C. At 500°C, 440 h are required to obtain the same degree of bonding, and at 300°C, bonding requires 1530 years. What is the activation energy for the diffusion bonding process? Does it appear that diffusion of gold or diffusion of silver controls the bonding rate? (Hint - note that rate is the reciprocal of time.)

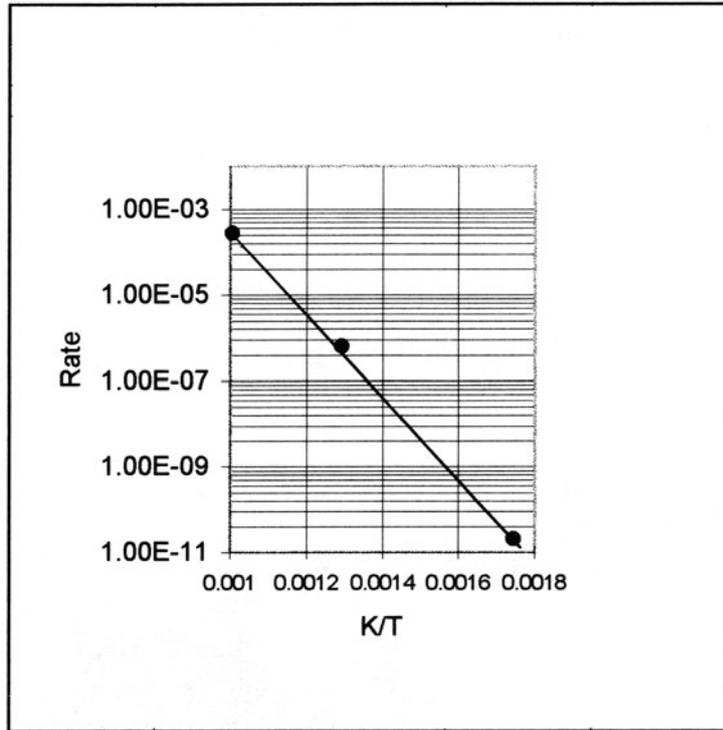
Temperature (°C)	Temperature (K)	1/T (K ⁻¹)	Time (s)	Rate (s ⁻¹)
700	973	0.001007	3600	0.278 x 10 ⁻³
500	773	0.001294	1.584 x 10 ⁶	0.631 x 10 ⁻⁶
300	573	0.001745	4.825 x 10 ¹⁰	0.207 x 10 ⁻¹⁰

$$\frac{0.278 \times 10^{-3}}{0.207 \times 10^{-10}} = \frac{\exp[-Q/(8.314)(973)]}{\exp[-Q/(8.314)(573)]} = \frac{\exp[-0.0001236Q]}{\exp[-0.000210Q]}$$

$$\ln(1.343 \times 10^7) = 16.413 = 8.64 \times 10^{-5}Q$$

$$Q = 190,000 \text{ J.mol}^{-1}$$

The activation energy for the diffusion of gold in silver is 190,000 J.mol⁻¹; thus the diffusion of gold appears to control the bonding rate.



Chapter 6 Mechanical Testing and Properties

6.1 A 4000 N force is applied to a 4 mm diameter nickel wire having a yield strength of 300 MPa and a tensile strength of 380 MPa. Determine (a) whether the wire will plastically deform and (b) whether the wire will experience necking.

(a) First determine the stress acting on the wire:

$$\sigma = F/A = 4000 \text{ N} / (\pi/4) (4 \text{ mm})^2 = 318 \text{ MPa}$$

Because σ is greater than the yield strength of 300 MPa, the wire will plastically deform.

(b) Because σ is less than the tensile strength of 380 MPa, no necking will occur.

6.2 A force of 100,000 N is applied to a 10 mm x 20 mm iron bar having a yield strength of 400 MPa and a tensile strength of 480 MPa. Determine (a) whether the bar will plastically deform and (b) whether the bar will experience necking.

(a) First determine the stress acting on the wire:

$$\sigma = F/A = 100,000 \text{ N} / (10 \text{ mm}) (20 \text{ mm}) = 500 \text{ N/mm}^2 = 500 \text{ MPa}$$

Because σ is greater than the yield strength of 400 MPa, the wire will plastically deform.

(b) Because σ is greater than the tensile strength of 480 MPa, the wire will also neck.

6.3 Calculate the maximum force that a 5 mm diameter rod of Al_2O_3 , having a yield strength of 240 MPa, can withstand with no plastic deformation.

$$F = \sigma A = (240 \text{ MPa}) (\pi/4) (5 \text{ mm})^2 = 4712 \text{ N}$$

6.4 A force of 20,000 N will cause a 10 mm x 10 mm bar of magnesium to stretch from 10 cm to 10.045 cm. Calculate the modulus of elasticity.

The strain ϵ is $\epsilon = (10.045 \text{ cm} - 10 \text{ cm}) / 10 \text{ cm} = 0.0045 \text{ cm/cm}$

The stress σ is $\sigma = 20,000 \text{ N} / (10 \text{ mm}) (10 \text{ mm}) = 200 \text{ N/mm}^2 = 200 \text{ MPa}$

$E = \sigma / \epsilon = 200 \text{ MPa} / 0.0045 \text{ cm/cm} = 44,444 \text{ MPa} = 44.4 \text{ GPa}$

6.5 A polymer bar's dimensions are 25 mm x 50 mm x 380 mm. The polymer has a modulus of elasticity of 4 GPa. What force is required to stretch the bar elastically to 387 mm?

The strain ϵ is $\epsilon = (387 \text{ mm} - 380 \text{ mm}) / (380 \text{ mm}) = 0.01842$

The stress σ is $\sigma = E\epsilon = (4 \text{ GPa})(0.01842) = 74 \text{ MPa}$

The force is then $F = \sigma A = (74 \text{ MPa})(25 \text{ mm})(50 \text{ mm}) = 92 \text{ kN}$

6.6 An aluminium plate 0.5 cm thick is to withstand a force of 50,000 N with no permanent deformation. If the aluminium has a yield strength of 125 MPa, what is the minimum width of the plate?

The area is $A = F/\sigma = 50,000 \text{ N} / 125 \text{ N/mm}^2 = 400 \text{ mm}^2$

The minimum width is $w = A/t = (400 \text{ mm}^2)(0.1 \text{ cm/mm})^2 / 0.5 \text{ cm} = 8 \text{ cm}$

6.7 A 75 mm diameter rod of copper is to be reduced to a 50 mm diameter rod by being pushed through an opening. To account for the elastic strain, what should be the diameter of the opening? The modulus of elasticity for the copper is 127 GPa and the yield strength is 275 MPa.

The strain is $\epsilon = \sigma/E = 275 \text{ MPa} / 127 \text{ GPa} = 0.00216$

The strain is also $\epsilon = (50 \text{ mm} - d_o) / d_o = 0.00216$

$$50 - d_o = 0.00216 d_o$$

$$d_o = 50 / 1.00216 = 49.89 \text{ mm}$$

The opening in the die must be smaller than the final diameter.

6.8 A 0.15 cm thick, 8 cm wide sheet of magnesium that is originally 5 m long is to be stretched to a final length of 6.2 m. What should be the length of the sheet before the applied stress is released? The modulus of elasticity of magnesium is 45 GPa and the yield strength is 200 MPa.

The strain is $\epsilon = \sigma/E = 200 \text{ MPa} / (45 \text{ GPa})(1000 \text{ MPa/GPa}) = 0.00444 \text{ cm/cm}$

$$\epsilon = (\ell_f - 6.2 \text{ m}) / 6.2 \text{ m} = 0.00444 \text{ m/m}$$

$$\ell_f = 6.2 + 0.0275 = 6.2275 \text{ m}$$

The sheet must be stretched extra long to account for elastic strain.

6.9 A steel cable 31 mm in diameter and 15.25 m long is to lift a 0.2 MN load. What is the length of the cable during lifting? The modulus of elasticity of the steel is 210 GPa.

The stress is $\sigma = F/A = \frac{(0.2 \text{ MN})}{(\pi/4)(31 \text{ mm})^2} = 265 \text{ MPa}$

The strain is $\epsilon = \sigma/E = 265 \text{ MPa} / 210 \text{ GPa} = 0.001262$

$$\epsilon = (\ell_f - 15.25 \text{ m}) / 15.25 \text{ m} = 0.001262$$

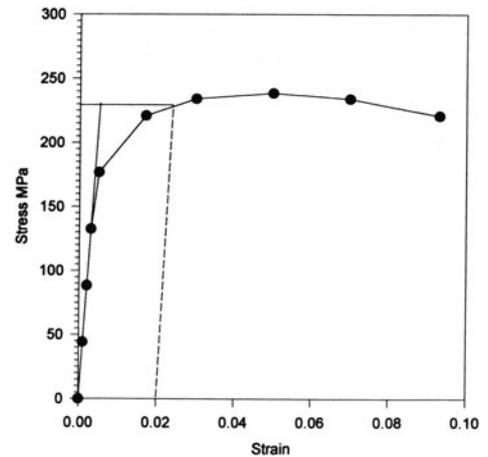
$$\ell_f = 15.2692 \text{ m}$$

6.10 The following data were collected from a 12 mm diameter test specimen of magnesium:

$$\sigma = F / (\pi/4) (12 \text{ mm})^2 = F/113.1$$

$$\epsilon = (\ell - 30)/30$$

Load (N)	Gage Length (mm)	Stress (MPa)	Strain (mm/mm)
0	30.0000	0	0.0
5000	30.0296	44.2	0.000987
10000	30.0592	88.4	0.001973
15000	30.0888	132.6	0.00296
20000	30.15	176.8	0.005
25000	30.51	221.0	0.017
26500	30.90	234.3	0.030
27000	31.50 (max load)	238.7	0.050
26500	32.10	234.3	0.070
25000	32.79 (fracture)	221.0	0.093



After fracture, the gage length is 32.61 mm and the diameter is 11.74 mm. Plot the data and calculate (a) the 0.2% offset yield strength, (b) the tensile strength, (c) the modulus of elasticity, (d) the %Elongation, (e) the %Reduction in area, (f) the engineering stress at fracture, (g) the true stress at fracture, and (h) the modulus of resilience.

(a) 0.2% offset yield strength = 186 MPa

(b) tensile strength = 238.7 MPa

(c) $E = (132.6 - 0) / (0.00296 - 0) = 44,800 \text{ MPa} = 44.8 \text{ GPa}$

(d) %Elongation = $\frac{(32.61 - 30)}{30} \times 100 = 8.7\%$

(e) %Reduction in area = $\frac{(\pi/4)(12)^2 - (\pi/4)(11.74)^2}{(\pi/4)(12)^2} \times 100 = 4.3\%$

(f) engineering stress at fracture = 221 MPa

(g) true stress at fracture = $25,000 \text{ N} / (\pi/4)(11.74)^2 = 231 \text{ MPa}$

(h) From the figure, yielding begins near 138 MPa psi. Thus:

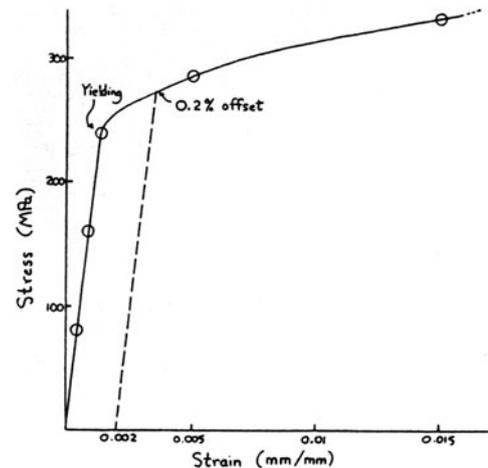
$$\frac{1}{2}(\text{yield strength})(\text{strain at yield}) = \frac{1}{2}(138)(0.00296) = 0.2 \text{ MPa}$$

6.11 The following data were collected from a 20 mm diameter test specimen of S.G. cast iron:

$$\sigma = F / (\pi/4) (20 \text{ mm})^2 = F/314.2$$

$$\epsilon = (\ell - 40)/40$$

Load (N)	Gage Length (mm)	Stress (MPa)	Strain (mm/mm)
0	40.0000	0	0.0
25000	40.0185	79.6	0.00046
50000	40.0370	159.2	0.000925
75000	40.0555	238.7	0.001388
90000	40.20	286.5	0.005
105000	40.60	334.2	0.015
120000	41.56	382.0	0.039
131000	44.00 (max load)	417.0	0.010
125000	47.52 (fracture)	397.9	0.188



After fracture, the gage length is 47.42 mm and the diameter is 18.35 mm. Plot the data and calculate (a) the 0.2% offset yield strength, (b) the tensile strength, (c) the modulus of elasticity, (d) the %Elongation, (e) the

%Reduction in area, (f) the engineering stress at fracture, (g) the true stress at fracture, and (h) the modulus of resilience.

- (a) 0.2% offset yield strength = 274 MPa
 (b) tensile strength = 417 MPa
 (c) $E = (238.7 - 0) / (0.001388 - 0) = 172,000 \text{ MPa} = 172 \text{ GPa}$
 (d) %Elongation = $\frac{(47.42 - 40)}{40} \times 100 = 18.55\%$
 (e) %Reduction in area = $\frac{(\pi/4)(20)^2 - (\pi/4)(18.35)^2}{(\pi/4)(20)^2} \times 100 = 15.8\%$
 (f) engineering stress at fracture = 397.9 MPa
 (g) true stress at fracture = $125,000 \text{ N} / (\pi/4)(18.35)^2 = 473 \text{ MPa}$
 (h) From the figure, yielding begins near 240 MPa. Thus:
 $\frac{1}{2}(\text{yield strength})(\text{strain at yield}) = \frac{1}{2}(240)(0.001388) = 0.17 \text{ MPa}$

6.12 A bar of Al_2O_3 that is 6 mm thick, 12 mm wide, and 225 mm long is tested in a three-point bending apparatus, with the supports located 150 mm apart. The deflection of the center of the bar is measured as a function of the applied load. The data are shown below. Determine the flexural strength and the flexural modulus.

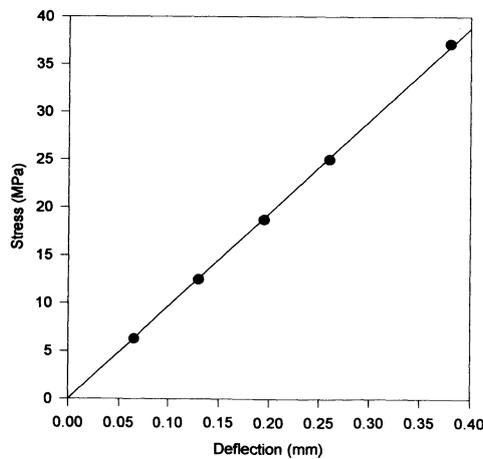
$$\begin{aligned} \text{stress} &= 3LF/2wh^2 \\ &= (3)(150 \text{ mm})F / (2)(12 \text{ mm})(6 \text{ mm})^2 \\ &= 3.125F \end{aligned}$$

Force (N)	Deflection (mm)	Stress (MPa)
64.500	0.065	6.270
128.500	0.130	12.500
193.000	0.195	18.760
257.500	0.260	25.030
382.500	0.380	37.180

The flexural strength is the stress at fracture, or 37.18 MPa.

The flexural modulus can be calculated from the linear curve; picking the first point as an example:

$$FM = \frac{FL^3}{4wh^3\delta} = \frac{(64.5 \text{ N})(150 \text{ mm})^3}{(4)(12 \text{ mm})(6 \text{ mm})^3(0.065 \text{ mm})} = 323 \text{ GPa}$$



6.13 A 10 mm diameter, 204 mm long titanium bar has a yield strength of 345 MPa, a modulus of elasticity of 110 GPa, and Poisson's ratio of 0.30. Determine the length and diameter of the bar when a 2.2 kN load is applied.

$$\text{The stress is } \sigma = F/A = 2.2 \text{ kN}/(\pi/4) (10 \text{ mm})^2 = 28 \text{ MPa}$$

The applied stress is much less than the yield strength; therefore Hooke's law can be used.

$$\text{The strain is } \epsilon = \sigma/E = 28 \text{ MPa} / (110 \text{ GPa}) = 0.0002545$$

$$\frac{l_f - l_o}{l_o} = \frac{l_f - 305 \text{ mm}}{305 \text{ mm}} = 0.0002545$$

$$l_f = 305.07763 \text{ mm}$$

$$\text{From Poisson's ratio, } \mu = -\epsilon_{\text{lat}} / \epsilon_{\text{long}} = 0.3$$

$$\epsilon_{\text{lat}} = - (0.3) (0.0002545) = - 0.0000763$$

$$\frac{d_f - d_o}{d_f} = \frac{d_f - 10 \text{ mm}}{10 \text{ mm}} = - 0.0000763$$

$$d_f = 9.99923 \text{ mm}$$

6.14 When a tensile load is applied to a 1.5-cm diameter copper bar, the diameter is reduced to 1.498-cm diameter. Determine the applied load, using the data in Table 6.3.

$$\text{From Table 6.3, } \mu = -\epsilon_{\text{lat}} / \epsilon_{\text{long}} = 0.36$$

$$\epsilon_{\text{lat}} = \frac{1.498 - 1.5}{1.5} = - 0.001333$$

$$\epsilon_{\text{long}} = -\epsilon_{\text{lat}} / \mu = - (-0.001333) / 0.36 = 0.0037 \text{ mm./mm.}$$

$$\sigma = E\epsilon = (124.8 \text{ GPa}) (1000 \text{ MPa/GPa}) (0.0037 \text{ mm./mm.}) = 462 \text{ MPa}$$

$$F = \sigma A = (462 \text{ MPa}) (\pi/4) (15 \text{ mm})^2 = 81,640 \text{ N}$$

6.15 A three-point bend test is performed on a block of ZrO₂ that is 200 mm long, 12 mm wide, and 6 mm thick and is resting on two supports 100 mm apart. When a force of 1780 N is applied, the specimen deflects 0.94 mm and breaks. Calculate (a) the flexural strength and (b) the flexural modulus, assuming that no plastic deformation occurs.

$$(a) \text{ flexural strength} = 3FL/2wh^2 = \frac{(3)(1780 \text{ N})(100 \text{ mm})}{(2)(12 \text{ mm})(6 \text{ mm})^2} = 618 \text{ MPa}$$

$$(b) \text{ flexural modulus} = FL^3/4wh^3\delta$$

$$= \frac{1780 \text{ N} (100 \text{ mm})^3}{(4)(12 \text{ mm})(6 \text{ mm})^3(0.94 \text{ mm})}$$

$$= 183 \text{ GPa}$$

6.16 A three-point bend test is performed on a block of silicon carbide that is 10 cm long, 1.5 cm wide, and 0.6 cm thick and is resting on two supports 7.5 cm apart. The sample breaks when a deflection of 0.09 mm is recorded. Calculate (a) the force that caused the fracture and (b) the flexural strength. The flexural modulus for silicon carbide is 480 GPa. Assume that no plastic deformation occurs.

(a) The force F required to produce a deflection of 0.09 mm is

$$F = (\text{flexural modulus}) (4wh^3\delta) / L^3$$

$$F = (480,000 \text{ MPa}) (4) (15 \text{ mm}) (6 \text{ mm})^3 (0.09 \text{ mm}) / (75 \text{ mm})^3$$

$$F = 1327 \text{ N}$$

$$(b) \text{ flexural strength} = 3FL/2wh^2 = (3) (1327 \text{ N}) (75 \text{ mm}) / (2) (15 \text{ mm}) (6 \text{ mm})^2 \\ = 276 \text{ MPa}$$

6.17 A thermosetting polymer containing glass beads is required to deflect 0.5 mm when a force of 500 N is applied. The polymer part is 2 cm wide, 0.5 cm thick, and 10 cm long. If the flexural modulus is 6.9 GPa, determine the minimum distance between the supports. Will the polymer fracture if its flexural strength is 85 MPa? Assume that no plastic deformation occurs.

The minimum distance L between the supports can be calculated from the flexural modulus.

$$L^3 = 4wh^3\delta (\text{flexural modulus}) / F$$

$$L^3 = (4) (20 \text{ mm}) (5 \text{ mm})^3 (0.5 \text{ mm}) (6.9 \text{ GPa}) (1000 \text{ MPa/GPa}) / 500 \text{ N}$$

$$L^3 = 69,000 \text{ mm}^3 \quad \text{or} \quad L = 41 \text{ mm}$$

The stress acting on the bar when a deflection of 0.5 mm is obtained is

$$\sigma = 3FL/2wh^2 = (3) (500 \text{ N}) (41 \text{ mm}) / (2) (20 \text{ mm}) (5 \text{ mm})^2 = 61.5 \text{ MPa}$$

The applied stress is less than the flexural strength of 85 MPa; the polymer is not expected to fracture.

6.18 The flexural modulus of alumina is 350 GPa and its flexural strength is 350 MPa. A bar of alumina 7.6 mm thick, 25 mm wide, and 250 mm long is placed on supports 175 mm apart. Determine the amount of deflection at the moment the bar breaks, assuming that no plastic deformation occurs.

The force required to break the bar is

$$F = 2wh^2(\text{flexural strength})/3L$$

$$F = (2) (25 \text{ mm}) (7.6 \text{ mm})^2 (350 \text{ MPa}) / (3) (175 \text{ mm}) = 1925 \text{ N}$$

The deflection just prior to fracture is

$$\delta = FL^3/4wh^3(\text{flexural modulus})$$

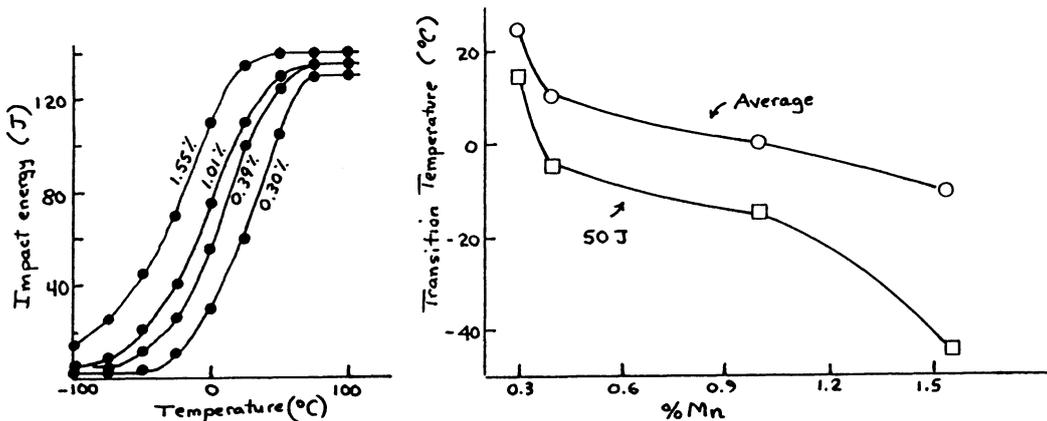
$$\delta = (1925 \text{ N}) (175 \text{ mm})^3 / (4) (25 \text{ mm}) (7.6 \text{ mm})^3 (350 \text{ GPa}) = 0.67 \text{ mm}$$

6.19 A Brinell hardness measurement, using a 10 mm diameter indenter and a 500 kg load, produces an indentation of 4.5 mm on an aluminium plate. Determine the Brinell hardness number HB of the metal.

$$HB = \frac{500 \text{ kg}}{(\pi/2) (10 \text{ mm}) [10 - \sqrt{10^2 - 4.5^2}]} = 29.8$$

6.20 The data below were obtained from a series of Charpy impact tests performed on four steels, each having a different manganese content. Plot the data and determine (a) the transition temperature (defined by the mean of the absorbed energies in the ductile and brittle regions) and (b) the transition temperature (defined as the temperature that provides 50 J absorbed energy). Plot the transition temperature versus manganese content and discuss the effect of manganese on the toughness of steel. What would be the minimum manganese allowed in the steel if a part is to be used at 0°C?

Test temperature °C	Impact energy (J)			
	0.30% Mn	0.39% Mn	1.01% Mn	1.55% Mn
-100	2	5	5	15
-75	2	5	7	25
-50	2	12	20	45
-25	10	25	40	70
0	30	55	75	110
25	60	100	110	135
50	105	125	130	140
75	130	135	135	140
100	130	135	135	140



(a) Transition temperatures defined by the mean of the absorbed energies are:

$$0.30\% \text{ Mn: mean energy} = 2 + (130 + 2)/2 = 68 \text{ J; } T = 27^\circ\text{C}$$

$$0.39\% \text{ Mn: mean energy} = 5 + (135 + 5)/2 = 75 \text{ J; } T = 10^\circ\text{C}$$

$$1.01\% \text{ Mn: mean energy} = 5 + (135 + 5)/2 = 75 \text{ J; } T = 0^\circ\text{C}$$

$$1.55\% \text{ Mn: mean energy} = 15 + (140 + 15)/2 = 92.5 \text{ J; } T = -12^\circ\text{C}$$

(b) Transition temperatures defined by 50 J are:

$$0.30\% \text{ Mn: } T = 15^\circ\text{C}$$

$$0.39\% \text{ Mn: } T = -5^\circ\text{C}$$

$$1.01\% \text{ Mn: } T = -15^\circ\text{C}$$

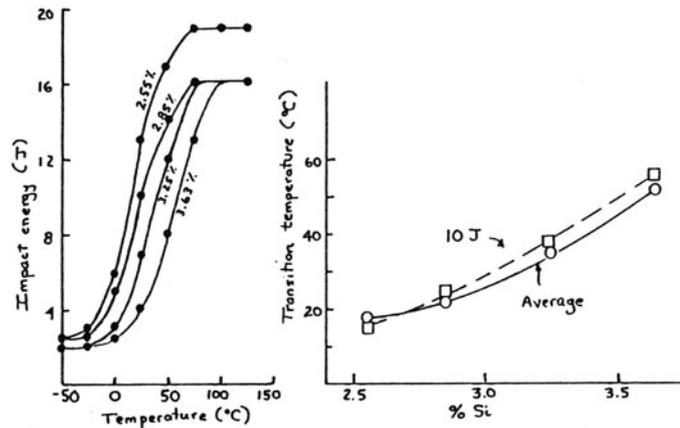
$$1.55\% \text{ Mn: } T = -45^\circ\text{C}$$

Increasing the manganese increases the toughness and reduces the transition temperature; manganese is therefore a desirable alloying element for improving the impact properties of the steel.

If the part is to be used at 25°C, we would want at least 1.0% Mn in the steel based on the mean absorbed energy criterion or 0.36% Mn based on the 50 J criterion.

6.21 The data below were obtained from a series of Charpy impact tests performed on four ductile cast irons, each having a different silicon content. Plot the data and determine (a) the transition temperature (defined by the mean of the absorbed energies in the ductile and brittle regions) and (b) the transition temperature (defined as the temperature that provides 10 J absorbed energy). Plot the transition temperature versus silicon content and discuss the effect of silicon on the toughness of the cast iron. What would be the maximum silicon allowed in the cast iron if a part is to be used at 25°C?

Test temperature °C	Impact energy (J)			
	2.55% Si	2.85% Si	3.25% Si	3.63% Si
- 50	2.5	2.5	2	2
- 25	3	2.5	2	2
0	6	5	3	2.5
25	13	10	7	4
50	17	14	12	8
75	19	16	16	13
100	19	16	16	16
125	19	16	16	16



(a) Transition temperatures defined by the mean of the absorbed energies are:

2.55% Si: mean energy = $2.5 + (19 + 2.5)/2 = 13.2$ J; $T = 26^\circ\text{C}$

2.85% Si: mean energy = $2.5 + (16 + 2.5)/2 = 11.8$ J; $T = 35^\circ\text{C}$

3.25% Si: mean energy = $2 + (16 + 2)/2 = 11$ J; $T = 45^\circ\text{C}$

3.63% Si: mean energy = $2 + (16 + 2)/2 = 11$ J; $T = 65^\circ\text{C}$

(b) Transition temperatures defined by 10 J are:

2.55% Si: $T = 15^\circ\text{C}$

2.85% Si: $T = 25^\circ\text{C}$

3.25% Si: $T = 38^\circ\text{C}$

3.63% Si: $T = 56^\circ\text{C}$

Increasing the silicon decreases the toughness and increases the transition temperature; silicon therefore reduces the impact properties of the cast iron.

If the part is to be used at 25°C, we would want a maximum of about 2.9% Si in the cast iron.

6.22 FCC metals are often recommended for use at low temperatures, particularly when any sudden loading of the part is expected. Explain.

FCC metals do not normally display a transition temperature; instead the impact energies decrease slowly with decreasing temperature and, in at least some cases (such as some aluminium alloys), the energies even increase at low temperatures. The FCC metals can obtain large ductilities, giving large areas beneath the true stress-strain curve.

6.23 A steel part can be made by powder metallurgy (compacting iron powder particles and sintering to produce a solid) or by machining from a solid steel block. Which part is expected to have the higher toughness? Explain.

Parts produced by powder metallurgy often contain considerable amounts of porosity due to incomplete sintering; the porosity provides sites at which cracks might easily nucleate. Parts machined from solid steel are less likely to contain flaws that would nucleate cracks, therefore improving toughness.

6.24 A number of aluminium-silicon alloys have a structure that includes sharp-edged plates of brittle silicon in the softer, more ductile aluminium matrix. Would you expect these alloys to be notch-sensitive in an impact test? Would you expect these alloys to have good toughness? Explain your answers.

The sharp-edged plates of the brittle silicon may act as stress-raisers, or notches, thus giving poor toughness to the alloy. The presence of additional notches, such as machining marks, will not have a significant effect, since there are already very large numbers of "notches" due to the microstructure. Consequently this type of alloy is expected to have poor toughness but is not expected to be notch-sensitive.

6.25 Alumina Al_2O_3 is a brittle ceramic with low toughness. Suppose that fibres of silicon carbide SiC, another brittle ceramic with low toughness, could be embedded within the alumina. Would doing this affect the toughness of the ceramic matrix composite? Explain. (These materials are discussed in later chapters.)

The SiC fibres may improve the toughness of the alumina matrix. The fibres may do so by several mechanisms. By introducing an interface (between the fibres and the matrix), a crack may be blocked; to continue growing, the crack may have to pass around the fibre, thus increasing the total energy of the crack and thus the energy that can be absorbed by the material. Or extra energy may be required to force the crack through the interface in an effort to continue propagating. In addition, the fibres may begin to pull out of the matrix, particularly if bonding is poor; the fibre pull-out requires energy, thus improving toughness. Finally, the fibres may bridge across the crack, helping to hold the material together and requiring more energy to propagate the crack.

6.26 A ceramic matrix composite contains internal flaws as large as 0.001 cm in length. The plane strain fracture toughness of the composite is 45 MPa√m and the tensile strength is 550 MPa. Will the flaw cause the composite to fail before the tensile strength is reached? Assume that $f = 1$.

Since the crack is internal, $2a = 0.001 \text{ cm} = 0.00001 \text{ m}$. Therefore
 $a = 0.000005 \text{ m}$

$$K_{Ic} = f\sigma\sqrt{\pi a} \quad \text{or} \quad \sigma = K_{Ic}/f\sqrt{\pi a}$$

$$\sigma = (45 \text{ MPa}\sqrt{\text{m}}) / (1)\sqrt{\pi(0.000005 \text{ m})} = 11,354 \text{ MPa}$$

The applied stress required for the crack to cause failure is much larger than the tensile strength of 550 MPa. Any failure of the ceramic should be expected due to the massive overload, not because of the presence of the flaws.

6.27 An aluminium alloy that has a plane strain fracture toughness of 27 MPa√m fails when a stress of 290 MPa is applied. Observation of the fracture surface indicates that fracture began at the surface of the part. Estimate the size of the flaw that initiated fracture. Assume that $f = 1.1$.

$$K_{Ic} = f\sigma\sqrt{\pi a} \quad \text{or} \quad a = (1/\pi) [K_{Ic}/f\sigma]^2$$

$$a = (1/\pi) [27 \text{ MPa}\sqrt{\text{m}} / (1.1)(290 \text{ MPa})]^2 = 2.28 \text{ mm}$$

6.28 A polymer that contains internal flaws 1 mm in length fails at a stress of 25 MPa. Determine the plane strain fracture toughness of the polymer. Assume that $f = 1$.

Since the flaws are internal, $2a = 1 \text{ mm} = 0.001 \text{ m}$; thus $a = 0.0005 \text{ m}$

$$K_{Ic} = f\sigma\sqrt{\pi a} = (1)(25 \text{ MPa})\sqrt{\pi(0.0005 \text{ m})} = 0.99 \text{ MPa}\sqrt{\text{m}}$$

6.29 A ceramic part for a jet engine has a yield strength of 520 MPa and a plane strain fracture toughness of 5.5 MPa√m. To be sure that the part does not fail, we plan to assure that the maximum applied stress is only one third the yield strength. We use a nondestructive test that will detect any internal flaws greater than 1.25 mm long. Assuming that $f = 1.4$, does our nondestructive test have the required sensitivity? Explain.

The applied stress is $\sigma = (1/3)(520 \text{ MPa}) = 173 \text{ MPa}$

$$a = (1/\pi) [K_{Ic}/f\sigma]^2 = (1/\pi) [5.5 \text{ MPa}\sqrt{\text{m}} / (1.4)(173 \text{ MPa})]^2$$

$$a = 0.164 \text{ mm}$$

The length of internal flaws is $2a = 0.328 \text{ mm}$.

Our nondestructive test can detect flaws as small as 1.25 mm long, which is not smaller than the critical flaw size required for failure. Thus our NDT test is not satisfactory.

6.30 A cylindrical tool steel specimen that is 150 mm long and 6 mm in diameter rotates as a cantilever beam and is to be designed so that failure never occurs. Assuming that the maximum tensile and compressive stresses are equal, determine the maximum load that can be applied to the end of the beam. (See Figure 6.19).

The stress must be less than the endurance limit, 410 MPa

$$\sigma = 10.18LF/d^3 \quad \text{or} \quad F = (\text{endurance limit})d^3/10.18L$$

$$F = (410 \text{ MPa})(6 \text{ mm})^3 / (10.18)(150 \text{ mm}) = 58 \text{ N}$$

6.31 A 2 cm diameter, 20 cm long bar of an acetal polymer (Figure 6.27) is loaded on one end and is expected to survive one million cycles of loading, with equal maximum tensile and compressive stresses, during its lifetime. What is the maximum permissible load that can be applied?

From the figure, we find that the fatigue strength must be 22 MPa in order for the polymer to survive one million cycles. Thus, the maximum load is

$$F = (\text{fatigue strength})d^3/10.18L$$

$$F = (22 \text{ MPa})(20 \text{ mm})^3 / (10.18)(200 \text{ mm}) = 86.4 \text{ N}$$

6.32 A cyclical load of 6.67 kN is to be exerted at the end of a 250 mm long aluminium beam (Figure 6.19). The bar must survive for at least 10^6 cycles. What is the minimum diameter of the bar?

From the figure, we find that the fatigue strength must be 240 MPa in order for the aluminium to survive 10^6 cycles. Thus, the minimum diameter of the bar is

$$d = \sqrt[3]{10.18LF/\text{fatigue strength}}$$

$$d = \sqrt[3]{(10.18)(250 \text{ mm})(6.67 \text{ kN})/240 \text{ MPa}} = 41.35 \text{ mm}$$

6.33 A cylindrical acetal polymer bar 20 cm long and 1.5 cm in diameter is subjected to a vibrational load at a frequency of 500 vibrations per minute with a load of 50 N. How many hours will the part survive before breaking? (See Figure 6.27)

The stress acting on the polymer is

$$\sigma = 10.18LF/d^3 = (10.18)(200 \text{ mm})(50 \text{ N}) / (15 \text{ mm})^3 = 30.16 \text{ MPa}$$

From the figure, the fatigue life at 30.16 MPa is about 2×10^5 cycles. Based on 500 cycles per minute, the life of the part is

$$\text{life} = 2 \times 10^5 \text{ cycles} / (500 \text{ cycles/min})(60 \text{ min/h}) = 6.7 \text{ h}$$

6.34 Suppose that we would like a part produced from the acetal polymer shown in Figure 6.27 to survive for one million cycles under conditions that provide for equal compressive and tensile stresses. What is the fatigue strength, or maximum stress amplitude, required? What are the maximum stress, the minimum stress, and the mean stress on the part during its use? What effect would the frequency of the stress application have on your answers? Explain.

From the figure, the fatigue strength at one million cycles is 22 MPa.

The maximum stress is +22 MPa, the minimum stress is -22 MPa, and the mean stress is 0 MPa.

A high frequency will cause heating of the polymer. As the temperature of the polymer increases, the fatigue strength will decrease. If the applied stress is not reduced, then the polymer will fail in a shorter time.

6.35 The high strength steel in Figure 6.21 is subjected to a stress alternating at 200 revolutions per minute between 600 MPa and 200 MPa (both tension). Calculate the growth rate of a surface crack when it reaches a length of 0.2 mm in both m/cycle and m/s. Assume that $f = 1.2$.

For the steel, $C = 1.62 \times 10^{-12}$ and $n = 3.2$. The change in the stress intensity factor ΔK is

$$\Delta K = f\Delta\sigma\sqrt{\pi a} = (1.2)(600 \text{ MPa} - 200 \text{ MPa})\sqrt{\pi(0.0002 \text{ m})} = 12.03 \text{ MPa}\sqrt{\text{m}}$$

The crack growth rate is

$$da/dN = 1.62 \times 10^{-12} (\Delta K)^{3.2}$$

$$da/dN = 1.62 \times 10^{-12} (12.03)^{3.2} = 4.638 \times 10^{-9} \text{ m/cycle}$$

$$da/dt = (4.638 \times 10^{-9} \text{ m/cycle})(200 \text{ cycles/min}) / 60 \text{ s/min}$$

$$da/dt = 1.55 \times 10^{-8} \text{ m/s}$$

6.36 The high-strength steel in Figure 6.21, which has a critical fracture toughness of 80 MPa $\sqrt{\text{m}}$, is subjected to an alternating stress varying from -900 MPa (compression) to +900 MPa (tension). It is to survive for 10^5 cycles before failure occurs. Calculate (a) the size of a surface crack required for failure to occur and (b) the largest initial surface crack size that will permit this to happen. Assume that $f = 1$.

(a) Only the tensile portion of the applied stress is considered in $\Delta\sigma$. Based on the applied stress of 900 MPa and the fracture toughness of 80 MPa $\sqrt{\text{m}}$, the size of a surface crack required for failure to occur is

$$K = f\sigma\sqrt{\pi a_c} \quad \text{or} \quad a_c = (1/\pi) [K/f\sigma]^2$$

$$a_c = (1/\pi) [80 \text{ MPa}\sqrt{\text{m}} / (1)(900 \text{ MPa})]^2 = 0.0025 \text{ m} = 2.5 \text{ mm}$$

(b) The largest initial surface crack tolerable to prevent failure within 10^5 cycles is

$$N = 10^5 \text{ cycles} = \frac{2[(0.0025 \text{ m})^{(2-3.2)/2} - a_i^{(2-3.2)/2}]}{(2-3.2)(1.62 \times 10^{-12})(1)^{3.2}(900)^{3.2}(\pi)^{3.2/2}}$$

$$10^5 = \frac{2[36.41 - (a_i)^{-0.60}]}{(-1.2)(1.62 \times 10^{-12})(1)(2.84 \times 10^9)(6.244)}$$

$$(a_i)^{-0.6} = 1760$$

$$a_i = 3.9 \times 10^{-6} \text{ m} = 0.0039 \text{ mm}$$

6.37 The acrylic polymer from which Figure 6.28 was obtained has a critical fracture toughness of 2 MPa $\sqrt{\text{m}}$. It is subjected to a stress alternating between -10 and +10 MPa. Calculate the growth rate of a surface crack when it reaches a length of 5×10^{-6} m if $f = 1.3$.

$\Delta\sigma = 10 \text{ MPa} - 0 = 10 \text{ MPa}$, since the crack doesn't propagate for compressive loads.

$$\Delta K = f\Delta\sigma\sqrt{\pi a} = (1.3)(10 \text{ MPa})\sqrt{\pi(5 \times 10^{-6} \text{ m})} = 0.0515 \text{ MPa}\sqrt{\text{m}}$$

From the graph, $da/dN = 3 \times 10^{-7} \text{ m/cycle}$

6.38 Calculate the constants "C" and "n" in the Equation 6.21 for the crack growth rate of an acrylic polymer. (See Figure 6.28.)

Let's pick two points on the graph:

$$\begin{aligned} da/dN &= 2 \times 10^{-6} \text{ m/cycle when } \Delta K = 0.1 \text{ MPa}\sqrt{\text{m}} \\ da/dN &= 1 \times 10^{-7} \text{ m/cycle when } \Delta K = 0.037 \text{ MPa}\sqrt{\text{m}} \end{aligned}$$

$$\frac{2 \times 10^{-6}}{1 \times 10^{-7}} = \frac{C(0.1)^n}{C(0.037)^n}$$

$$20 = (0.1 / 0.037)^n = (2.703)^n$$

$$\ln(20) = n \ln(2.703) \quad 2.9957 = 0.994n \quad n = 3.01$$

$$2 \times 10^{-6} = C(0.1)^{3.01} = 0.000977C \quad C = 2.047 \times 10^{-3}$$

6.39 The acrylic polymer from which Figure 6.28 was obtained is subjected to an alternating stress between 15 MPa and 0 MPa. The largest surface cracks initially detected by nondestructive testing are 0.001 mm in length. If the critical fracture toughness of the polymer is 2 MPa√m, calculate the number of cycles required before failure occurs. Let $f = 1.2$. (Hint: use the results of Problem 6.41.)

From Problem 6.41, $C = 2.047 \times 10^{-3}$ and $n = 3.01$

The critical flaw size a_c is

$$a_c = (1/\pi) [K_{Ic} / f\sigma]^2 = (1/\pi) [(2 \text{ MPa}\sqrt{\text{m}}) / (1.2)(15 \text{ MPa})]^2$$

$$a_c = 0.00393 \text{ m} = 3.93 \text{ mm}$$

Then

$$N = \frac{2[(0.00393 \text{ m})^{(2-3.01)/2} - (0.000001 \text{ m})^{(2-3.01)/2}]}{(2-3.01)(2.047 \times 10^{-3})(1.2)^{3.01}(15 \text{ MPa})^{3.01}(\pi)^{3.01/2}}$$

$$N = \frac{2(16.3995 - 1071.52)}{(-1.01)(2.047 \times 10^{-3})(1.7312)(3467.65)(5.6)} = 30.36 \text{ cycles}$$

6.40 Verify that integration of $da/dN = C(\Delta K)^n$ will give Equation 6.23.

$$dN = (1/cf^n\Delta\sigma^n\pi^{n/2})(da/a^{n/2}) \quad \text{or} \quad N = (1/cf^n\Delta\sigma^n\pi^{n/2})\int (da/a^{n/2})$$

$$\text{since } \int a^p da = [1/(1+p)](a^{p+1})$$

$$\text{then if } p = -n/2, \int da/a^{n/2} = \frac{1}{1-n/2} [a^{-n/2+1}]_{a_i}^{a_c}$$

$$= (2/2-n) [a_c^{(2-n)/2} - a_i^{(2-n)/2}]$$

$$\text{thus } N = \frac{2[a_c^{(2-n)/2} - a_i^{(2-n)/2}]}{(2-n)cf^n\Delta\sigma^n\pi^{n/2}}$$

6.41 The activation energy for self-diffusion in copper is 0.2 MJ/mol. A copper specimen creeps at 0.002 mm.mm⁻¹.h⁻¹ when a stress of 100 MPa is applied at 600°C. If the creep rate of copper is dependent on self-diffusion, determine the creep rate if the temperature is 800°C.

The creep rate is governed by an Arrhenius relationship of the form rate = A exp(-Q/RT). From the information given,

$$\frac{x}{0.002} = \frac{A \exp[0.2/(8.314)(800+273)]}{A \exp[0.2/(8.314)(600+273)]} = \frac{1.834 \times 10^{-10}}{1.078 \times 10^{-12}}$$

$$x = (0.002) (1.834 \times 10^{-10} / 1.078 \times 10^{-12}) = 0.34 \text{ mm/mm.h}$$

6.42 When a stress of 130 MPa is applied to a material heated to 900°C, rupture occurs in 25,000 h. If the activation energy for rupture is 0.15 MJ/mol, determine the rupture time if the temperature is reduced to 800°C.

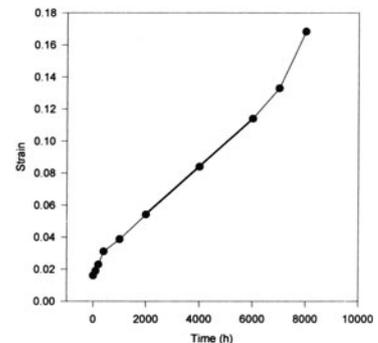
The rupture time is related to temperature by an Arrhenius relationship of the form $t_r = A \exp(+Q/RT)$; the argument of the exponential is positive because the rupture time is inversely related to the rate. From the information given

$$\frac{t_r}{25,000 \text{ h}} = \frac{A \exp[0.15/(8.314)(800+273)]}{A \exp[0.15/(8.314)(900+273)]} = \frac{20.06 \times 10^6}{4.78 \times 10^6}$$

$$t_r = (25,000) (20.06 \times 10^6 / 4.78 \times 10^6) = 104,916 \text{ h}$$

6.43 The following data were obtained from a creep test for a specimen having an initial gage length of 50 mm and an initial diameter of 15 mm. The initial stress applied to the material is 67 MPa. The diameter of the specimen after fracture is 13.2 mm.

Gauge length (mm)	Time (h)	Strain
50.800	0.000	0.016
50.950	100.000	0.019
51.150	200.000	0.023
51.560	400.000	0.031
51.940	1000.000	0.039
52.710	2000.000	0.054
54.200	4000.000	0.084
55.700	6000.000	0.114
56.650	7000.000	0.133
58.420	8000.000	0.168



Determine (a) the load applied to the specimen during the test, (b) the approximate length of time during which linear creep occurs, (c) the creep rate in mm.mm⁻¹.h⁻¹ and mm %/h, and (d) the true stress acting on the specimen at the time of rupture.

(a) The load is $F = \sigma A = (67 \text{ MPa}) (\pi/4) (15 \text{ mm})^2 = 11840 \text{ N}$

(b) The plot of strain versus time is linear between approximately 400 and 6000 hours, or a total of 5600 hours.

(c) From the graph, the strain rate is the slope of the linear portion of the curve.

$$\Delta\epsilon/\Delta t = \frac{0.114 - 0.031}{6000 - 400} = 1.48 \times 10^{-5} \text{ mm.mm.}^{-1}\text{h}^{-1} = 1.48 \times 10^{-3} \text{ \%}/\text{h}$$

(d) At the time of rupture, the force is still 11840 N, but the diameter is reduced to 13.2 mm. The true stress is therefore

$$\sigma_t = F/A = 11840 \text{ N} / (\pi/4) (13.2 \text{ mm})^2 = 86 \text{ MPa}$$

6.44 A stainless steel is held at 705°C under different loads. The following data are obtained:

Applied Stress (MPa)	Rupture Time (h)	Creep Rate (%/h)
106.9	1200	0.022
128.2	710	0.068
147.5	300	0.201
160.0	110	0.332

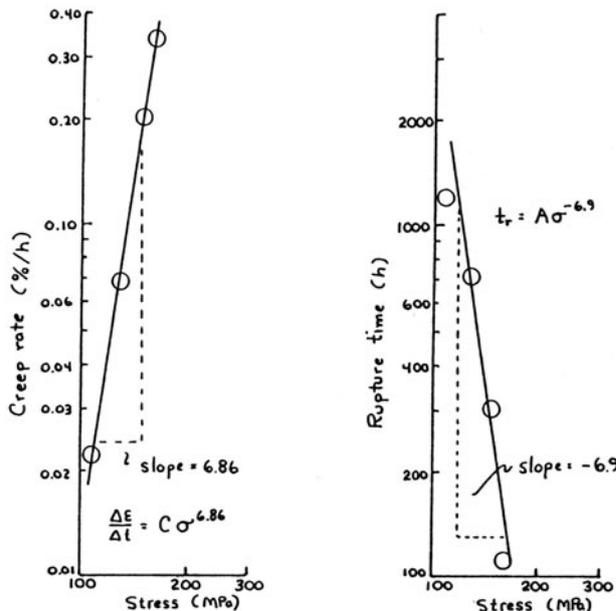
Determine the exponents "n" and "m" in Equations 6.25 and 6.26 that describe the dependence of creep rate and rupture time on applied stress.

Plots describing the effect of applied stress on creep rate and on rupture time are shown below. In the first plot, the creep rate is given by $\Delta\epsilon/\Delta t = C\sigma^n$ and the graph is a log-log plot. In the second plot, rupture time is given by $t_r = A\sigma^m$, another log-log plot.

The exponents "n" and "m" are the slopes of the two graphs. In this case,

$$n = 6.86$$

$$m = -6.9$$



6.45 Using the data in Figure 6.25(a) for an iron-chromium-nickel alloy, determine the activation energy Q_r and the constant "m" for rupture in the temperature range 980 to 1090°C.

The appropriate equation is $t_r = K\sigma^m \exp(Q_r/RT)$.

From Figure 6.25(a), we can determine the rupture time versus temperature for a fixed stress, say $\sigma = 1$ MPa:

$$\begin{aligned} t_r &= 2,400 \text{ h} && \text{at } 1090^\circ\text{C} = 1363 \text{ K} \\ t_r &= 14,000 \text{ h} && \text{at } 1040^\circ\text{C} = 1313 \text{ K} \\ t_r &= 100,000 \text{ h} && \text{at } 980^\circ\text{C} = 1253 \text{ K} \end{aligned}$$

From this data, the equation becomes $t_r = K' \exp(Q_r/RT)$ and we can find Q_r by simultaneous equations or graphically.

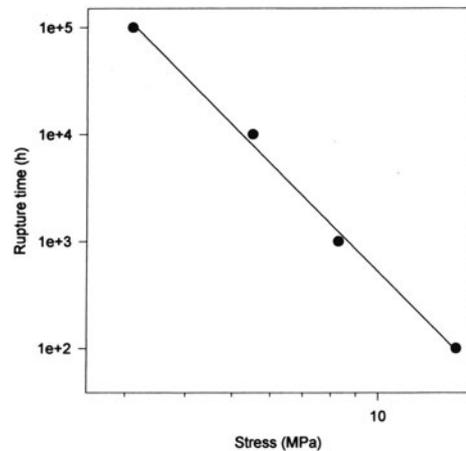
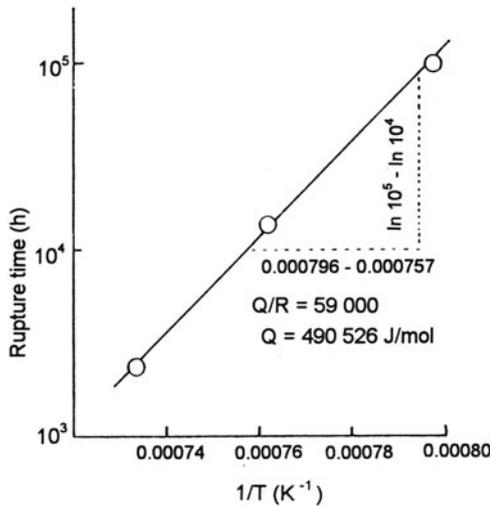
$$Q_r = 490,526 \text{ J/mol}$$

We can also determine the rupture time versus applied stress for a constant temperature, say 1090°C:

$$\begin{aligned} t_r &= 10^5 \text{ h} && \text{for } \sigma = 3.1 \text{ MPa} \\ t_r &= 10^4 \text{ h} && \text{for } \sigma = 5.5 \text{ MPa} \\ t_r &= 10^3 \text{ h} && \text{for } \sigma = 8.27 \text{ MPa} \\ t_r &= 10^2 \text{ h} && \text{for } \sigma = 14.48 \text{ MPa} \end{aligned}$$

With this approach, the equation becomes $t_r = K''\sigma^m$, where "m" is obtained graphically or by simultaneous equations:

$$m = 4.55$$

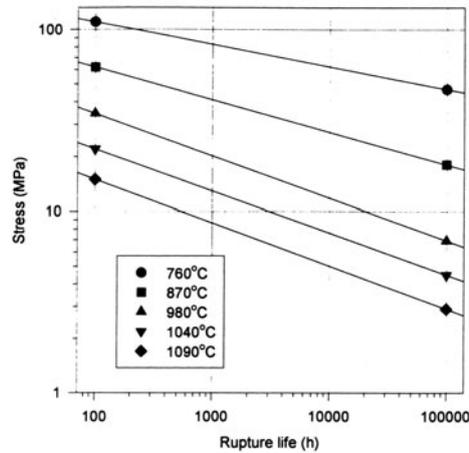


6.46 A 25 mm diameter bar of an iron-chromium-nickel alloy is subjected to a load of 11 kN. How many days will the bar survive without rupturing at 980°C? [See Figure 6.25(a)]

The stress is $\sigma = F/A = 11,000 \text{ N} / (\pi/4) (25 \text{ mm})^2 = 22.4 \text{ MPa}$

From the graph, the rupture time is approximately 700 h / 24 h/day = 29 days

Stress-rupture curves for an iron-chromium-nickel alloy



6.47 A 5 mm x 20 mm bar of an iron-chromium-nickel alloy is to operate at 1040°C for 10 years without rupturing. What is the maximum load that can be applied? [See Figure 6.25(a)]

The operating time is (10 years)(365 days/year)(24 h/day) = 87,600 h

From the graph, the stress must be less than 4 MPa. The load is then

$$F = \sigma A = (4 \text{ MPa})(5 \text{ mm})(20 \text{ mm}) = 400 \text{ N}$$

6.48 An iron-chromium-nickel alloy is to withstand a load of 6670 N at 760°C for 6 years. Calculate the minimum diameter of the bar. [See Figure 6.25(a)]

The operating time is (6 years)(365 days/year)(24 h/day) = 52,560 h

From the graph, the stress must be less than 50 MPa. The minimum diameter of the bar is then

$$d = \sqrt{(4/\pi)(F/\sigma)} = \sqrt{(4/\pi)(6670 \text{ N}/50 \text{ MPa})} = 13 \text{ mm}$$

6.49 A 30 mm diameter bar of an iron-chromium-nickel alloy is to operate for 5 years under a load of 17.8 kN. What is the maximum operating temperature? [See Figure 6.25(a)]

The operating time is (5 years)(365 days/year)(24 h/day) = 43,800 h

The stress is $\sigma = F/A = 17,800 \text{ N} / (\pi/4)(30 \text{ mm})^2 = 25.2 \text{ MPa}$

From the figure, the temperature must be below 850°C in order for the bar to survive five years at 25.2 MPa.

6.50 A 25 mm x 50 mm ductile cast iron bar must operate for 9 years at 650°C. What is the maximum load that can be applied? [See Figure 6.25(b)]

The operating time is (9 year)(365 days/year)(24 h/day) = 78,840 h.

The temperature is $650 + 273 = 923 \text{ K}$

$$LM = (923/1000)[36 + 0.78 \ln(78,840)] = 41.35$$

From the graph, the stress must be no more than about 7 MPa. The load is then

$$F = \sigma A = (7 \text{ MPa}) (25 \text{ mm}) (50 \text{ mm}) = 8750 \text{ N}$$

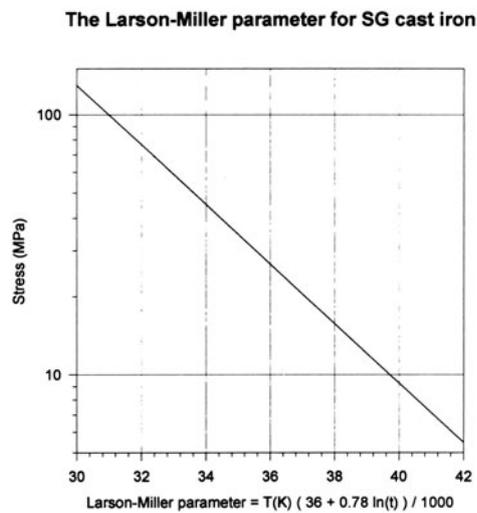
6.51 A ductile cast iron bar is to operate at a stress of 40 MPa for 1 year. What is the maximum allowable temperature? [See Figure 6.25(b)]

The operating time is (1 year) (365 days/year) (24 h/day) = 8760 h

From the graph, the Larson-Miller parameter must be 34.4 at a stress of 40 MPa. Thus

$$34.4 = (T / 1000) [36 + 0.78 \ln(8760)] = 0.043T$$

$$T = 800\text{K} = 527^\circ\text{C}$$



Chapter 7 Strain Hardening and Annealing

7.1 A 12.5 mm diameter metal bar with a 50 mm gauge length is subjected to a tensile test. The following measurements are made.

Force (kN)	Gauge length (mm)	Diameter (mm)
122	56.14	12.19
120	62.05	11.60
114	68.57	11.03

Determine the strain hardening coefficient for the metal. Is the metal most likely to be FCC, BCC, or HCP? Explain.

Force (N)	Gauge length (m)	Diameter (m)	True stress (MN.m ⁻²)	True strain
122000	56.14 x 10 ⁻³	12.19 x 10 ⁻³	1045	0.116
120000	62.05 x 10 ⁻³	11.60 x 10 ⁻³	1135	0.216
114000	68.57 x 10 ⁻³	11.03 x 10 ⁻³	1193	0.316

$$\sigma_t = K\epsilon_t^n \quad \text{or} \quad \ln \sigma_t = \ln k + n \ln \epsilon_t$$

$$\begin{aligned} \ln(1045 \times 10^6) &= \ln k + n \ln(0.116) = 20.7673 = \ln k - n (2.1542) \\ \ln(1193 \times 10^6) &= \ln k + n \ln(0.316) = \underline{20.8997} = \ln k - n (1.1520) \\ &\quad -0.1324 = -1.0022 n \end{aligned}$$

$$n = 0.13 \quad \text{which is in the range of BCC metals}$$

7.2 A 15 mm diameter metal bar with a 30 mm gauge length is subjected to a tensile test. The following measurements are made.

Force (N)	Gauge length (mm)	Diameter (mm)
16,240	36.642	12.028
19,066	44.754	10.884
19,273	54.663	9.848

Determine the strain hardening coefficient for the metal. Is the metal most likely to be FCC, BCC, or HCP? Explain.

Force (N)	Gauge length (m)	Diameter (m)	True stress (MN.m ⁻²)	True strain
16240	36.642 x 10 ⁻³	12.028 x 10 ⁻³	143	0.200
19066	44.754 x 10 ⁻³	10.884 x 10 ⁻³	205	0.400
19273	54.663 x 10 ⁻³	9.848 x 10 ⁻³	253	0.600

$$\sigma_t = K\epsilon_t^n \quad \text{or} \quad \ln \sigma_t = \ln K + n \ln \epsilon_t$$

$$\begin{aligned} \ln (143 \times 10^6) &= \ln K + n \ln 0.2 = 18.7783 = \ln K - n (1.6094) \\ \ln (253 \times 10^6) &= \ln K + n \ln 0.6 = \underline{19.3489} = \ln K - n (0.5108) \\ &\quad -0.5706 = -1.0986 n \end{aligned}$$

$$n = 0.52$$

A strain hardening coefficient of 0.52 is typical of FCC metals.

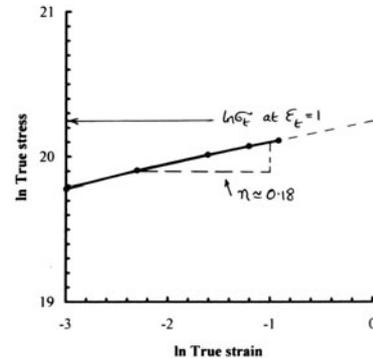
7.3 A true stress-true strain curve is shown in Figure 7.21. Determine the strain hardening coefficient for the metal.

$$\sigma_t = K\epsilon_t^n$$

ϵ_t	σ_t
0.05	390 MN.m ⁻²
0.10	440 MN.m ⁻²
0.20	490 MN.m ⁻²
0.30	520 MN.m ⁻²
0.40	540 MN.m ⁻²

From graph: $K = 623 \text{ MN.m}^{-2}$

$$n = 0.18$$



7.4 A Cu-30% Zn alloy bar has a strain hardening coefficient of 0.50. The bar, which has an initial diameter of 10 mm and an initial gauge length of 30 mm, fails at an engineering stress of 120 MN.m⁻². After fracture, the gauge length is 35 mm and the diameter is 9.26 mm. No necking occurred. Calculate the true stress when the true strain is 0.05.

$$\epsilon_t = \ln(l_f/l_0) = \ln(35 \times 10^{-3}/30 \times 10^{-3}) = 0.154$$

$$\sigma_E = 120 \text{ MN.m}^{-2} = \frac{F}{(\pi/4)(10 \times 10^{-3})^2}$$

$$F = 9425 \text{ N}$$

$$\sigma_t = \frac{9425 \text{ N}}{(\pi/4)(9.26 \times 10^{-3} \text{ m})^2} = 139.95 \text{ MN.m}^{-2}$$

$$\sigma_t = K(0.154)^{0.5} = 139.95 \text{ MN.m}^{-2} \quad \text{or} \quad K = 356.6 \text{ MN.m}^{-2}$$

The true stress at $\epsilon_t = 0.05$ is:

$$\sigma_t = 356.6 (0.05)^{0.5} \quad \text{or} \quad \sigma_t = 79.7 \text{ MN.m}^{-2}$$

7.5 The Frank-Read source shown in Figure 7.3(e) has created four dislocation loops from the original dislocation line. Estimate the total dislocation line present in the photograph and determine the percent increase in the length of dislocations produced by the deformation.

If the length of the original dislocation line is 1 mm on the photograph, then we can estimate the circumference of the dislocation loops. The loops are not perfect circles, so we might measure the smallest and largest diameters, then use the average:

$$\text{first loop: } D_{\text{small}} = 10 \text{ mm}, D_{\text{large}} = 14 \text{ mm}; D_{\text{avg}} = 12 \text{ mm} \\ \text{circumference} = 12.0\pi$$

$$\text{second loop: } D_{\text{small}} = 18 \text{ mm}; D_{\text{large}} = 20 \text{ mm}; D_{\text{avg}} = 19 \text{ mm} \\ \text{circumference} = 19.0\pi$$

$$\text{third loop: } D_{\text{small}} = 28 \text{ mm}; D_{\text{large}} = 30 \text{ mm}; D_{\text{avg}} = 29 \text{ mm} \\ \text{circumference} = 29.0\pi$$

$$\text{fourth loop: } D_{\text{small}} = 42 \text{ mm}; D_{\text{large}} = 45 \text{ mm}; D_{\text{avg}} = 43.5 \text{ mm} \\ \text{circumference} = 43.5\pi$$

Therefore in the photograph itself:

$$\text{total length} = 1 + (12.0 + 19.0 + 29.0 + 43.5)\pi = 326 \text{ mm}$$

The magnification in the photograph is 30,000. Therefore:

$$\text{total length} = 326 / 30,000 = 0.0109 \text{ mm}$$

The original dislocation line is $1 \text{ mm} / 30,000 = 3.33 \times 10^{-5} \text{ mm}$

$$\% \text{ increase} = (0.0109 - 0.0000333) / (3.33 \times 10^{-5} \text{ mm}) \times 100 = 32,630\%$$

7.6 A 6 mm thick copper plate is to be cold worked 63%. Find the final thickness.

$$63 = \frac{6 - t_f}{6} \times 100\% \quad \text{or} \quad t_f = 2.22 \text{ mm}$$

7.7 A 6 mm diameter copper bar is to be cold worked 63%. Find the final diameter.

$$63 = \frac{(6)^2 - d_f^2}{(6)^2} \times 100\% \quad \text{or} \quad d_f^2 = 13.32 \quad \text{or} \quad d_f = 3.65 \text{ mm}$$

7.8 A 50 mm diameter copper rod is reduced to 37.5 mm diameter, then reduced again to a final diameter of 25 mm. In a second case, the 50 mm diameter rod is reduced in one step from 50 mm to 25 mm diameter. Calculate the %CW for both cases.

$$\% \text{ CW} = \frac{(50)^2 - (25)^2}{(50)^2} \times 100 = 75\% \text{ in both cases}$$

7.9 A 3105 aluminium plate is reduced from 45 mm to 30 mm. Determine the final properties of the plate. (See Figure 7.22)

$$\% \text{ CW} = \frac{45 - 30}{45} \times 100\% = 33.3\%$$

$$TS = 162 \text{ MN.m}^{-2} \quad YS = 137 \text{ MN.m}^{-2} \quad \%E = 5\%$$

7.10 A Cu-30% Zn brass bar is reduced from 25 mm diameter to a 10 mm diameter. Determine the final properties of the bar. (See Figure 7.23.)

$$CW = \frac{(25)^2 - (10)^2}{(25)^2} \times 100 = 84\%$$

$$TS = 725 \text{ MN.m}^{-2} \quad YS = 470 \text{ MN.m}^{-2} \quad \%E = 1\%$$

(Ts value obtained by extrapolating plot.)

7.11 A 3105 aluminium bar is reduced from a 25 mm diameter, to a 20 mm diameter, to a 15 mm diameter, to a final 10 mm diameter. Determine the %CW and the properties after each step of the process. Calculate the total percent cold work. (See Figure 7.22.)

If we calculated the percent deformation in each step separately, we would find that 36% deformation is required to go from 25 mm to 20 mm. The deformation from 20 mm to 15 mm (using 20 mm as the initial

diameter) is 43.75%, and the deformation from 15 mm to 10 mm (using 15 mm as the initial diameter) is 55.6%. If we added these three deformations, the total would be 135.35%. This would not be correct. Instead, we must always use the original 25 mm diameter as our starting point. The following table summarises the actual deformation and properties after each step.

	$\frac{TS}{MN.m^{-2}}$	$\frac{YS}{MN.m^{-2}}$	%E
$\frac{(25)^2 - (20)^2}{(25)^2} \times 100 = 36\%$	164	140	5
$\frac{(25)^2 - (15)^2}{(25)^2} \times 100 = 64\%$	185	168	3
$\frac{(25)^2 - (10)^2}{(25)^2} \times 100 = 84\%$	200	185	2

The total percent cold work is actually 84%, not the 135.35%.

7.12 We want a copper bar to have a tensile strength of at least 480 MN.m⁻² and a final diameter of 9.5 mm. What is the minimum diameter of the original bar? (See Figure 7.6.)

%CW ≥ 50% to achieve the minimum tensile strength

$$50 = \frac{d_o^2 - (9.5)^2}{d_o^2} \times 100$$

$$d_o^2 = 180.5 \quad \text{or} \quad d_o = 13.4 \text{ mm}$$

7.13 We want a Cu-30% Zn brass plate originally 30 mm thick to have a yield strength greater than 345 MN.m⁻² and a %Elongation of at least 10%. What range of final thicknesses must be obtained? (See Figure 7.23.)

YS > 345 MN.m⁻² requires CW > 20%
 %E > 10% requires CW < 35%

$$\frac{30 - t_f}{30} = 0.20 \quad \frac{30 - t_f}{30} = 0.35$$

$$t_f = 24 \text{ mm} \quad t_f = 19.5 \text{ mm}$$

$$t_f = 19.5 \text{ to } 24 \text{ mm}$$

7.14 We want a copper sheet to have at least 345 MN.m⁻² yield strength and at least 10% Elongation, with a final thickness of 30 mm. What range of original thicknesses must be used? (See Figure 7.6.)

YS > 345 MN.m⁻² requires CW ≥ 30%
 %E > 10% requires CW ≤ 30%

$$\frac{t_o - 30}{t_o} = 0.30$$

$$t_o = 42.8 \text{ mm}$$

Result shows that the yield strength and % Elongation requirements necessitate 30% CW.

7.15 A 3105 aluminium plate previously cold worked 20% is 50 mm thick. It is then cold worked further to 33 mm. Calculate the total percent cold work and determine the final properties of the plate. (See Figure 7.22.)

The original thickness (before the 20% cold work) must have been:

$$\frac{t_o - 50}{t_o} = 0.20 \quad t_o = 62.5 \text{ mm}$$

The total cold work is then based on the prior 62.5 mm thickness:

$$CW = \frac{62.5 - 33}{62.5} \times 100\% = 47\% \quad \begin{array}{l} TS = 175 \text{ MN.m}^{-2} \\ YS = 152 \text{ MN.m}^{-2} \\ \%E = 4\% \end{array}$$

7.16 An aluminium-lithium strap 6 mm thick and 50 mm wide is to be cut from a rolled sheet, as described in Figure 7.9. The strap must be able to support a 150 kN load without plastic deformation. Determine the range of orientations from which the strap can be cut from the rolled sheet.

$$\sigma = \frac{150 \times 10^3}{(6 \times 10^{-3})(50 \times 10^{-3})} \geq 500 \text{ MN.m}^{-2}$$

The properties can be obtained at angles of 0 to 4° from the rolling direction of the sheet.

Note: if load to be supported was 145 kN then properties can be obtained at angles of 0° to about 20° from the rolling direction.

7.17 We want to draw a 7.5 mm diameter copper wire having a yield strength of 140 MN.m⁻² into a 6 mm diameter wire. (a) Find the draw force, assuming no friction. (b) Will the drawn wire break during the drawing process? Show. (See Figure 7.6)

(a) Before drawing (0% CW), the yield strength is 140 MN.m⁻²

$$CW = \frac{(7.5)^2 - (6)^2}{(7.5)^2} = 36\% \quad \begin{array}{l} \text{which gives } YS = 375 \text{ MN.m}^{-2} \\ \text{in the drawn wire} \end{array}$$

(b) The force needed to draw the original wire is :

$$140 \text{ MN.m}^{-2} = F / (\pi/4) (7.5 \times 10^{-3})^2 \quad \text{or} \quad F = 6185 \text{ N}$$

(c) The stress acting on the drawn wire is:

$$\sigma = 6185 / (\pi/4) (6 \times 10^{-3})^2 = 219 \text{ MN.m}^{-2} < 375 \text{ MN.m}^{-2}$$

Since the actual stress (219 MN.m⁻²) acting on the drawn wire is less than the yield strength (375 MN.m⁻²) of the drawn wire, the wire will not break during manufacturing.

7.18 A 3105 aluminium wire is to be drawn to give a 1-mm diameter wire having a yield strength of 140 MN.m⁻². (a) Find the original diameter of the wire, (b) calculate the draw force required, and (c) determine whether the as-drawn wire will break during the process. (See Figure 7.22)

(a) We need to cold work 35% to obtain the required yield strength:

$$\frac{d_o^2 - 1^2}{d_o^2} = 0.35 \quad d_o = \sqrt{1/0.65} = 1.24 \text{ mm}$$

(b) The initial yield strength of the wire (with 0% cold work) is 50 MN.m^{-2} , so the force required to deform the initial wire is:

$$F = 50 \times 10^6 [(\pi/4) (1.24 \times 10^{-3})^2] = 60 \text{ N}$$

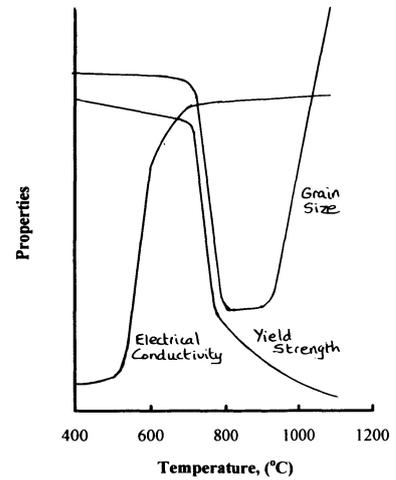
(c) The stress acting on the drawn wire (which has a smaller diameter but is subjected to the same drawing force) is:

$$\sigma = \frac{60 \text{ N}}{(\pi/4) (1 \times 10^{-3} \text{ m})^2} = 76.4 \text{ MN.m}^{-2}$$

Since the actual stress is less than the 140 MN.m^{-2} yield strength of the drawn wire, the process will be successful and the wire will not break.

7.19 The following data were obtained when a cold worked metal was annealed. (a) Estimate the recovery, recrystallisation, and grain growth temperatures. (b) Recommend a suitable temperature for a stress relief heat treatment. (c) Recommend a suitable temperature for a hot-working process. (d) Estimate the melting temperature of the alloy.

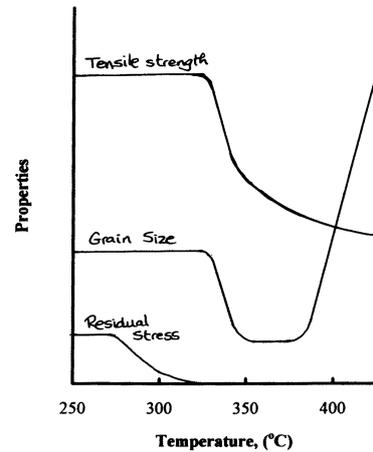
Annealing Temperature (°C)	Electrical Conductivity ($\text{ohm}^{-1} \cdot \text{m}^{-1}$)	Yield Strength (MN.m^{-2})	Grain Size (mm)
400	3.04×10^7	86	0.10
500	3.05×10^7	85	0.10
600	3.36×10^7	84	0.10
700	3.45×10^7	83	0.098
800	3.46×10^7	52	0.030
900	3.46×10^7	47	0.031
1000	3.47×10^7	44	0.070
1100	3.47×10^7	42	0.120



- (a) Electrical conductivity begins to increase between 500 and 600°C ; yield strength and grain size both decrease between 700 and 800°C ; grain size begins to increase between 900 and 1000°C .
Therefore
recovery temperature $\approx 500^\circ\text{C}$
recrystallisation temperature $\approx 750^\circ\text{C}$
grain growth temperature $\approx 950^\circ\text{C}$
- (b) Stress relieve above recovery temperature but below recrystallisation temperature: 700°C
- (c) Hot work above recrystallisation temperature but below the grain growth temperature. A temperature of about 900°C might be appropriate.
- (d) Recrystallisation temperature $\approx 0.4 T_m$
 $\therefore 0.4 T_m \approx 1023 \text{ K}$
 $T_m \approx 2558 \text{ K} = 2285^\circ\text{C}$

7.20 The following data were obtained when a cold worked metal was annealed. (a) Estimate the recovery, recrystallisation, and grain growth temperatures. (b) Recommend a suitable temperature for obtaining a high strength, high electrical conductivity wire. (c) Recommend a suitable temperature for a hot-working process. (d) Estimate the melting temperature of the alloy.

Annealing Temperature (°C)	Residual Stresses (MN.m ⁻²)	Tensile Strength (MN.m ⁻²)	Grain Size (mm)
250	145	360	0.076
275	145	360	0.076
300	35	360	0.076
325	0	360	0.076
350	0	235	0.025
375	0	200	0.025
400	0	185	0.089
425	0	170	0.183



- (a) recovery temperature $\approx 280^{\circ}\text{C}$
 recrystallisation temperature $\approx 330^{\circ}\text{C}$
 grain growth temperature $\approx 380^{\circ}\text{C}$
- (b) For a high strength, high conductivity wire, we want to heat into the recovery range. A suitable temperature might be 320°C .
- (c) Hot working temperature = 375°C
- (d) $0.4 T_{mp} \approx 603 \text{ K}$
 $T_{mp} \approx 603 / 0.4 = 1508 \text{ K} = 1235^{\circ}\text{C}$

7.21 A titanium alloy contains a very fine dispersion of tiny Er_2O_3 particles. What will be the effect of these particles on the grain growth temperature and the size of the grains at any particular annealing temperature? Explain.

These particles, by helping pin the grain boundaries, will increase the grain growth temperature and decrease the grain size.

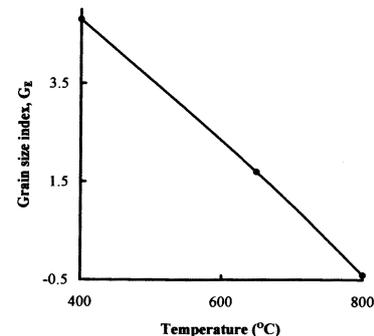
7.22 Determine the grain size Index G_B for each of the micrographs in Figure 7.17 and plot the grain size number versus the annealing temperature.

The approximate number of grains per square millimetre in each photomicrograph at 75x is:

400°C: 0.0403 grains/mm²
 so, $m = (75/1)^2 (0.0403) = 226.6875$
 $m = 2^{(G_B+3)}$
 $\log(226.6875) = (G_B + 3) \log(2)$
 $G_B = (2.3554 - 0.9031) / (0.301)$
 $G_B = 4.8$

650°C: 4.65×10^{-3} grains/mm²
 $m = (75/1)^2 (4.65 \times 10^{-3}) = 26.1563$
 $\log(26.1563) = (G_B + 3) \log(2)$
 $G_B = 1.7$

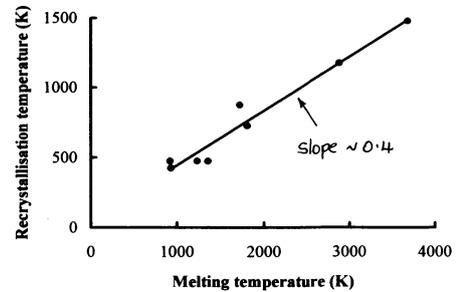
800°C: 1.085×10^{-3} grains/mm²
 $m = (75/1)^2 (1.085 \times 10^{-3}) = 6.1031$
 $\log(6.1031) = (G_B + 3) \log(2)$
 $G_B = -0.4$



7.23 Using the data in Table 7.3, plot the recrystallisation temperature versus the melting temperature of each metal, using absolute temperatures (Kelvin). Measure the slope and compare with the expected relationship between these two temperatures. Is our approximation a good one?

Converting the recrystallisation and melting temperatures to Kelvin, we can obtain the graph shown. The relationship of $T_r = 0.4T_m$ (K) is very closely followed.

	T_m	T_r
Al	933 K	423 K
Mg	923 K	473 K
Ag	1235 K	473 K
Cu	1358 K	473 K
Fe	1811 K	723 K
Ni	1726 K	873 K
Mo	2883 K	1173 K
W	3683 K	1473 K



7.24 We wish to produce a 7.5 mm thick plate of 3105 aluminium having a tensile strength of at least 170 MN.m^{-2} and a %Elongation of at least 5%. The original thickness of the plate is 75 mm. The maximum cold work in each step is 80%. Describe the cold working and annealing steps required to make this product. Compare this process with that you would recommend if you could do the initial deformation by hot working. (See Figure 7.22).

For $TS \geq 170 \text{ MN.m}^{-2}$ $CW \geq 40\%$; For $\%E \geq 5\%$ $CW \leq 40\%$
 \therefore required $CW = 40\%$

$$\frac{t_i - 7.5}{t_i} = 0.40$$

$$\text{or } t_i = 12.5 \text{ mm}$$

Cold Work/anneal treatment

CW 70% from 75 mm to 22.5 mm
 anneal
 CW 44.4% from 22.5mm to 12.5 mm
 anneal
 CW 40% from 12.5mm to 7.5 mm

Hot work treatment

HW 83.3% from 75 mm to 12.5 mm
 CW 40% from 12.5 mm to 7.5 mm

7.25 We wish to produce a 5 mm diameter wire of copper having a minimum yield strength of 400 MN.m^{-2} and a minimum %Elongation of 5%. The original diameter of the rod is 50 mm and the maximum cold work in each step is 80%. Describe the cold working and annealing steps required to make this product. Compare this process with that you would recommend if you could do the initial deformation by hot working. (See Figure 7.6).

For $YS > 400 \text{ MN.m}^{-2}$ $CW \geq 40\%$; For $\%E > 5$ $CW \geq 44\%$
 \therefore pick $CW = 42\%$, the middle of the allowable range

$$\frac{d_i^2 - (5)^2}{d_i^2} = 0.42 \quad \text{or} \quad d_i = \sqrt{25/0.58} = 6.56 \text{ mm}$$

Cold Work/anneal treatment

CW 75% from 50 to 25 mm-diameter
 anneal
 CW 75% from 25 to 12.5 mm
 anneal
 CW 72.5% from 12.5 to 6.56 mm
 anneal
 CW 42% from 6.56 to 5 mm

Hot work treatment

HW 98.3% from 50 to 6.56 mm
 CW 42% from 6.56 to 5 mm

Chapter 8 Principles of Solidification Strengthening and Processing

8.1 Suppose that liquid nickel is undercooled until homogeneous nucleation occurs. Calculate (a) the critical radius of the nucleus required, and (b) the number of nickel atoms in the nucleus. Assume that the lattice parameter of the solid FCC nickel is 0.356 nm.

From Table 8.1, $\Delta T = 480^\circ\text{C}$

$$r^* = \frac{2\sigma T_m}{\Delta H_f \Delta T} = \frac{(2)(255 \times 10^{-3} \text{ J.m}^{-2})(1453 + 273)}{(2756 \times 10^6 \text{ J.m}^{-3})(480)} = 6.65 \times 10^{-10} \text{ m}$$

$$a_o = 0.356 \text{ nm}$$

$$V_{uc} = 4.5118 \times 10^{-29} \text{ m}^3$$

$$V_{nucleus} = (4\pi/3)(6.65 \times 10^{-10} \text{ m})^3 = 1.232 \times 10^{-27} \text{ m}^3$$

$$\text{number of unit cells/nucleus} = \frac{1.232 \times 10^{-27}}{4.5118 \times 10^{-29}} = 27.3$$

$$\text{atoms per nucleus} = (4 \text{ atoms/cell})(27.3 \text{ cells}) = 109 \text{ atoms}$$

8.2 Suppose that liquid iron is undercooled until homogeneous nucleation occurs. Calculate (a) the critical radius of the nucleus required, and (b) the number of iron atoms in the nucleus. Assume that the lattice parameter of the solid BCC iron is 0.292 nm.

$$r^* = \frac{2\sigma T_m}{\Delta H_f \Delta T} = \frac{(2)(204 \times 10^{-3} \text{ J.m}^{-2})(1538 + 273)}{(1737 \times 10^6 \text{ J.m}^{-3})(420)} = 1.013 \times 10^{-9} \text{ m}$$

$$V_{nucleus} = (4\pi/3)(10.013 \times 10^{-9})^3 = 4.352 \times 10^{-27} \text{ m}^3$$

$$V_{uc} = (2.92 \times 10^{-10})^3 = 2.490 \times 10^{-29} \text{ m}^3$$

$$\text{number of unit cells/nucleus} = \frac{4.352 \times 10^{-27}}{2.490 \times 10^{-29}} = 175$$

$$\text{atoms per nucleus} = (175 \text{ cells})(2 \text{ atoms/cell}) = 350 \text{ atoms}$$

8.3 Suppose that solid nickel was able to nucleate homogeneously with an undercooling of only 22°C . How many atoms would have to group together spontaneously for this to occur? Assume that the lattice parameter of the solid FCC nickel is 0.356 nm.

$$r^* = \frac{2\sigma T_m}{\Delta H_f \Delta T} = \frac{(2)(255 \times 10^{-3} \text{ J.m}^{-2})(1453 + 273)}{(2756 \times 10^6 \text{ J.m}^{-3})(22)} = 1.452 \times 10^{-8} \text{ m}$$

$$V_{uc} = 4.5118 \times 10^{-29} \text{ m}^3 \text{ (see Problem 8.1)}$$

$$V_{nuc} = (4\pi/3)(1.452 \times 10^{-8} \text{ m})^3 = 1.282 \times 10^{-23} \text{ m}^3$$

$$\text{number of unit cells/nucleus} = 1.282 \times 10^{-23} / 4.5118 \times 10^{-29} = 2.84 \times 10^5$$

$$\text{atoms per nucleus} = (4 \text{ atoms/cells})(2.84 \times 10^5 \text{ cell}) = 1.136 \times 10^6$$

8.4 Suppose that solid iron was able to nucleate homogeneously with an undercooling of only 15°C. How many atoms would have to group together spontaneously for this to occur? Assume that the lattice parameter of the solid BCC iron is 0.292 nm.

$$r^* = \frac{2\sigma T_m}{\Delta H_f \Delta T} = \frac{(2)(204 \times 10^{-3} \text{ J.m}^{-2})(1538 + 273)}{(1737 \times 10^6 \text{ J.m}^{-3})(15)} = 2.836 \times 10^{-8} \text{ m}$$

$$V_{uc} = 2.490 \times 10^{-29} \text{ m}^3 \quad (\text{see Problem 8.2})$$

$$V_{nuc} = (4\pi/3)(2.836 \times 10^{-8} \text{ m})^3 = 9.554 \times 10^{-23} \text{ m}^3$$

$$\text{number of unit cells/nucleus} = \frac{9.554 \times 10^{-23}}{2.490 \times 10^{-29}} = 3.837 \times 10^6$$

$$\text{atoms per nucleus} = (2 \text{ atoms/cells})(3.837 \times 10^6 \text{ cell}) = 7.674 \times 10^6$$

8.5 Calculate the fraction of solidification that occurs dendritically when iron nucleates (a) at 10°C undercooling, (b) at 100°C undercooling, and (c) homogeneously. The specific heat of iron is $5.78 \times 10^6 \text{ J.m}^{-3} \cdot \text{°C}^{-1}$.

$$f = \frac{c\Delta T}{\Delta H_f} = \frac{(5.78 \times 10^6 \text{ J.m}^{-3} \cdot \text{°C}^{-1})(10\text{°C})}{1737 \times 10^6 \text{ J.m}^{-3}} = 0.0333$$

$$\frac{c\Delta T}{\Delta H_f} = \frac{(5.78 \times 10^6 \text{ J.m}^{-3} \cdot \text{°C}^{-1})(100\text{°C})}{1737 \times 10^6 \text{ J.m}^{-3}} = 0.333$$

$$\frac{c\Delta T}{\Delta H_f} = \frac{(5.78 \times 10^6 \text{ J.m}^{-3} \cdot \text{°C}^{-1})(420\text{°C})}{1737 \times 10^6 \text{ J.m}^{-3}} = \text{ALL DENDRITICALLY}$$

8.6 Calculate the fraction of solidification that occurs dendritically when silver nucleates (a) at 10°C undercooling, (b) at 100°C undercooling, and (c) homogeneously. The specific heat of silver is $3.25 \times 10^6 \text{ J.m}^{-3} \cdot \text{°C}^{-1}$.

$$f = \frac{c\Delta T}{\Delta H_f} = \frac{(3.25 \times 10^6 \text{ J.m}^{-3} \cdot \text{°C}^{-1})(10\text{°C})}{965 \times 10^6 \text{ J.m}^{-3}} = 0.0337$$

$$\frac{c\Delta T}{\Delta H_f} = \frac{(3.25 \times 10^6 \text{ J.m}^{-3} \cdot \text{°C}^{-1})(100\text{°C})}{965 \times 10^6 \text{ J.m}^{-3}} = 0.337$$

$$\frac{c\Delta T}{\Delta H_f} = \frac{(3.25 \times 10^6 \text{ J.m}^{-3} \cdot \text{°C}^{-1})(250\text{°C})}{965 \times 10^6 \text{ J.m}^{-3}} = 0.842$$

8.7 Analysis of a nickel casting suggests that 28% of the solidification process occurred in a dendritic manner. Calculate the temperature at which nucleation occurred. The specific heat of nickel is $4.1 \times 10^6 \text{ J.m}^{-3} \cdot \text{°C}^{-1}$.

$$f = \frac{c\Delta T}{\Delta H_f} = \frac{(4.1 \times 10^6 \text{ J.m}^{-3} \cdot \text{°C}^{-1})(\Delta T)}{2756 \times 10^6 \text{ J.m}^{-3}} = 0.28$$

$$(1.488 \times 10^{-3})(\Delta T) = 0.28$$

$$\Delta T = 188\text{°C} \quad \text{or} \quad T_n = 1453 - 188 = 1265\text{°C}$$

8.8 A 50 mm cube solidifies in 4.6 min. Calculate (a) the mould constant in Chvorinov's rule and (b) the solidification time for a 12 mm x 12 mm x 150 mm bar cast under the same conditions. Assume that $n = 2$.

a) We can find the volume and surface area of the cube:

$$V = (50)^3 = 1.25 \times 10^5 \text{ mm}^3 \quad A = 6(50)^2 = 1.5 \times 10^4 \text{ mm}^2 \quad t = B(V/A)^2$$

$$t = 4.6 = \frac{B(1.25 \times 10^5)^2}{(1.5 \times 10^4)^2}$$

$$B = 4.6 / (8.333)^2 = 0.066 \text{ min. mm}^{-2} = 3.97 \text{ s. mm}^{-2}$$

b) For the bar, assuming that $B = 3.97 \text{ s. mm}^{-2}$:

$$V = (12)(12)(150) = 2.16 \times 10^4 \text{ mm}^3$$

$$A = 2(12)(12) + 4(12)(150) = 7.488 \times 10^3 \text{ mm}^2$$

$$t = (3.97)(2.16 \times 10^4 / 7.488 \times 10^3)^2 = 33 \text{ s.}$$

8.9 A 50 mm diameter sphere solidifies in 1050 s. Calculate the solidification time for a 3 mm x 100 mm x 200 mm plate cast under the same conditions. Assume that $n = 2$.

$$t = B(V/A)^2$$

$$t = 1050 \text{ s} = B \frac{(4\pi/3)(25)^3}{4\pi(25)^2} = B[65450/7854]^2 \quad \text{or} \quad B = 15.12 \text{ s. mm}^{-2}$$

$$t = \frac{(15.12)(3 \times 100 \times 200)^2}{[2(3)(200) + 2(3)(100) + 2(100)(200)]^2}$$

$$= 15.12[6 \times 10^4 / 4.18 \times 10^4]^2 = 31.15 \text{ s}$$

8.10 Find the constants B and n in Chvorinov's rule by plotting the following data on a log-log plot:

Casting dimensions (mm)	Solidification time (min)	time (s)
12 x 200 x 300	1.03	61.8
50 x 75 x 250	4.53	271.8
60 cube	2.80	168.0
25 x 100 x 225	2.43	145.8

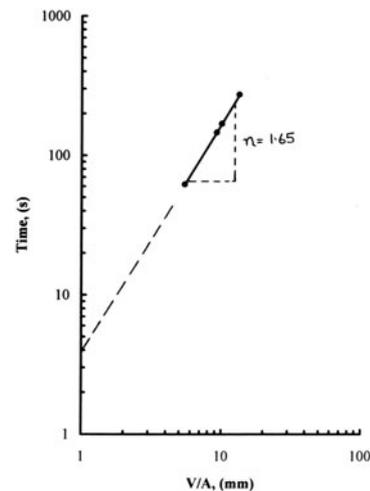
Volume (mm ³)	Area (mm ²)	V/A (mm)
720000	132000	5.454
937500	70000	13.393
216000	21600	10.000
562500	61250	9.184

$$n = \text{gradient} = \frac{2.4343 - 1.7910}{1.1269 - 0.7367} = 1.65$$

$$\log t = \log B \text{ when } \log V/A = 0$$

$$\therefore \log B = 0.6$$

$$\therefore B = 4.0 \text{ s. mm}^{-2}$$

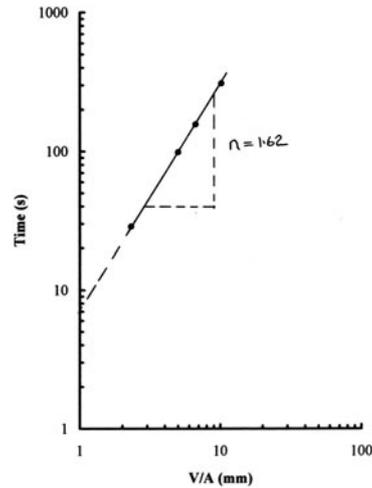


8.11 Find the constants B and n in Chvorinov's rule by plotting the following data on a log-log plot.

Casting dimensions (cm)	Solidification time (s)
10 x 10 x 60	28.58
20 x 40 x 40	98.30
40 x 40 x 40	155.89
80 x 60 x 50	306.15

V (mm ³)	A (mm ²)	V/A (mm)
6000	2600	2.31
32000	6400	5.00
64000	9600	6.67
240000	23600	10.17

From the graph, we find that
 $B = 7.2 \text{ s} \cdot \text{mm}^{-2}$ and $n = 1.62$



8.12 A 75 mm diameter casting was produced. The times required for the solid-liquid interface to reach different distances beneath the casting surface were measured and are shown in the following table.

Distance from surface (mm)	Time (s)	\sqrt{t}
2.5	32.6	5.71
7.5	73.5	8.57
12.5	130.6	11.43
20.0	225.0	15.00
25.0	334.9	18.30

Determine (a) the time at which solidification begins at the surface and (b) the time at which the entire casting is expected to be solid. (c) Suppose the centre of the casting actually solidified in 720 s. Explain why this time might differ from the time calculated in part b.

We could plot d versus \sqrt{t} , as shown, finding $\sqrt{t}_{\text{surface}}$ from where the plot intersects the x-axis and \sqrt{t}_{centre} where the plot intersects $d = 37.5$ mm. Or we could take two of the data points and solve for c and k .

$$d = (k\sqrt{t}) - c$$

$$2.5 = (k\sqrt{32.6}) - c$$

$$12.5 = (k\sqrt{130.6}) - c$$

$$-10.0 = k[\sqrt{32.6} - \sqrt{130.6}] = -5.718 k$$

$$k = 1.75$$

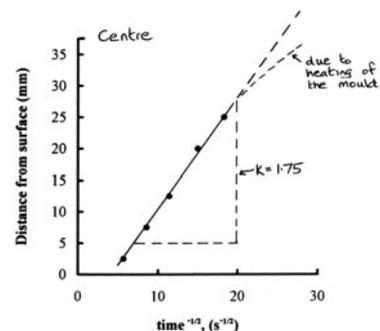
$$c = (1.75\sqrt{32.6}) - 2.5 = 7.48$$

a) $d = 0 = (1.75\sqrt{t}) - 7.48$

$$t_{\text{surface}} = (7.48/1.75)^2 = 18.4 \text{ s}$$

b) $37.5 = (1.75\sqrt{t}) - 7.48$

$$t_{\text{centre}} = (44.98/1.75)^2 = 661 \text{ s}$$



c) The mould gets hot during the solidification process, and consequently heat is extracted from the casting more slowly. This in turn changes the constants in the equation and increases the time required for complete solidification.

8.13 Figure 8.6(b) shows a photograph of an aluminium alloy. Estimate (a) the secondary dendrite arm spacing and (b) the local solidification time for that area of the casting.

(a) The distance between adjacent dendrite arms can be measured. Although most people doing these measurements will arrive at slightly different numbers, the author's calculations obtained from four different primary arms are:

$$\begin{aligned} 16 \text{ mm} / 6 \text{ arms} &= 2.67 \text{ mm} \\ 9 \text{ mm} / 5 \text{ arms} &= 1.80 \text{ mm} \\ 13 \text{ mm} / 7 \text{ arms} &= 1.85 \text{ mm} \\ 18 \text{ mm} / 9 \text{ arms} &= 2.00 \text{ mm} \\ \text{average} &= 2.08 \text{ mm} \end{aligned}$$

Dividing by the magnification of x50:

$$SDAS = 2.08 \text{ mm} / 50 = 4.16 \times 10^{-2} \text{ mm}$$

(b) From Figure 8.7, we find that LST = 90 s

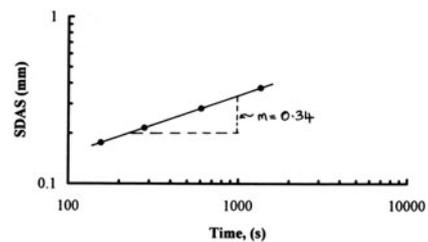
8.14 Figure 8.22 shows a photograph of FeO dendrites that have precipitated from a ceramic glass (an undercooled liquid). Estimate the secondary dendrite arm spacing.

We can find 13 SDAS along a 35 mm distance on the photomicrograph. The magnification of the photomicrograph is x450, while we want the actual length (at magnification x 1). Thus:

$$SDAS = (13 \text{ SDAS} / 35 \text{ mm}) (1/450) = 8.25 \times 10^{-4} \text{ mm}$$

8.15 Find the constants K and m relating the secondary dendrite arm spacing to the local solidification time by plotting the following data on a log-log plot:

Solidification Time (s)	SDAS (mm)
156	0.176
282	0.216
606	0.282
1356	0.374



From the slope of the graph:

$$m = 34/100 = 0.34$$

We can then pick a point off the graph (say SDAS = 0.225 mm when LST = 300 s) and calculate "K":

$$SDAS = Kt_s^m$$

$$0.225 = K(300)^{0.34} = 6.954 K$$

$$K = \frac{0.225}{6.954} = 0.032 \text{ s.}$$

8.16 Figure 8.23 shows dendrites in a titanium powder particle that has been rapidly solidified. Assuming that the size of the titanium dendrites is related to solidification time by the same relationship as in aluminium, estimate the solidification time of the powder particle.

The secondary dendrite arm spacing can be estimated from the photomicrograph at several locations. The author's calculations, derived from measurements at three locations, are

$$11 \text{ mm} / 8 \text{ arms} = 1.375 \text{ mm}$$

$$13 \text{ mm} / 8 \text{ arms} = 1.625 \text{ mm}$$

$$13 \text{ mm} / 8 \text{ arms} = 1.625 \text{ mm}$$

$$\text{average} = 1.540 \text{ mm}$$

Dividing by the magnification of 2200:

$$SDAS = (1.540 \text{ mm}) (1/2200) = 7 \times 10^{-4} \text{ mm}$$

The relationship between SDAS and solidification time for aluminium is (from Example 8.3):

$$SDAS = R t_s^m = 8 \times 10^{-3} t^{0.42} = 7 \times 10^{-4}$$

$$t^{0.42} = 0.0875$$

$$t = (0.0875)^{1/0.42} = 0.003 \text{ s}$$

8.17 The secondary dendrite arm spacing in an electron beam weld of copper is 9.5×10^{-3} mm. Estimate the solidification time of the weld.

From Figure 8.7, we can determine the equation relating SDAS and solidification time for copper:

$$m = 20/50 = 0.40 \quad R = 4 \times 10^{-2} \text{ mm}$$

Then for the copper weld:

$$9.5 \times 10^{-3} = 4 \times 10^{-2} (LST)^{0.40}$$

$$0.2375 = (LST)^{0.38} \quad \therefore -0.6243 = 0.40 \log(LST)$$

$$\log(LST) = -1.5607 \quad \therefore LST = 0.027 \text{ s}$$

8.18 A cooling curve is shown in Figure 8.24. Determine (a) the pouring temperature, (b) the solidification temperature, (c) the superheat, (d) the cooling rate just before solidification begins, (e) the total solidification time, (f) the local solidification time, and (g) the probable identity of the metal. (h) If the cooling curve was obtained at the centre of the casting sketched in the figure, determine the mould constant, assuming that $n = 2$.

$$(a) T_{\text{pour}} = 475^\circ\text{C}$$

$$(e) t_s = 470 \text{ s}$$

$$(b) T_{\text{sol}} = 320^\circ\text{C}$$

$$(f) LST = 470 - 130 = 340 \text{ s}$$

$$(c) \Delta T_s = 475 - 320 = 155^\circ\text{C}$$

$$(g) \text{Cd}$$

$$(d) \quad \Delta T/\Delta t = \frac{475 - 320}{130 - 0} = 1.2 \text{ } ^\circ\text{C/s} \quad (h) \quad t_s = B(V/A)^2$$

$$470 = B[60000/76000]^2$$

$$B = 7.54 \text{ s.mm}^{-2}$$

8.19 A cooling curve is shown in Figure 8.25. Determine (a) the pouring temperature, (b) the solidification temperature, (c) the superheat, (d) the cooling rate just before solidification begins, (e) the total solidification time, (f) the local solidification time, (g) the undercooling, and (h) the probable identity of the metal. (i) If the cooling curve was obtained at the centre of the casting sketched in the figure, determine the mould constant, assuming $n = 2$.

$$(a) \quad T_{\text{pour}} = 900^\circ\text{C} \quad (e) \quad t_s = 9.7 \text{ min}$$

$$(b) \quad T_{\text{sol}} = 420^\circ\text{C} \quad (f) \quad LST = 9.7 - 2.25 = 7.45 \text{ min}$$

$$(c) \quad \Delta T_s = 900 - 420 = 480^\circ\text{C} \quad (g) \quad 420 - 360 = 60^\circ\text{C}$$

$$(d) \quad \Delta T/\Delta t = \frac{900 - 400}{1.6 - 0} = 312 \text{ } ^\circ\text{C/min} \quad (h) \quad \text{Zn}$$

$$(i) \quad t_s = B[V/A]^2$$

$$\therefore 9.7 \times 60 = B[125000/15000]^2$$

$$\therefore B = 8.38 \text{ s.mm}^{-2}$$

8.20 Figure 8.26 shows the cooling curves obtained from several locations within a cylindrical aluminium casting. Determine the local solidification times and the SDAS at each location, then plot the tensile strength versus distance from the casting surface. Would you recommend that the casting be designed so that a large or small amount of material must be machined from the surface during finishing? Explain.

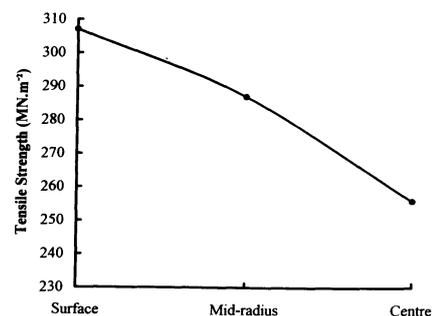
The local solidification times (LST) can be found from the cooling curves and can be used to find the expected SDAS values from Figure 8.7. The SDAS values can then be used to find the tensile strength, using Figure 8.8.

$$\text{Surface: } LST = 10 \text{ s} \Rightarrow SDAS = 1.5 \times 10^{-2} \text{ mm} \Rightarrow TS = 307 \text{ MN.m}^{-2}$$

$$\text{Midradius: } LST = 100 \text{ s} \Rightarrow SDAS = 5 \times 10^{-2} \text{ mm} \Rightarrow TS = 287 \text{ MN.m}^{-2}$$

$$\text{Centre: } LST = 500 \text{ s} \Rightarrow SDAS = 10 \times 10^{-2} \text{ mm} \Rightarrow TS = 256 \text{ MN.m}^{-2}$$

You prefer to machine as little material off the surface of the casting as possible; the surface material has the finest structure and highest strength; any excessive machining simply removes the "best" material.



8.21 Calculate the volume, diameter, and height of the cylindrical riser required to prevent shrinkage in a 100 mm x 250 mm x 500 mm casting if the H/D of the riser is 1.5.

$$V_c = 100 \times 250 \times 500 = 1.25 \times 10^7 \text{ mm}^3$$

$$A_c = (2)(100)(250) + (2)(100)(500) + (2)(250)(500) = 4 \times 10^5 \text{ mm}^2$$

$$V_R = (\pi/4)D^2H = 0.375\pi D^3$$

$$A_R = 2(\pi/4)D^2 + \pi DH = 2\pi D^2$$

$$(V/A)_R \geq (V/A)_c$$

$$\therefore \frac{0.375\pi D^3}{2\pi D^2} \geq \frac{1.25 \times 10^7}{4 \times 10^5}$$

$$0.1875D \geq 31.25$$

$$\text{i.e. } D \geq 166.7 \text{ mm, } H = 250 \text{ mm, } V = 5456336 \text{ mm}^3$$

8.22 Calculate the volume, diameter, and height of the cylindrical riser required to prevent shrinkage in a 25 mm x 150 mm x 150 mm casting if the H/D of the riser is 1.0.

$$V_c = 25 \times 150 \times 150 = 562500 \text{ mm}^3$$

$$A_c = 4(25)(150) + 2(150)(150) = 60000 \text{ mm}^2$$

$$V_R = (\pi/4)D^2H = (\pi/4)D^3$$

$$A_R = 2(\pi/4)D^2 + \pi DH = (\pi/2)D^2 + \pi D^2 \\ = 1.5\pi D^2$$

$$\text{Need } (V/A)_R \geq (V/A)_c$$

$$\frac{0.25\pi D^3}{1.5\pi D^2} \geq \frac{562500}{60000}$$

$$0.1667D \geq 79.37$$

$$D \geq 56.2 \text{ mm}$$

$$\therefore H = 56.2 \text{ mm}$$

$$V = 139412 \text{ mm}^3$$

8.23 Figure 8.27 shows a cylindrical riser attached to a casting. Compare the solidification times for each casting section and the riser and determine whether the riser will be effective.

$$(V/A)_{thin} = \frac{(75)(150)(200)}{(75)(150) + 2(75)(200) + 2(150)(200)} = 22.22$$

$$(V/A)_{thick} = \frac{(150)(150)(150)}{(150)(75) + 5(150)(150) - (\pi/4)(75)^2} = 28.28$$

$$(V/A)_{riser} = \frac{(\pi/4)(75)^2(175)}{\pi(75)(175) + (\pi/4)(75)^2} = 16.93$$

Note that the riser area in contact with the casting is not included in either the riser or casting surface area; no heat is lost across this interface. In a like manner, the area of contact between the thick and thin portions of the casting are not included in the calculation of the casting area.

The riser will not be effective; the thick section of the casting has the largest V/A ratio and therefore requires the longest solidification time. Consequently the riser will be completely solid before the thick section is solidified; no liquid metal will be available to compensate for the solidification shrinkage.

8.24 Figure 8.28 shows a cylindrical riser attached to a casting. Compare the solidification times for each casting section and the riser and determine whether the riser will be effective.

$$(V/A)_{thick} = \frac{(100)(100)(100)}{5(100)(100) + 1(50)(100)} = \frac{1000000}{55000} = 18.2$$

$$(V/A)_{thin} = \frac{(50)(50)(100)}{3(50)(100) + 2(50)(50)} = \frac{250000}{20000} = 12.5$$

$$(V/A)_R = \frac{(\pi/4)(100^2)(200)}{\pi(100)(200) + 2(\pi/4)100^2} = \frac{1570796}{78540} = 20$$

The area between the thick and thin sections of the casting are not included in calculating casting area; no heat is lost across this interface.

The riser will not be effective; the thin section has the smallest V/A ratio and therefore freezes first. Even though the riser has the longest solidification time, the thin section isolates the thick section from the riser, preventing liquid metal from feeding from the riser to the thick section. Shrinkage will occur in the thick section.

8.25 A 100 mm diameter sphere of liquid copper is allowed to solidify, producing a spherical shrinkage cavity in the centre of the casting. Compare the volume and diameter of the shrinkage cavity in the copper casting to that obtained when a 100 mm sphere of liquid iron is allowed to solidify.

$$\% \text{ shrinkage Cu} : 5.1\% \quad \text{Fe: } 3.4\% \quad r_{sphere} = 100/2 = 50 \text{ mm}$$

$$\text{Cu: } V_{shrinkage} = (4\pi/3)(50)^3(0.051) = 26703 \text{ mm}^3$$

$$(4\pi/3)r^3 = 26703 \text{ mm}^3 \quad \text{or} \quad r = 18.54 \text{ mm}$$

$$d_{pore} = 37.08 \text{ mm}$$

$$\text{Fe: } V_{shrinkage} = (4\pi/3)(50)^3(0.034) = 17802.4 \text{ mm}^3$$

$$(4\pi/3)r^3 = 17802.4 \text{ mm}^3 \quad \text{or} \quad r = 16.2 \text{ mm}$$

$$d_{pore} = 32.4 \text{ mm}$$

8.26 A 100 mm cube of a liquid metal is allowed to solidify. A spherical shrinkage cavity with a diameter of 37.8 mm is observed in the solid casting. Determine the percent volume change that occurs during solidification.

$$V_{liquid} = (100 \text{ mm})^3 = 1000000 \text{ mm}^3$$

$$V_{shrinkage} = (4\pi/3)(37.8/2)^3 = 28279.6 \text{ mm}^3$$

$$V_{solid} = 1000000 - 28279.6 = 971720.3 \text{ mm}^3$$

$$\% \text{Volume change} = \frac{1000000 - 971720.3}{1000000} \times 100 = 2.8\%$$

8.27 A 20 mm x 40 mm x 60 mm magnesium casting is produced. After cooling to room temperature, the casting is found to weigh 80 g. Determine (a) the

volume of the shrinkage cavity at the centre of the casting and (b) the percent shrinkage that must have occurred during solidification.

The density of the magnesium is 1.738 Mg.m^{-3}

$$a) \quad V_{\text{initial}} = (20)(40)(60) = 48000 \text{ mm}^3 \equiv \text{volume of fully sound casting.}$$

$$V_{\text{final}} = 80 \text{ g} / 1.738 \times 10^{-3} \text{ g mm}^{-3} = 46030 \text{ mm}^3 \\ \equiv \text{volume metal excluding volume shrinkage present} \\ \therefore \text{volume of shrinkage} = 48000 - 46030 = 1970 \text{ mm}^3$$

$$b) \quad \% \text{shrinkage} = \frac{48000 - 46030}{48000} \times 100\% = 4.1\%$$

8.28 A 200 mm x 50 mm x 250 mm iron casting is produced and, after cooling to room temperature, is found to weigh 19 kg. Determine (a) the percent shrinkage that must have occurred during solidification and (b) the number of shrinkage pores in the casting if all of the shrinkage occurs as pores with a diameter of 1.27 mm.

The density of the iron is 7.87 Mg.m^{-3}

$$a) \quad V_{\text{actual}} = \frac{19000 \text{ g}}{7.87 \times 10^{-3} \text{ g.mm}^3} = 2414231 \text{ mm}^3$$

$$V_{\text{intended}} = (200)(50)(250) = 2500000 \text{ mm}^3$$

$$\text{shrinkage} = \frac{2500000 - 2414231}{2500000} \times 100\% = 3.4\%$$

$$b) \quad V_{\text{pores}} = 2500000 - 2414231 = 85769 \text{ mm}^3$$

$$r_{\text{pores}} = (1.27 \text{ mm}/2) = 0.635 \text{ mm}$$

$$\text{No of pores} = \frac{85769 \text{ mm}^3}{(4\pi/3)(0.635 \text{ mm})^3} = 79969 \text{ pores}$$

8.29 Liquid magnesium is poured into a 20 mm x 20 mm x 240 mm mould and, as a result of directional solidification, all of the solidification shrinkage occurs along the length of the casting. Determine the length of the casting immediately after solidification is completed.

$$V_{\text{initial}} = (20)(20)(240) = 96000 \text{ mm}^3$$

$$\% \text{ contraction} = 4 \quad \text{or} \quad 0.04 \times 96000 = 3840 \text{ mm}^3$$

$$V_{\text{final}} = 96000 - 3840 = 92160 \text{ mm}^3 = (20)(20)(L)$$

$$\text{Length } (L) = 230.4 \text{ mm}$$

8.30 A liquid cast iron has a density of 7.65 Mg.m^{-3} . Immediately after solidification, the density of the solid cast iron is found to be 7.71 Mg.m^{-3} . Determine the percent volume change that occurs during solidification. Does the cast iron expand or contract during solidification?

$$\frac{1/7.65 - 1/7.71}{1/7.65} \times 100\% = \frac{0.1307 \text{ m}^3 - 0.1297 \text{ m}^3}{0.1307 \text{ m}^3} \times 100\% = 0.78\%$$

The casting contracts.

8.31 From Figure 8.16, find the solubility of hydrogen in liquid aluminium just before solidification begins when the partial pressure of hydrogen is 1 atm. Determine the solubility of hydrogen (in mm³/g Al) at the same temperature if the partial pressure was reduced to 0.01 atm.

Solubility at 1 atm: 4.6 mm³ H₂/g Al

Call solubility at 0.01 atm = x H₂/g Al

$$\frac{4.6}{x} = \frac{K/1}{K/0.01}$$

$$x = 4.6/0.01 = 0.46 \text{ mm}^3/\text{g Al}$$

8.32 The solubility of hydrogen in liquid aluminium at 715°C is found to be 10 mm³/g Al. If all of this hydrogen precipitated as gas bubbles during solidification and remained trapped in the casting, calculate the volume percent gas in the solid aluminium.

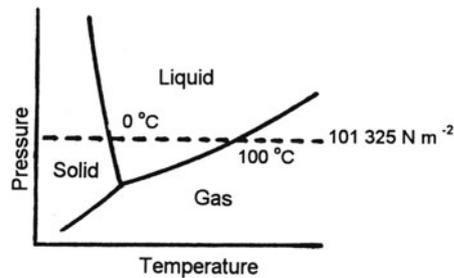
There are 10 mm³H₂ in 1 g Al

$$\text{so, } 10 \text{ mm}^3\text{H}_2 \text{ in } \frac{1}{2.699 \times 10^{-3}} \text{ mm}^3\text{Al}$$

$$\text{so, } 0.02699 \text{ mm}^3\text{H}_2 \text{ in } 1 \text{ mm}^3 \text{ Al} = 2.699\%.$$

Chapter 9 Solid Solution Strengthening and Phase Equilibrium

9.1 The triple point for water occurs at 709 Pa and 0.0075°C. Using this information and your knowledge of the behavior of water at atmospheric pressure of 101,325 Pa, construct a schematic unary phase diagram.



9.2 The unary phase diagram for SiO₂ is shown in Figure 14-6. Locate the triple point where solid, liquid, and vapour coexist and give the temperature and the type of solid present. What do the other "triple" points indicate?

a) The solid-liquid-vapour triple point occurs at 1713°C; the solid phase present at this point is β-cristobalite.

(b) The other triple points describe the equilibrium between two solids and a vapour phase.

9.3 Based on Hume-Rothery's conditions, which of the following systems would be expected to display unlimited solid solubility? Explain.

- | | | | |
|-----------|-----------|-----------|-----------|
| (a) Au-Ag | (b) Al-Cu | (c) Al-Au | (d) U-W |
| (e) Mo-Ta | (f) Nb-W | (g) Mg-Zn | (h) Mg-Cd |

(a)	$r_{Au} = 0.1442$ $r_{Ag} = 0.1445$ $\Delta r = 0.2\%$	$v = +1$ $v = +1$	FCC FCC Yes	(e)	$r_{Mo} = 0.1363$ $r_{Ta} = 0.143$ $\Delta r = 4.7\%$	$v = +4$ $v = +5$	BCC BCC No
(b)	$r_{Al} = 0.1432$ $r_{Cu} = 0.1278$ $\Delta r = 10.7\%$	$v = +3$ $v = +1$	FCC FCC No	(f)	$r_{Nb} = 0.1426$ $r_{W} = 0.1371$ $\Delta r = 3.9\%$	$v = +4$ $v = +4$	BCC BCC Yes
(c)	$r_{Al} = 0.1432$ $r_{Au} = 0.1442$ $\Delta r = 0.7\%$	$v = +3$ $v = +1$	FCC FCC No	(g)	$r_{Mg} = 0.1604$ $r_{Zn} = 0.1332$ $\Delta r = 17\%$	$v = +2$ $v = +2$	HCP HCP No
(d)	$r_U = 0.138$ $r_W = 0.1371$ $\Delta r = 0.7\%$	$v = +4$ $v = +4$	Ortho FCC No	(h)	$r_{Mg} = 0.1604$ $r_{Cd} = 0.1490$ $\Delta r = 7.1\%$	$v = +2$ $v = +2$	HCP HCP Yes

The Au-Ag, Mo-Ta, and Mg-Cd systems have the required radius ratio, the same crystal structures, and the same valences. Each of these might be expected to display complete solid solubility. [The Au-Ag and Mo-Ta do have isomorphous phase diagrams. In addition, the Mg-Cd alloys all solidify like isomorphous alloys; however a number of solid state phase transformations complicate the diagram.]

9.4 Suppose 1 at% of the following elements is added to copper without exceeding the solubility limit. Which one would be expected to give the higher strength alloy? Is any of the alloying elements expected to have unlimited solid solubility in copper?

- (a) Au (b) Mn (c) Sr (d) Si (e) Co

For copper: $r_{Cu} = 0.1278 \text{ nm}$

(a) Au:	$r = 0.1442$	$\Delta r = \frac{r_{Au} - r_{Cu}}{r_{Cu}} = +12.8\%$	May be Unlimited Solubility.
(b) Mn:	$r = 0.112$	$\Delta r = -12.4\%$	Different structure.
(c) Sr:	$r = 0.2151$	$\Delta r = +68.3\%$	Highest Strength
(d) Si:	$r = 0.1176$	$\Delta r = -8.0\%$	Different structure.
(e) Co:	$r = 0.1253$	$\Delta r = -2.0\%$	Different structure.

The Cu-Sr alloy would be expected to be strongest (largest size difference). The Cu-Au alloy satisfies Hume-Rothery's conditions and might be expected to display complete solid solubility -- in fact it freezes like an isomorphous series of alloys, but a number of solid state transformations occur at lower temperatures.

9.5 Suppose 1 at% of the following elements is added to aluminium without exceeding the solubility limit. Which one would be expected to give the least reduction in electrical conductivity? Is any of the alloy elements expected to have unlimited solid solubility in aluminium?

- (a) Li (b) Ba (c) Be (d) Cd (e) Ga

For aluminium: $r = 0.1432 \text{ nm}$ (FCC structure with valence of 3)

(a) Li:	$r = 0.1519$	$\Delta r = 6.1\%$	BCC	valence = 1
(b) Ba:	$r = 0.2176$	$\Delta r = -52.0\%$	BCC	valence = 2
(c) Be:	$r = 0.1143$	$\Delta r = -20.2\%$	HCP	valence = 2
(d) Cd:	$r = 0.149$	$\Delta r = 4.1\%$	HCP	valence = 2
(e) Ga:	$r = 0.1218$	$\Delta r = 14.9\%$	Orthorhombic	valence = 3

The cadmium would be expected to give the smallest reduction in electrical conductivity, since the Cd atoms are most similar in size to the aluminium atoms.

None are expected to have unlimited solid solubility, due either to difference in valence, atomic radius or crystal structure.

9.6 Which of the following oxides is expected to have the largest solid solubility in Al_2O_3 ?

- (a) Al_2O_3 (b) Cr_2O_3 (c) Fe_2O_3 (d) Ti_2O_3

The ionic radius of $Al^{3+} = 0.051 \text{ nm}$

(a) $r_y = 0.089$	$\Delta r = \frac{0.063 - 0.051}{0.051} \times 100 = 74.5\%$
(b) $r_{Cr} = 0.063$	$\Delta r = 23.5\%$

$$(c) r_{Fe} = 0.064 \quad \Delta r = 25.5\%$$

$$(d) r_{Ti} = 0.068 \quad \Delta r = 33.3\%$$

We would expect Cr_2O_3 to have a high solubility in Al_2O_3 ; in fact, they are completely soluble in one another.

9.7 Determine the liquidus temperature, solidus temperature, and freezing range for the following NiO-MgO ceramic compositions. [See Figure 9.8(b)]

(a) NiO-30 mol% MgO
(c) NiO-60 mol% MgO

(b) NiO-45 mol% MgO
(d) NiO-85 mol% MgO

$$(a) \quad T_L = 2330^\circ C \quad T_S = 2150^\circ C \quad FR = 180^\circ C$$

$$(b) \quad T_L = 2460^\circ C \quad T_S = 2250^\circ C \quad FR = 210^\circ C$$

$$(c) \quad T_L = 2570^\circ C \quad T_S = 2380^\circ C \quad FR = 190^\circ C$$

$$(d) \quad T_L = 2720^\circ C \quad T_S = 2610^\circ C \quad FR = 110^\circ C$$

9.8 Determine the liquidus temperature, solidus temperature, and freezing range for the following MgO-FeO ceramic compositions. (See Figure 9.18)

(a) MgO-25 wt% FeO
(c) MgO-65 wt% FeO

(b) MgO-45 wt% FeO
(d) MgO-80 wt% FeO

$$(a) \quad T_L = 2600^\circ C \quad T_S = 2230^\circ C \quad FR = 370^\circ C$$

$$(b) \quad T_L = 2340^\circ C \quad T_S = 1900^\circ C \quad FR = 440^\circ C$$

$$(c) \quad T_L = 2000^\circ C \quad T_S = 1610^\circ C \quad FR = 390^\circ C$$

$$(d) \quad T_L = 1750^\circ C \quad T_S = 1480^\circ C \quad FR = 270^\circ C$$

9.9 Determine the phases present, the compositions of each phase, and the amount of each phase in mol% for the following NiO-MgO ceramics at $2400^\circ C$. [See Figure 9.8(b)]

(a) NiO-30 mol% MgO
(c) NiO-60 mol% MgO

(b) NiO-45 mol% MgO
(d) NiO-85 mol% MgO

$$(a) \quad L: \text{NiO-30 mol\% MgO} \quad 100\% L$$

$$(b) \quad L: \quad 38\% \text{ MgO} \quad \%L = \frac{62 - 45}{62 - 38} \times 100\% = 70.8\%$$

$$S: \quad 62\% \text{ MgO} \quad \%L = \frac{45 - 38}{62 - 38} \times 100\% = 29.2\%$$

$$(c) \quad L: \quad 38\% \text{ MgO} \quad \%L = \frac{62 - 60}{62 - 38} \times 100\% = 8.3\%$$

$$S: \quad 62\% \text{ MgO} \quad \%L = \frac{60 - 38}{62 - 38} \times 100\% = 91.7\%$$

$$(d) \quad S: \quad 85\% \text{ MgO} \quad 100\% S$$

9.10 Determine the phases present, the compositions of each phase, and the amount of each phase in wt% for the following MgO-FeO ceramics at 2000°C. (See Figure 9.18.)

(a) MgO-25 wt% FeO
(c) MgO-60 wt% FeO

(b) MgO-45 wt% FeO
(d) MgO-80 wt% FeO

(a) S: 25% FeO

100% S

(b) S: 39% FeO

$$\%S = \frac{65 - 45}{65 - 39} \times 100\% = 76.9\%$$

L: 65% FeO

$$\%L = \frac{45 - 39}{65 - 39} \times 100\% = 23.1\%$$

(c) S: 39% FeO

$$\%S = \frac{65 - 60}{65 - 39} \times 100\% = 19.2\%$$

L: 65% MgO

$$\%L = \frac{60 - 39}{65 - 39} \times 100\% = 80.8\%$$

(d) S: 80% MgO

100% L

9.11 Consider an alloy of 65 wt% Cu and 35 wt% Al. Calculate the composition of the alloy in at%.

$$\text{At\% Cu} = \frac{65/63.54}{(65/63.54) + (35/26.981)} \times 100\% = 44.1\%$$

$$\text{At\% Al} = \frac{35/26.981}{(65/63.54) + (35/26.981)} \times 100\% = 55.9\%$$

9.12 Consider a ceramic composed of 30 mol% MgO and 70 mol% FeO. Calculate the composition of the ceramic in wt%.

$$MW_{\text{MgO}} = 24.312 + 16 = 40.312 \text{ g.mol}^{-1} \quad MW_{\text{FeO}} = 55.847 + 16 = 71.847 \text{ g.mol}^{-1}$$

$$\text{wt\% MgO} = \frac{(30)(40.312)}{(30)(40.312) + (70)(71.847)} \times 100\% = 19.4\%$$

$$\text{wt\% FeO} = \frac{(70)(71.847)}{(30)(40.312) + (70)(71.847)} \times 100\% = 80.6\%$$

9.13 A NiO-20 mol% MgO ceramic is heated to 2200°C. Determine (a) the composition of the solid and liquid phases in both mol% and wt% and (b) the amount of each phase in both mol% and wt%. (c) Assuming that the density of the solid is 6.32 Mg.m⁻³ and that of the liquid is 7.14 Mg.m⁻³, determine the amount of each phase in vol%.

$$MW_{\text{MgO}} = 24.312 + 16 = 40.312 \text{ g.mol}^{-1} \quad MW_{\text{NiO}} = 58.71 + 16 = 74.71 \text{ g.mol}^{-1}$$

$$(a) \text{ L: } 15 \text{ mol\% MgO} \quad \text{wt\% MgO} = \frac{(15)(40.312)}{(15)(40.312) + (85)(74.71)} \times 100\% = 8.69\%$$

$$\text{S: } 38 \text{ mol\% MgO} \quad \text{wt\% MgO} = \frac{(38)(40.312)}{(38)(40.312) + (62)(74.71)} \times 100\% = 24.85\%$$

$$(b) \text{ mol\% L} = \frac{38 - 20}{38 - 15} \times 100\% = 78.26\% \quad \text{mol\% S} = 21.74\%$$

The original composition, in wt% MgO, is:

$$\frac{(20)(40.312)}{(20)(40.312) + (80)(74.71)} \times 100\% = 11.9\%$$

$$\text{wt\% L} = \frac{24.85 - 11.9}{24.85 - 8.69} \times 100\% = 80.1\% \quad \text{wt\% S} = 19.9\%$$

$$(c) \text{ Vol\% L} = \frac{80.1/7.14}{(80.1/7.14) + (19.9/6.32)} \times 100\% = 78.1\% \quad \text{Vol\% S} = 21.9\%$$

9.14 A Nb-60 wt% W alloy is heated to 2800°C. Determine (a) the composition of the solid and liquid phases in both wt% and at% and (b) the amount of each phase in both wt% and at%. (c) Assuming that the density of the solid is 16.05 Mg.m⁻³ and that of the liquid is 13.91 Mg.m⁻³, determine the amount of each phase in vol%. (See Figure 9.19)

$$(a) \text{ L: } 49 \text{ wt\% W} \quad \text{at\% W} = \frac{49/183.85}{(49/183.85) + (51/92.91)} \times 100\% = 32.7\%$$

$$\alpha: \quad 70 \text{ wt\% W} \quad \text{at\% W} = \frac{(70/183.85)}{(70/183.85) + (30/92.91)} \times 100\% = 54.1\%$$

$$(b) \text{ wt\% L} = \frac{70 - 60}{70 - 49} \times 100\% = 47.6\% \quad \text{wt\% } \alpha = 52.4\%$$

The original composition, in wt% MgO, is:

$$\frac{60 / 183.85}{(60 / 183.85) + (40 / 92.91)} \times 100\% = 43.1\%$$

$$\text{at\% L} = \frac{54.1 - 43.1}{54.1 - 32.7} \times 100\% = 51.4\% \quad \text{wt\% } \alpha = 48.6\%$$

$$(c) \text{ Vol\% L} = \frac{47.6/13.91}{(47.6/13.91) + (52.4/16.05)} \times 100\% = 51.2\% \quad \text{Vol\% } \alpha = 48.8\%$$

9.15 How many grams of nickel must be added to 500 grams of copper to produce an alloy that has a liquidus temperature of 1350°C. What is the ratio of the number of nickel atoms to copper atoms in this alloy?

We need 60 wt% Ni to obtain the correct liquidus temperature.

$$\%Ni = 60 = \frac{x}{x + 500 \text{ g}} \times 100\% \quad \text{or} \quad x = 750 \text{ g Ni}$$

$$\frac{\text{Ni atoms}}{\text{Cu atoms}} = \frac{(750 \text{ g})(N_A) / 58.71 \text{ g.mol}^{-1}}{(500 \text{ g})(N_A) / 63.54 \text{ g.mol}^{-1}} = 1.62$$

9.16 How many grams of nickel must be added to 500 grams of copper to produce an alloy that contains 50 wt% α at 1300°C?

At 1300°C, the composition of the two phases in equilibrium are

$$\text{L: } 46 \text{ wt\% Ni} \quad \text{and} \quad \alpha: \quad 58 \text{ wt\% Ni}$$

The alloy required to give 50% α is then

$$\frac{x - 46}{58 - 46} \times 100 = 50\% \alpha \quad \text{or} \quad x = 52 \text{ wt\% Ni}$$

The number of grams of Ni must be:

$$\frac{x}{x + 500} \times 100\% = 52 \quad \text{or} \quad x = 541.7 \text{ g Ni}$$

9.17 How many grams of MgO must be added to 1 kg of NiO to produce a ceramic that has a solidus temperature of 2200°C?

$$MW_{MgO} = 40.312 \text{ g.mol}^{-1} \quad MW_{NiO} = 74.71 \text{ g.mol}^{-1}$$

38 mol% MgO is needed to obtain the correct solidus temperature.

$$\text{wt} \% \text{ MgO} = \frac{(38)(40.312)}{(38)(40.312) + (62)(74.71)} \times 100\% = 24.9\%$$

The number of grams required is:

$$\frac{x}{x + 1000} \times 100\% = 24.9\% \quad \text{or} \quad x = 332 \text{ g of MgO}$$

9.18 How many grams of MgO must be added to 1 kg of NiO to produce a ceramic that contains 25 mol% solid at 2400°C?

$$\begin{array}{ll} L: & 38 \text{ mol}\% \text{ MgO} \quad MW_{MgO} = 40.312 \text{ g.mol}^{-1} \\ S: & 62 \text{ mol}\% \text{ MgO} \quad MW_{NiO} = 74.71 \text{ g.mol}^{-1} \end{array}$$

$$\frac{x - 38}{62 - 38} \times 100\% = 25\%S \quad \text{or} \quad x = 44 \text{ mol}\% \text{ MgO}$$

$$\text{wt}\% \text{ MgO} = \frac{(44)(40.312)}{(44)(40.312) + (56)(74.71)} \times 100\% = 29.77\%$$

The number of grams of MgO is then:

$$\frac{x}{x + 1000} \times 100\% = 29.77\% \quad \text{or} \quad x = 424 \text{ g MgO}$$

9.19 We would like to produce a solid MgO-FeO ceramic that contains equal mol percentages of MgO and FeO at 1200°C. Determine the wt% FeO in the ceramic. (See Figure 9.18)

$$\begin{array}{l} \text{Only solid is present at } 1200^\circ\text{C.} \\ MW_{MgO} = 40.312 \text{ g.mol}^{-1} \\ MW_{FeO} = 71.847 \text{ g.mol}^{-1} \end{array}$$

$$50 \text{ mol}\% \text{ FeO: } \frac{(50)(71.847)}{(50)(40.312) + (50)(71.847)} = 64.1 \text{ wt}\% \text{ FeO}$$

9.20 We would like to produce a MgO-FeO ceramic that is 30 wt% solid at 2000°C. Determine the original composition of the ceramic in wt%. (See Figure 9.18)

$$L: 65 \text{ wt}\% \text{ FeO} \quad S: 38 \text{ wt}\% \text{ FeO}$$

$$30 \text{ wt}\% = \frac{65 - x}{65 - 38} \times 100\% \quad \text{or} \quad x = 56.9 \text{ wt}\% \text{ FeO}$$

9.21 A Nb-W alloy held at 2800°C is partly liquid and partly solid. (a) If possible, determine the composition of each phase in the alloy. (b) If possible, determine the amount of each phase in the alloy. (See Figure 9.19.)

(a) L: 49 wt% W α : 70 wt% W

(b) Not possible unless we know the original composition of the alloy.

9.22 A Nb-W alloy contains 55% α at 2600°C. Determine (a) the composition of each phase and (b) the original composition of the alloy. (See Figure 9.19.)

(a) L: 22 wt% W α : 42 wt% W

(b) $0.55 = \frac{x - 22}{42 - 22}$ or $x = 33 \text{ wt\% W}$

9.23 Suppose a 0.5 tonne bath of a Nb-40 wt% W alloy is held at 2800°C. How many grammes of tungsten can be added to the bath before any solid forms? How many grammes of tungsten must be added to cause the entire bath to be solid? (See Figure 9.19)

Solid starts to form at 2800°C when 49 wt% W is in the alloy. In 0.5 tonnes of the original Nb-40% W alloy, there are $(0.4)(500) = 200 \text{ kg W}$ and 300 kg Nb. The total amount of tungsten that must be in the final alloy is:

$$0.49 = \frac{x}{x + 300} \quad \text{or} \quad x = 271 \text{ kg W total}$$

or $271 - 200 = 71$ additional kg of W must be added or 71000 g.

To be completely solid at 2800°C, the alloy must contain 70 wt% W. The total amount of tungsten required in the final alloy is:

$$0.70 = \frac{x}{x + 300} \quad \text{or} \quad x = 700 \text{ kg W total}$$

or $700 - 200 = 500$ additional kilogrammes of W must be added
= 500,000 g of W.

9.24 A fibre-reinforced composite material is produced, in which tungsten fibres are embedded in a Nb matrix. The composite is composed of 70 vol% tungsten. (a) Calculate the wt% of tungsten fibres in the composite. (b) Suppose the composite is heated to 2600°C and held for several years. What happens to the fibres? Explain. (See Figure 9.19)

$$\begin{aligned} \text{(a) } \text{wt\%} &= \frac{(70 \text{ m}^3)(19.254 \text{ Mg.m}^{-3})}{(70 \text{ m}^3)(19.254 \text{ Mg.m}^{-3}) + (30 \text{ m}^3)(8.57 \text{ Mg.m}^{-3})} \times 100 \\ &= 83.98 \text{ wt\% W} \end{aligned}$$

(b) The fibres will dissolve. Since the W and Nb are completely soluble in one another, and the temperature is high enough for rapid diffusion, a single solid solution will eventually be produced.

9.25 Suppose a crucible made of pure nickel is used to contain 500 g of liquid copper at 1150°C. Describe what happens to the system as it is held at this temperature for several hours. Explain.

Cu dissolves Ni until the Cu contains enough Ni that it solidifies completely. When 10% Ni is dissolved, freezing begins:

$$0.10 = \frac{x}{x + 500} \quad \text{or} \quad x = 55.5 \text{ g Ni}$$

When 18% Ni dissolved, the bath is completely solid:

$$0.18 = \frac{x}{x + 500} \quad \text{or} \quad x = 109.8 \text{ g Ni}$$

Copper will diffuse in the solid solution (see chapter 5) until equilibrium is reached.

9.26 Equal moles of MgO and FeO are combined and melted. Determine (a) the liquidus temperature, the solidus temperature, and the freezing range of the ceramic and (b) determine the phase(s) present, their composition(s), and their amount(s) at 1800°C. (See Figure 9.18.)

$$MW_{MgO} = 40.312 \text{ g.mol}$$

$$MW_{FeO} = 71.847 \text{ g.mol}$$

$$\text{wt\% FeO} = \frac{(1 \text{ mol FeO})(71.847 \text{ g.mol})}{(1 \text{ mol FeO})(71.847) + (1 \text{ mol MgO})(40.312)} = 64.1\%$$

$$(a) \quad T_{Liq} = 2000^\circ\text{C} \quad T_s = 1620^\circ\text{C} \quad FR = 380^\circ\text{C}$$

$$(b) \quad L: 75\% \text{ FeO} \quad S: 50\% \text{ FeO}$$

$$\%L = \frac{64.1 - 50}{75 - 50} \times 100\% = 56.4\% \quad \%S = 43.6\%$$

9.27 Suppose 75 cm³ of Nb and 45 cm³ of W are combined and melted. Determine (a) the liquidus temperature, the solidus temperature, and the freezing range of the alloy and (b) determine the phase(s) present, their composition(s), and their amount(s) at 2800°C. (See Figure 9.19.)

$$\text{wt\% W} = \frac{(45 \times 10^{-6} \text{ m}^3)(19.254 \text{ Mg.m}^{-3})}{(45 \times 10^{-6})(19.254) + (75 \times 10^{-6})(8.57)} \times 100 = 57.4 \text{ wt\% W}$$

$$(a) \quad T_{Liq} = 2900^\circ\text{C} \quad T_{sol} = 2690^\circ\text{C} \quad FR = 210^\circ\text{C}$$

$$(b) \quad L: 49\%W \quad \%L = \frac{70 - 57.4}{70 - 49} = 60\%$$

$$\alpha: 70\%W \quad \% \alpha = 40\%$$

9.28 A NiO-60 mol% MgO ceramic is allowed to solidify. Determine (a) the composition of the first solid to form and (b) the composition of the last liquid to solidify under equilibrium conditions.

$$(a) \quad 1st \alpha: 80\% \text{ MgO} \quad (b) \quad Last L: 35\% \text{ MgO}$$

9.29 A Nb-35% W alloy is allowed to solidify. Determine (a) the composition of the first solid to form and (b) the composition of the last liquid to solidify under equilibrium conditions. (See Figure 9.19)

$$(a) \quad 1st \alpha: 55\% W \quad (b) \quad Last L: 18\% W$$

9.30 For equilibrium conditions and a MgO-65 wt% FeO ceramic, determine (a) the liquidus temperature, (b) the solidus temperature, (c) the freezing range, (d) the composition of the first solid to form during solidification, (e) the composition of the last liquid to solidify, (f) the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 1800°C, and (g)

the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 1600°C. (See Figure 9.18)

- (a) *Liquidus* = 2000°C
- (b) *Solidus* = 1605°C
- (c) *Freezing range* = 2000 - 1605 = 395°C
- (d) *First solid*: 40% FeO
- (e) *Last liquid*: 88 % FeO
- (f) L: 75% FeO %L = $\frac{65 - 51}{75 - 51} \times 100\% = 58\%$

- α: 51% FeO %α = 42%
- (g) α: 65% FeO 100% α

9.31 For the nonequilibrium conditions shown for the MgO-65 wt% FeO ceramic, determine (a) the liquidus temperature, (b) the nonequilibrium solidus temperature, (c) the freezing range, (d) the composition of the first solid to form during solidification, (e) the composition of the last liquid to solidify, (f) the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 1800°C, and (g) the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 1600°C. (See Figure 9.18)

- (a) *Liquidus* = 2000°C
- (b) *Solidus* = 1450°C
- (c) *Freezing range* = 2000 - 1450 = 550°C
- (d) *First solid*: 40% FeO
- (e) *Last liquid*: 92% FeO
- (f) L: 75% FeO %L = $\frac{65 - 46}{75 - 46} \times 100\% = 65.5\%$
S: 46% FeO %S = 34.5%
- (g) L: 88% FeO %L = $\frac{65 - 55}{88 - 55} \times 100\% = 30.3\%$
S: 55% FeO %S = 69.7%

9.32 For equilibrium conditions and a Nb-80 wt% W alloy, determine (a) the liquidus temperature, (b) the solidus temperature, (c) the freezing range, (d) the composition of the first solid to form during solidification, (e) the composition of the last liquid to solidify, (f) the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 3000°C, and (g) the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 2800°C. (see Figure 9.19)

- (a) *Liquidus* = 3100°C
- (b) *Solidus* = 2920°C
- (c) *Freezing range* = 3100 - 2920 = 180°C
- (d) *First solid*: 90% W
- (e) *Last liquid*: 64% W
- (f) L: 70% W %L = $\frac{85 - 80}{85 - 70} \times 100\% = 33.3\%$
α: 85% W %α = 66.7%
- (g) α: 80% W 100% α

9.33 For the nonequilibrium conditions shown for the Nb-80 wt% W alloy, determine (a) the liquidus temperature, (b) the nonequilibrium solidus temperature, (c) the freezing range, (d) the composition of the first solid to form during solidification, (e) the composition of the last liquid to solidify, (f) the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 3000°C, and (g) the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 2800°C. (See Figure 9.19)

- (a) Liquidus = 3100°C
 (b) Solidus = 2720°C
 (c) Freezing range = 3100 - 2720 = 380°C
 (d) First solid: 90% W
 (e) Last liquid: 40% W
 (f) L: 70% W %L = $\frac{88 - 80}{88 - 70} \times 100\% = 44.4\%$
 α: 88% W %α = 55.6%
- (g) L: 50% W %L = $\frac{83 - 80}{83 - 50} \times 100\% = 9.1\%$
 α: 83% W %α = 90.9%

9.34 Figure 9.20 shows the cooling curve for a NiO-MgO ceramic. Determine (a) the liquidus temperature, (b) the solidus temperature, (c) the freezing range, (d) the pouring temperature, (e) the superheat, (f) the local solidification time, (g) the total solidification time, and (h) the composition of the ceramic.

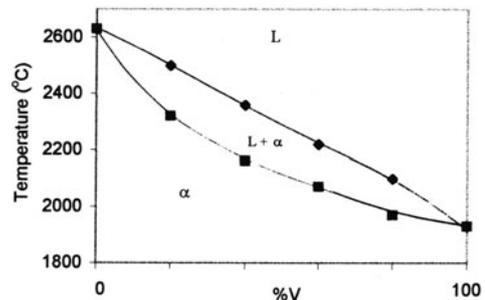
- (a) Liquidus = 2690°C
 (b) Solidus = 2570°C
 (c) Freezing range = 2690 - 2570 = 120°C
 (d) Pouring temperature = 2775°C
 (e) Superheat = 2775 - 2690 = 85°C
 (f) Local solidification time = 27 - 5 = 22 min
 (g) Total solidification time = 27 min
 (h) 80% MgO

9.35 Figure 9.21 shows the cooling curve for a Nb-W alloy. Determine (a) the liquidus temperature, (b) the solidus temperature, (c) the freezing range, (d) the pouring temperature, (e) the superheat, (f) the local solidification time, (g) the total solidification time, and (h) the composition of the alloy.

- (a) Liquidus = 2900°C
 (b) Solidus = 2710°C
 (c) Freezing range = 2900 - 2710 = 190°C
 (d) Pouring temperature = 2990°C
 (e) Superheat = 2990 - 2900 = 90°C
 (f) Local solidification time = 340 - 40 = 300 s
 (g) Total solidification time = 340 min
 (h) 60% W

9.36 Cooling curves are shown in Figure 9.22 for several Mo-V alloys. Based on these curves, construct the Mo-V phase diagram.

% V	T_{Liquidus}	T_{Solidus}
0% V	2630°C	
20% V	2500°C	2320°C
40% V	2360°C	2160°C
60% V	2220°C	2070°C
80% V	2100°C	1970°C
100% V	1930°C	



Chapter 10 Dispersion Strengthening by Solidification

10.1 A hypothetical phase diagram is shown in Figure 10.30. (a) Are any intermetallic compounds present? If so, identify them and determine whether they are stoichiometric or nonstoichiometric. (b) Identify the solid solutions present in the system. Is either material A or B allotropic? Explain. (c) Identify the three-phase reactions by writing down the temperature, the reaction in equation form, the composition of each phase in the reaction, and the name of the reaction.

(a) θ = non-stoichiometric

(b) α , η , γ , and β ; material B is allotropic, existing in three different forms at different temperatures

(c) 1100°C : $\gamma + L \rightarrow \beta$; peritectic; L: 82% B γ : 97% B β : 90% B
 900°C : $L_1 \rightarrow L_2 + \alpha$; monotectic; L_1 : 28% B L_2 : 50% B α : 5% B
 680°C : $L \rightarrow \alpha + \beta$; eutectic; L: 60% B α : 5% B β : 90% B
 600°C : $\alpha + \beta \rightarrow \theta$; peritectoid; α : 5% B β : 80% B θ : 37% B
 300°C : $\beta \rightarrow \theta + \eta$; eutectoid; β : 90% B θ : 40% B η : 95% B

10.2 The Cu-Zn phase diagram is shown in Figure 13.10. (a) Are any intermetallic compounds present? If so, identify them and determine whether they are stoichiometric or nonstoichiometric. (b) Identify the solid solutions present in the system. (c) Identify the three-phase reactions by writing down the temperature, the reaction in equation form, and the name of the reaction.

(a) β , β' , γ , δ , ϵ : all nonstoichiometric.

(b) α , θ

(c) 900°C : $\alpha + L \rightarrow \beta$; peritectic
 830°C : $\beta + L \rightarrow \gamma$; peritectic
 700°C : $\gamma + L \rightarrow \delta$; peritectic
 600°C : $\delta + L \rightarrow \epsilon$; peritectic
 550°C : $\delta \rightarrow \gamma + \epsilon$; eutectoid
 420°C : $\epsilon + L \rightarrow \theta$; peritectic
 250°C : $\beta' \rightarrow \alpha + \gamma$; eutectoid

10.3 A portion of the Al-Cu phase diagram is shown in Figure 11.5. (a) Determine the formula for the θ compound. (b) Identify the three-phase reaction by writing down the temperature, the reaction in equation form, the composition of each phase in the reaction, and the name of the reaction.

$$(a) \quad \theta \text{ at } 54\% \text{ Cu}; \quad \frac{54 \text{ g} / 63.54 \text{ g.mol}^{-1}}{54/63.54 + 46/26.981} = 33 \text{ at\% Cu}; \quad \text{CuAl}_2$$

(b) $548^\circ\text{C}; L \rightarrow \alpha + \theta$; eutectic; $L: 33.2\% \text{ Cu}, \alpha: 5.65\% \text{ Cu}, \theta: 52.5\% \text{ Cu}$.

10.4 The Al-Li phase diagram is shown in Figure 13.6. (a) Are any intermetallic compounds present? If so, identify them and determine whether they are stoichiometric or nonstoichiometric. Determine the formula for each compound. (b) Identify the three-phase reactions by writing down the temperature, the reaction in equation form, the composition of each phase in the reaction, and the name of the reaction.

(a) β is non-stoichiometric @ 21 wt% Li:

$$\text{at\% Li} = \frac{21 \text{ g} / 6.94 \text{ g.mol}^{-1}}{21/6.94 + 79/26.981} \times 100\% = 50 \text{ at\% Li} \therefore \text{AlLi}$$

γ is stoichiometric @ 34 wt% Li:

$$\text{at\% Li} = \frac{34 \text{ g} / 6.94 \text{ g.mol}^{-1}}{34/6.94 + 66/26.981} \times 100\% = 66.7\% \text{ Li} \therefore \text{AlLi}_2$$

(b) $600^\circ\text{C}: L \rightarrow \alpha + \beta$ eutectic $L: 9.9\% \text{ Li} \quad \alpha: 4\% \text{ Li} \quad \beta: 20.4\% \text{ Li}$

$510^\circ\text{C}: \beta + L \rightarrow \gamma$ peritectic $\beta: 25\% \text{ Li} \quad L: 47\% \text{ Li} \quad \gamma: 34\% \text{ Li}$

$170^\circ\text{C}: L \rightarrow \gamma + \alpha(L_1)$ eutectic $L: 98\% \text{ Li} \quad \gamma: 34\% \text{ Li} \quad \alpha(L_1): 99\% \text{ Li}$

10.5 An intermetallic compound is found for 38 wt% Sn in the Cu-Sn phase diagram. Determine the formula for the compound.

$$\text{at\% Sn} = \frac{38 \text{ g} / 118.69 \text{ g.mol}^{-1}}{38/118.69 + 62/63.54} = 0.25 \quad \text{or} \quad \text{SnCu}_3$$

10.6 An intermetallic compound is found for 10 wt% Si in the Cu-Si phase diagram. Determine the formula for the compound.

$$\text{at\% Si} = \frac{10 \text{ g} / 28.08 \text{ g.mol}^{-1}}{10/28.08 + 90/63.54} = 0.20 \quad \text{or} \quad \text{SiCu}_4$$

10.7 Using the phase rule, predict and explain how many solid phases will form in an eutectic reaction in a ternary (three component) phase diagram, assuming that the pressure is fixed.

$$F = C - P + 1$$

$$\text{At the eutectic, } F = 0, C = 3 \quad 0 = 3 - P + 1 \quad \text{or} \quad P = 4$$

Therefore, $L \rightarrow \alpha + \beta + \gamma$ and 3 solid phases form.

10.8 Consider a Pb-15% Sn alloy. During solidification, determine (a) the composition of the first solid to form, (b) the liquidus temperature, solidus temperature, solvus temperature, and freezing range of the alloy, (c) the amounts and compositions of each phase at 260°C , (d) the amounts and compositions of each phase at 183°C , and (e) the amounts and compositions of each phase at 25°C .

(a) 8% Sn

- (b) $liquidus = 290^{\circ}C$, $solidus = 240^{\circ}C$,
 $solvus = 170^{\circ}C$, $freezing\ range = 50^{\circ}C$
- (c) $L: 30\% Sn$ $\alpha: 12\% Sn$; $\%L = \frac{15 - 12}{30 - 12} \times 100\% = 17\%$ $\%\alpha = 83\%$
- (d) $\alpha: 15\% Sn$ $100\% \alpha$
- (e) $\alpha: 2\% Pb$ $\beta: 100\% Sn$ $\%\alpha = \frac{100 - 15}{100 - 2} \times 100 = 87\%$ $\%\beta = 13\%$

10.9 Consider an Al-12% Mg alloy (Figure 13.3). During solidification, determine (a) the composition of the first solid to form, (b) the liquidus temperature, solidus temperature, solvus temperature, and freezing range of the alloy, (c) the amounts and compositions of each phase at 525°C, (d) the amounts and compositions of each phase at 450°C, and (e) the amounts and compositions of each phase at 25°C.

- (a) 2.5% Mg
- (b) $liquidus = 600^{\circ}C$, $solidus = 470^{\circ}C$,
 $solvus = 400^{\circ}C$, $freezing\ range = 130^{\circ}C$
- (c) $L: 26\% Mg$ $\alpha: 7\% Mg$; $\%\alpha = \frac{26 - 12}{26 - 7} \times 100\% = 74\%$ $\%L = 26\%$
- (d) $\alpha: 12\% Mg$ $100\% \alpha$
- (e) $\alpha: 1\% Mg$ $\beta: 34\% Mg$ $\%\alpha = \frac{34 - 12}{34 - 1} \times 100\% = 67\%$ $\%\beta = 33\%$

10.10 Consider a Pb-35% Sn alloy. Determine (a) if the alloy is hypoeutectic or hypereutectic, (b) the composition of the first solid to form during solidification, (c) the amounts and compositions of each phase at 184°C, (d) the amounts and compositions of each phase at 182°C, (e) the amounts and compositions of each microconstituent at 182°C, and (f) the amounts and compositions of each phase at 25°C.

- (a) hypoeutectic (b) 14% Sn
- (c) $\alpha: 19\% Sn$ $L: 61.9\% Sn$ $\%\alpha = \frac{61.9 - 35}{61.9 - 19} \times 100\% = 63\%$ $\%L = 37\%$
- (d) $\alpha: 19\% Sn$ $\beta: 97.5\% Sn$ $\%\alpha = \frac{97.5 - 35}{97.5 - 19} \times 100\% = 80\%$ $\%\beta = 20\%$
- (e) primary $\alpha: 19\% Sn$ $\%primary\ \alpha = 63\%$
eutectic: 61.9% Sn $\%eutectic = 37\%$
- (f) $\alpha: 2\% Sn$ $\beta: 100\% Sn$ $\%\alpha = \frac{100 - 35}{100 - 2} \times 100\% = 66\%$ $\%\beta = 34\%$

10.11 Consider a Pb-70% Sn alloy. Determine (a) if the alloy is hypoeutectic or hypereutectic, (b) the composition of the first solid to form during solidification, (c) the amounts and compositions of each phase at 184°C, (d) the amounts and compositions of each phase at 182°C, (e) the amounts and compositions of each microconstituent at 182°C, and (f) the amounts and compositions of each phase at 25°C.

- (a) hypereutectic (b) 98% Sn
- (c) $\beta: 97.5\% Sn$ $L: 61.9\% Sn$ $\%\beta = \frac{70 - 61.9}{97.5 - 61.9} \times 100\% = 22.8\%$ $\%L = 77.2\%$

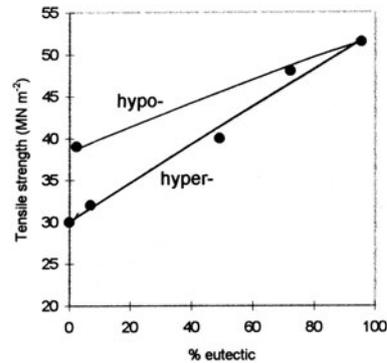
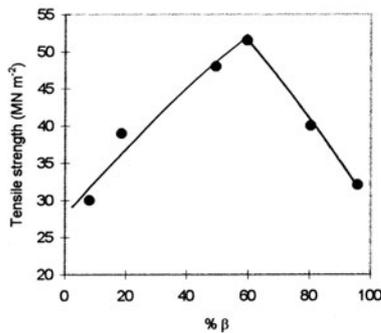
(d) $\alpha: 19\% \text{ Sn}$ $\beta: 97.5\% \text{ Sn}$ $\% \alpha = \frac{97.5 - 70}{97.5 - 19} \times 100\% = 35\%$ $\% \beta = 65\%$

(e) primary $\beta: 97.5\% \text{ Sn}$ $\% \text{primary } \beta = 22.8\%$
 eutectic: $61.9\% \text{ Sn}$ $\% \text{eutectic} = 77.2\%$

(f) $\alpha: 2\% \text{ Sn}$ $\beta: 100\% \text{ Sn}$ $\% \alpha = \frac{100 - 70}{100 - 2} \times 100\% = 30\%$ $\% \beta = 70\%$

10.12 Calculate the total $\% \beta$ and the $\% \text{eutectic}$ microconstituent at room temperature for the following lead-tin alloys: 10% Sn, 20% Sn, 50% Sn, 60% Sn, 80% Sn, and 95% Sn. Using Figure 10.20, plot the strength of the alloys versus the $\% \beta$ and the $\% \text{eutectic}$ and explain your graphs.

	$\% \beta$	$\% \text{eutectic}$
10% Sn	$\frac{10 - 2}{99 - 2} = 8.2\%$	0%
20% Sn	$\frac{20 - 2}{99 - 2} = 18.6\%$	$\frac{20 - 19}{61.9 - 19} = 2.3\%$
50% Sn	$\frac{50 - 2}{99 - 2} = 49.5\%$	$\frac{50 - 19}{61.9 - 19} = 72.3\%$
60% Sn	$\frac{60 - 2}{99 - 2} = 59.8\%$	$\frac{60 - 19}{61.9 - 19} = 95.6\%$
80% Sn	$\frac{80 - 2}{99 - 2} = 80.4\%$	$\frac{97.5 - 80}{97.5 - 61.9} = 49.2\%$
95% Sn	$\frac{95 - 2}{99 - 2} = 95.9\%$	$\frac{97.5 - 95}{97.5 - 61.9} = 7.0\%$



10.13 Consider an Al-4% Si alloy. Determine (a) if the alloy is hypoeutectic or hypereutectic, (b) the composition of the first solid to form during solidification, (c) the amounts and compositions of each phase at 578°C, (d) the amounts and compositions of each phase at 576°C, (e) the amounts and compositions of each microconstituent at 576°C, and (f) the amounts and compositions of each phase at 25°C.

(a) hypoeutectic

(b) 1% Si

(c) $\alpha: 1.65\% \text{ Si}$ $L: 12.6\% \text{ Si}$ $\% \alpha = \frac{12.6 - 4}{12.6 - 1.65} = 78.5\%$ $\% L = 21.5\%$

$$(d) \quad \alpha: 1.65\% \text{ Si} \quad \beta: 99.83\% \text{ Si} \quad \% \alpha = \frac{99.83 - 4}{99.83 - 1.65} = 97.6\% \quad \% \beta = 2.4\%$$

$$(e) \quad \begin{array}{ll} \text{primary } \alpha: 1.65\% \text{ Si} & \% \text{primary } \alpha = 78.5\% \\ \text{eutectic: } 12.6\% \text{ Si} & \% \text{eutectic} = 21.5\% \end{array}$$

$$(f) \quad \alpha: 0\% \text{ Si} \quad \beta: 100\% \text{ Si} \quad \% \alpha = \frac{100 - 4}{100 - 0} = 96\% \quad \% \beta = 4\%$$

10.14 Consider a Al-25% Si alloy. Determine (a) if the alloy is hypoeutectic or hypereutectic, (b) the composition of the first solid to form during solidification, (c) the amounts and compositions of each phase at 578°C, (d) the amounts and compositions of each phase at 576°C, (e) the amounts and compositions of each microconstituent at 576°C, and (f) the amounts and compositions of each phase at 25°C.

(a) hypereutectic

(b) 100% Si

$$(c) \quad \beta: 99.83\% \text{ Si} \quad L: 12.6\% \text{ Si} \quad \%L = \frac{99.83 - 25}{99.83 - 12.6} = 85.8\% \quad \% \beta = 14.2\%$$

$$(d) \quad \alpha: 1.65\% \text{ Si} \quad \beta: 99.83\% \text{ Si} \quad \% \alpha = \frac{99.83 - 25}{99.83 - 1.65} = 76.2\% \quad \% \beta = 23.8\%$$

$$(e) \quad \begin{array}{ll} \text{primary } \beta: 99.83\% \text{ Si} & \% \text{primary } \beta = 14.2\% \\ \text{eutectic: } 12.6\% \text{ Si} & \% \text{eutectic} = 85.8\% \end{array}$$

$$(f) \quad \alpha: 0\% \text{ Si} \quad \beta: 100\% \text{ Si} \quad \% \alpha = \frac{100 - 25}{100 - 0} = 75\% \quad \% \beta = 25\%$$

10.15 A Pb-Sn alloy contains 45% α and 55% β at 100°C. Determine the composition of the alloy. Is the alloy hypoeutectic or hypereutectic?

$$\% \alpha = 45 = \frac{98.0 - x}{98.0 - 5} \times 100 \quad \text{or} \quad x = 56.15\% \text{ Sn} \quad \text{Hypoeutectic}$$

10.16 An Al-Si alloy contains 85% α and 15% β at 500°C. Determine the composition of the alloy. Is the alloy hypoeutectic or hypereutectic?

$$\% \alpha = 85 = \frac{100 - x}{100 - 1} \times 100 \quad \text{or} \quad x = 15.85\% \text{ Si} \quad \text{Hypereutectic}$$

10.17 A Pb-Sn alloy contains 23% primary α and 77% eutectic microconstituent. Determine the composition of the alloy.

$$\% \text{ primary } \alpha = 23 = \frac{61.9 - x}{61.9 - 19} \times 100 \quad \text{or} \quad x = 52\% \text{ Sn}$$

10.18 An Al-Si alloy contains 15% primary β and 85% eutectic microconstituent. Determine the composition of the alloy.

$$\% \text{ eutectic} = 85 = \frac{100 - x}{100 - 12.6} \times 100 \quad \text{or} \quad x = 25.71\% \text{ Si}$$

10.19 Determine the maximum solubility for the following cases.

(a) lithium in aluminium (Figure 13.6)

- (b) aluminium in magnesium (Figure 13.8)
- (c) copper in zinc (Figure 13.10)
- (d) carbon in γ -iron (Figure 11.13)

- (a) 4% Li dissolves in aluminium
- (b) 12.7% Al dissolves in magnesium
- (c) 3% Cu dissolves in zinc
- (d) 2.11% C dissolves in γ -iron

10.20 Determine the maximum solubility for the following cases.

- (a) magnesium in aluminium (Figure 13.3)
- (b) zinc in copper (Figure 13.10)
- (c) beryllium in copper (Figure 13.10)
- (d) Al_2O_3 in MgO (Figure 14.9)

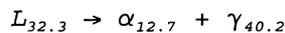
- (a) 14.9% Mg dissolves in aluminium
- (b) 40% Zn dissolves in copper
- (c) 2.5% Be dissolves in copper
- (d) 18% Al_2O_3 dissolves in MgO

10.21 Observation of a microstructure shows that there is 28% eutectic and 72% primary β in an Al-Li alloy (Figure 13.6). (a) Determine the composition of the alloy and whether it is hypoeutectic or hypereutectic. (b) How much α and β are in the eutectic microconstituent?

$$(a) \quad 28 = \frac{20.4 - x}{20.4 - 9.9} \times 100 \quad \text{or} \quad x = 17.46\% \text{ Li} \quad \text{Hypereutectic}$$

$$(b) \quad \% \alpha_{\text{Eut}} = \frac{20.4 - 9.9}{20.4 - 4} \times 100\% = 64\% \quad \text{and} \quad \% \beta_{\text{Eut}} = 36\%$$

10.22 Write the eutectic reaction that occurs, including the compositions of the three phases in equilibrium, and calculate the amount of α and β in the eutectic microconstituent in the Mg-Al system, Figure 13.8.



$$\therefore \% \alpha_{\text{Eut}} = \frac{40.2 - 32.3}{40.2 - 12.7} \times 100\% = 28.7\% \quad \text{and} \quad \% \gamma_{\text{Eut}} = 71.3\%$$

10.23 Calculate the total amount of α and β and the amount of each microconstituent in a Pb-50% Sn alloy at 182°C. What fraction of the total α in the alloy is contained in the eutectic microconstituent?

$$\alpha_{\text{total}} = \frac{97.5 - 50}{97.5 - 19} \times 100\% = 60.5\% \quad \beta_{\text{Total}} = 39.5\%$$

$$\alpha_{\text{Primary}} = \frac{61.9 - 50}{61.9 - 19} \times 100\% = 27.7\% \quad \text{Eut} = 72.3\%$$

$$\alpha_{\text{in eutectic}} = 60.5 - 27.7 = 32.8\%$$

$$f = 32.8/60.5 = 0.54$$

10.24 Figure 10.31 shows a cooling curve for a Pb-Sn alloy. Determine (a) the pouring temperature, (b) the superheat, (c) the liquidus temperature, (d) the eutectic temperature, (e) the freezing range, (f) the local solidification time, (g) the total solidification time, and (h) the composition of the alloy.

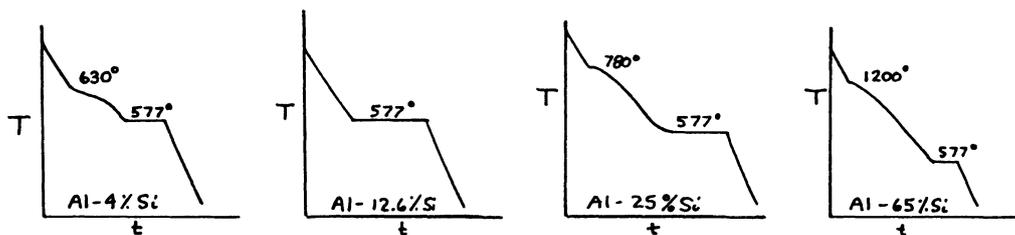
- (a) pouring temperature = 360°C
- (b) superheat = 360 - 250 = 110°C
- (c) liquidus temperature = 250°C
- (d) eutectic temperature = 183°C
- (e) freezing range = 250 - 183 = 67°C
- (f) local solidification time = 600 - 110 = 490 s
- (g) total solidification time = 600 s
- (h) approximately 32% Sn

10.25 Figure 10.32 shows a cooling curve for an Al-Si alloy. Determine (a) the pouring temperature, (b) the superheat, (c) the liquidus temperature, (d) the eutectic temperature, (e) the freezing range, (f) the local solidification time, (g) the total solidification time, and (h) the composition of the alloy.

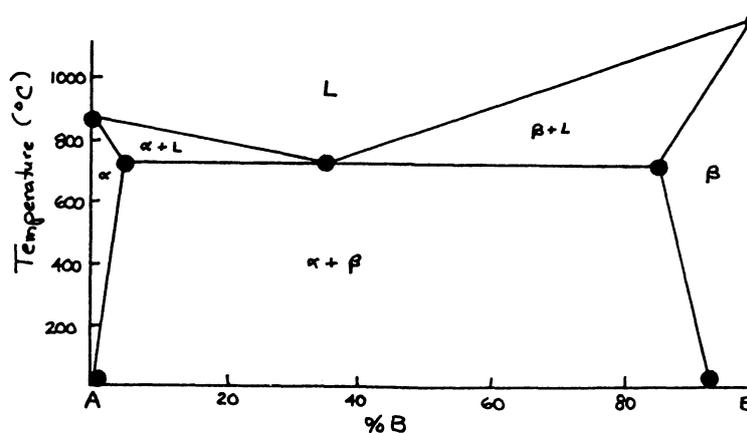
- (a) pouring temperature = 1150°C
- (b) superheat = 1150 - 1000 = 150°C
- (c) liquidus temperature = 1000°C
- (d) eutectic temperature = 577°C
- (e) freezing range = 1000 - 577 = 423°C
- (f) local solidification time = 11.5 - 1 = 10.5 min
- (g) total solidification time = 11.5 min
- (h) approximately 45% Si

10.26 Draw the cooling curves, including appropriate temperatures, expected for the following Al-Si alloys.

- (a) Al-4% Si (b) Al-12.6% Si (c) Al-25% Si (d) Al-65% Si

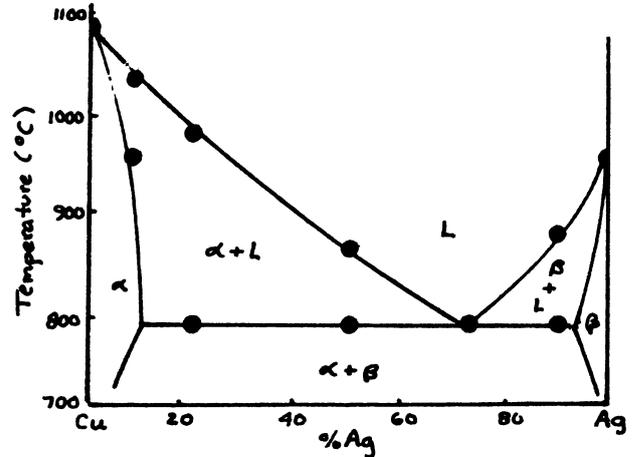


10.27 Based on the following observations, construct a phase diagram. Element A melts at 850°C and element B melts at 1200°C. Element B has a maximum solubility of 5% in element A, and element A has a maximum solubility of 15% in element B. The number of degrees of freedom from the phase rule is zero when the temperature is 725°C and there is 35% B present. At room temperature 1% B is soluble in A and 7% A is soluble in B.



10.28 Cooling curves are obtained for a series of Cu-Ag alloys, (Figure 10.33). Use this data to produce the Cu-Ag phase diagram. The maximum solubility of Ag in Cu is 7.9% and the maximum solubility of Cu in Ag is 8.8%. The solubilities at room temperature are near zero.

	T_{lig}	T_{sol}
0% Ag	$\rightarrow 1085^{\circ}\text{C}$	
8% Ag	$\rightarrow 1030^{\circ}\text{C}$	950°C
20% Ag	$\rightarrow 975^{\circ}\text{C}$	780°C
50% Ag	$\rightarrow 860^{\circ}\text{C}$	780°C
71.9% Ag	$\rightarrow 780^{\circ}\text{C}$	780°C
90% Ag	$\rightarrow 870^{\circ}\text{C}$	780°C
100% Ag	$\rightarrow 961^{\circ}\text{C}$	



10.29 The $\text{SiO}_2\text{-Al}_2\text{O}_3$ phase diagram is included in Figure 14.38. A refractory is required to contain molten metal at 1900°C . (a) Will pure Al_2O_3 be a potential candidate? Explain. (b) Will Al_2O_3 contaminated with 1% SiO_2 be a candidate? Explain.

(a) Yes. $T_m = 2040^{\circ}\text{C} > 1900^{\circ}\text{C}$ No liquid will form.

(b) No. Some liquid will form.

$$\%L = \frac{100 - 99}{100 - 80} \times 100\% = 5\% L$$

This liquid will weaken the refractory.

10.30 Consider the ternary phase diagram shown in Figures 10.28 and 10.29. Determine the liquidus temperature, the first solid to form, and the phases present at room temperature for the following compositions.

(a) A-30%B-20%C (b) A-10%B-25%C (c) A-60%B-10%C

(a) $T_{\text{Liq}} = 220^{\circ}\text{C}$; β ; $\alpha + \gamma + \beta$

(b) $T_{\text{Liq}} = 330^{\circ}\text{C}$; α ; $\alpha + \gamma$

(c) $T_{\text{Liq}} = 390^{\circ}\text{C}$; β ; $\alpha + \beta$

10.31 Consider the ternary phase diagram shown in Figures 10.28 and 29. Determine the liquidus temperature, the first solid to form, and the phases present at room temperature for the following compositions.

(a) A-5%B-80%C (b) A-50%B-5%C (c) A-30%B-35%C

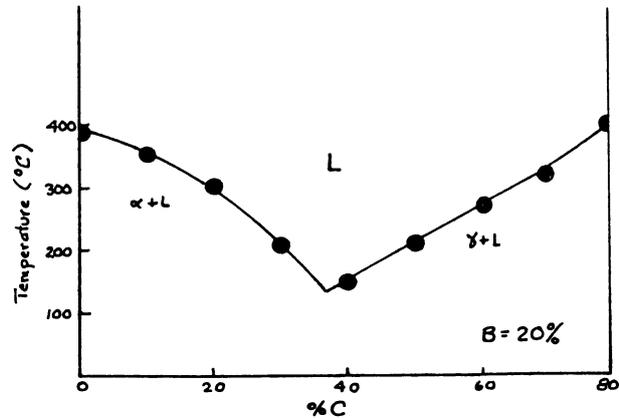
(a) $T_{\text{Liq}} = 390^{\circ}\text{C}$; γ ; $\alpha + \gamma$

(b) $T_{\text{Liq}} = 330^{\circ}\text{C}$; β ; $\alpha + \beta$

(c) $T_{\text{Liq}} = 290^{\circ}\text{C}$; β ; $\alpha + \beta + \gamma$

10.32 Consider the liquidus plot in Figure 10.28. (a) For a constant 20%B, draw a graph showing how the liquidus temperature changes from 80%A-20%B-0%C to 0%A-20%B-80%C, (b) What is the composition of the ternary eutectic in this system? (c) Estimate the temperature at which the ternary eutectic reaction occurs.

%A	%B	%C	$T_{liquidus}$
80	20	0	390°C
70	20	10	355°C
60	20	20	300°C
50	20	30	210°C
40	20	40	150°C
30	20	50	210°C
20	20	60	270°C
10	20	70	320°C
0	20	80	400°C

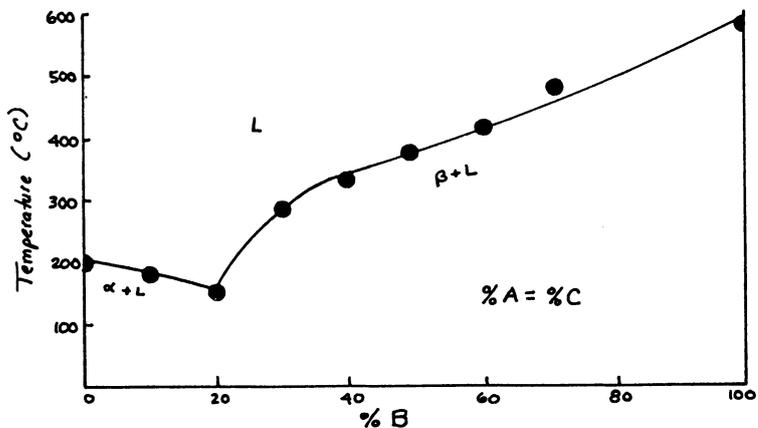


(b) The composition of the ternary eutectic is about 40% A - 20% B - 40% C

(c) The ternary eutectic temperature is about 150°C

10.33 From the liquidus plot in Figure 10.28, prepare a graph of liquidus temperature versus percent B for a constant ratio of materials A and C (that is, from pure B to 50%A-50%C on the liquidus plot). Material B melts at 580°C.

A	B	C	Temperature (°C)
50	0	50	200°C
45	10	45	180°C
40	20	40	150°C
35	30	35	280°C
30	40	30	330°C
25	50	25	375°C
20	60	20	415°C
15	70	15	485°C
0	100	0	580°C



Chapter 11 Dispersion Strengthening by Phase Transformation and Heat Treatment

11.1 Determine the constants c and n in Equation 11.2 that describe the rate of crystallization of polypropylene at 140°C . (See Figure 11.29)

$$f = 1 - \exp(-ct^n) \qquad T = 140^{\circ}\text{C} = 413 \text{ K}$$

We can rearrange the equation and eliminate the exponential by taking natural logarithms of both sides of the rearranged equation. We can then note that $\ln(1-f)$ versus t is a power equation; if these terms are plotted on a log-log plot, we should obtain a linear relationship, as the graph of the data below indicates. Note that in setting up the equation for plotting, we switch the minus sign from the right hand to the left hand side, since we don't have negative numbers on the log-log paper.

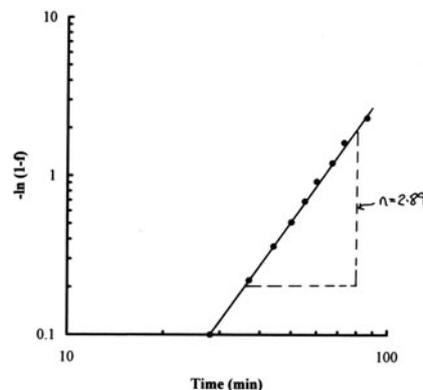
$1 - f = \exp(-ct^n)$	f	t (min)	$-\ln(1-f)$
$\ln(1-f) = -ct^n$	0.1	28	0.1
$\ln[-\ln(1-f)] = \ln(ct^n)$	0.2	37	0.22
$\ln[-\ln(1-f)] = \ln(c) + n \ln(t)$	0.3	44	0.36
	0.4	50	0.51
	0.5	55	0.69
	0.6	60	0.92
	0.7	67	1.20
	0.8	73	1.61
	0.9	86	2.302

A log-log plot of " $-\ln(1-f)$ " versus " t " is shown. From the graph, we find that the slope $n = 2.89$ and the constant c can be found from one of the points from the curve:

if $f = 0.5$, $t = 55$. Then

$$1 - 0.5 = \exp[-c(55)^{2.89}]$$

$$c = 6.47 \times 10^{-6}$$



11.2 Determine the constants c and n in Equation 11.2 that describe the rate of recrystallisation of copper at 135°C . (See Figure 11.2)

$$f = 1 - \exp(-ct^n)$$

$$T = 135^{\circ}\text{C} = 408 \text{ K}$$

We can rearrange the equation and eliminate the exponential by taking natural logarithms of both sides of the rearranged equation. We can then note that $\ln(1-f)$ versus t is a power equation and should give a linear relationship in a log-log plot. Note that in setting up the equation for plotting, we switch the minus sign from the right hand to the left hand side, since we don't have negative numbers on the log-log paper.

$$1 - f = \exp(-ct^n)$$

$$\ln(1-f) = -ct^n$$

$$\ln[-\ln(1-f)] = \ln(ct^n)$$

$$\ln[-\ln(1-f)] = \ln(c) + \ln(t)$$

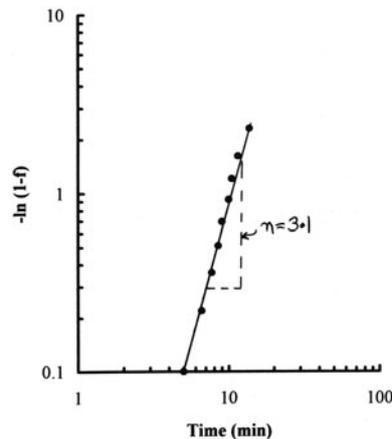
f	t (min)	$-\ln(1-f)$
0.1	5.0	0.10
0.2	6.6	0.22
0.3	7.7	0.36
0.4	8.5	0.51
0.5	9.0	0.69
0.6	10.0	0.92
0.7	10.5	1.20
0.8	11.5	1.61
0.9	13.7	2.30

A log-log plot of " $-\ln(1-f)$ " versus " t " is shown. From the graph, we find that the slope $n = 3.1$ and the constant c can be found from one of the points from the curve:

If $f = 0.6$, then $t = 10$. Then

$$1 - 0.6 = \exp[-c(10)^{3.1}]$$

$$c = 7.28 \times 10^{-4}$$



11.3 Determine the activation energy for crystallisation of polypropylene, using the curves in Figure 11.29.

We can determine how the rate (equal to $1/\tau$) changes with temperature:

$$\text{rate} = 1/\tau = A \exp(-Q/RT)$$

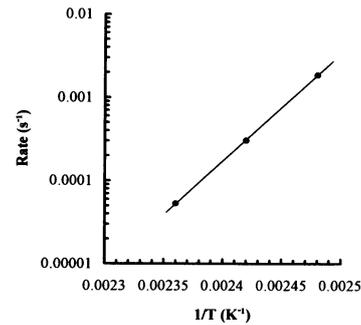
$1/\tau \text{ (s}^{-1}\text{)}$	$1/T \text{ (K}^{-1}\text{)}$
$1/(9 \text{ min})(60 \text{ s/min}) = 1.85 \times 10^{-3}$	$1/(130 + 273) = 2.48 \times 10^{-3}$
$1/(55 \text{ min})(60 \text{ s/min}) = 3.03 \times 10^{-4}$	$1/(140 + 273) = 2.42 \times 10^{-3}$
$1/(316 \text{ min})(60 \text{ s/min}) = 5.27 \times 10^{-5}$	$1/(150 + 273) = 2.36 \times 10^{-3}$

From the semilog graph of rate versus reciprocal temperature, we find that the slope is:

$$Q/R = \frac{\ln(10^{-3}) - \ln(7 \times 10^{-5})}{0.00246 - 0.00237}$$

$$Q/R = 29,547$$

$$Q = 245,654 \text{ J.mol}^{-1}$$



11.4 (a) Recommend an artificial age hardening heat treatment for a Cu-1.2% Be alloy (see Figure 13.10). Include appropriate temperatures. (b) Compare the amount of the γ_2 precipitate that forms by artificial aging at 400°C with the amount of the precipitate that forms by natural aging.

(a) For the Cu-1.2% Be alloy, the peritectic temperature is 870°C; above this temperature, liquid may form. The solvus temperature is about 530°C. Therefore:

- 1) Solution treat between 530°C and 870°C
(780°C is typical for beryllium copper alloys)
- 2) Quench
- 3) Age below 530°C (330°C is typical for these alloys)

(b) We can perform lever law calculations at 400°C and at room temperature. The solubility of Be in Cu at 400°C is about 0.7% Be and that at room temperature is about 0.2% Be:

$$\gamma_2 \text{ (at } 400^\circ \text{C)} = \frac{1.2 - 0.7}{11.7 - 0.7} \times 100 = 4.5\%$$

$$\gamma_2 \text{ (room T)} = \frac{1.2 - 0.2}{12 - 0.2} \times 100 = 8.5\%$$

11.5 Suppose that age hardening is possible in the Al-Mg system (see Figure 13.3). (a) Recommend an artificial age hardening heat treatment for each of the following alloys and (b) compare the amount of the β precipitate that forms from your treatment of each alloy. (c) Testing of the alloys after the heat treatment reveals that little strengthening occurs as a result of the heat treatment. Which of the requirements for age hardening is likely not satisfied?

(c) A-55% B: almost all θ is formed. The alloy is expected to be very brittle.

(d) A-87% B: the alloy cools from a two-phase ($\beta + \theta$) region to a one-phase (β) region, opposite of what we need for age hardening.

(e) A-95% B: the alloy is single phase (β) at all temperatures and thus cannot be age hardened.

11.8 Figure 11.1 shows the sigmoidal curve for the transformation of austenite. Determine the constants c and n in Equation 11.2 for this reaction. By comparing this figure with the TTT diagram, Figure 11.19, estimate the temperature at which this transformation occurred.

f	$1 - f$	$-\ln(1-f)$	t (s)
0.25	0.75	0.288	63 s
0.50	0.50	0.69	110 s
0.75	0.25	1.39	164 s

From the log-log plot of $-\ln(1-f)$ versus " t ", we find that the slope $n = 1.58$ and since $t = 110$ s when $f = 0.5$,

$$0.5 = 1 - \exp[-c(110)^{1.58}]$$

$$c = 4.12 \times 10^{-4}$$

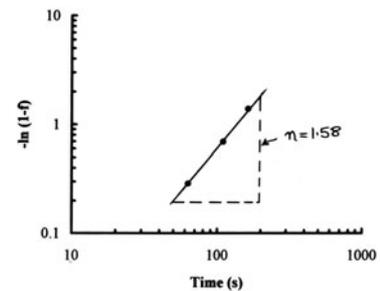


Figure 11.1 shows that the transformation begins at about 20 s and ends at about 720 s. Based on the TTT diagram (Figure 11.19), the transformation temperature must be about 680°C.

11.9 For an Fe-0.35%C alloy, determine (a) the temperature at which austenite first begins to transform on cooling, (b) the primary microconstituent that forms, (c) the composition and amount of each phase present at 728°C, (d) the composition and amount of each phase present at 726°C, and (e) the composition and amount of each microconstituent present at 726°C.

From Figure 11.3,

- (a) 795°C (b) primary α -ferrite
- (c) α : 0.0218% C $\% \alpha = \frac{0.77 - 0.35}{0.77 - 0.0218} \times 100 = 56.1\%$
- γ : 0.77% C $\% \gamma = 43.9\%$
- (d) α : 0.0218% C $\% \alpha = \frac{6.67 - 0.35}{6.67 - 0.0218} \times 100 = 95.1\%$
- Fe_3C : 6.67% C $\% \text{Fe}_3\text{C} = 4.9\%$
- (e) primary α : 0.0218 %C $\% \text{ primary } \alpha = 56.1\%$
- pearlite: 0.77 %C $\% \text{ Pearlite} = 43.9\%$

11.10 For an Fe-1.15%C alloy, determine (a) the temperature at which austenite first begins to transform on cooling, (b) the primary microconstituent that forms, (c) the composition and amount of each phase present at 728°C, (d) the composition and amount of each phase present at 726°C, and (e) the composition and amount of each microconstituent present at 726°C.

(a) 880°C (b) primary Fe₃C

(c) Fe₃C : 6.67% C %Fe₃C = $\frac{1.15 - 0.77}{6.67 - 0.77} \times 100 = 6.4\%$

γ : 0.77% C %γ = 93.6%

(d) α : 0.0218% C %α = $\frac{6.67 - 1.15}{6.67 - 0.0218} \times 100 = 83\%$

Fe₃C : 6.67% C %Fe₃C = 17%

(e) primary Fe₃C : 6.67 %C % primary Fe₃C = 6.4%

pearlite: 0.77 %C % Pearlite = 93.6%

11.11 A steel contains 8% cementite and 92% ferrite at room temperature. Estimate the carbon content of the steel. Is the steel hypoeutectoid or hypereutectoid?

Call x the carbon content of the steel.

$$\alpha = 0.92 = \frac{6.67 - x}{6.67 - 0} \quad x = 0.53\% \text{ C} \quad \text{Hypoeutectoid}$$

11.12 A steel contains 18% cementite and 82% ferrite at room temperature. Estimate the carbon content of the steel. Is the steel hypoeutectoid or hypereutectoid?

$$\alpha = 0.82 = \frac{6.67 - x}{6.67 - 0} \quad x = 1.20\% \text{ C} \quad \text{Hypereutectoid}$$

11.13 A steel contains 18% pearlite and 82% primary ferrite at room temperature. Estimate the carbon content of the steel. Is the steel hypoeutectoid or hypereutectoid?

$$\text{primary } \alpha = 0.82 = \frac{0.77 - x}{0.77 - 0.0} \quad x = 0.139\% \text{ C} \quad \text{Hypoeutectoid}$$

11.14 A steel contains 94% pearlite and 6% primary cementite at room temperature. Estimate the carbon content of the steel. Is the steel hypoeutectoid or hypereutectoid?

$$\text{Pearlite} = 0.94 = \frac{6.67 - x}{6.67 - 0.77} \quad x = 1.124\% \text{ C} \quad \text{Hypereutectoid}$$

11.15 A steel contains 55% α and 45% γ at 750°C. Estimate the carbon content of the steel.

$$\alpha = 0.02\% \text{ C} \quad \text{and} \quad \gamma = 0.6\% \text{ C} \quad (\text{from the tie line at } 750^\circ \text{C})$$

$$\% \alpha = 0.55 = \frac{0.6 - x}{0.6 - 0.02} \times 100 \quad x = 0.281\% \text{ C}$$

11.16 A steel contains 96% γ and 4% Fe_3C at 800°C . Estimate the carbon content of the steel.

$$\gamma = 0.92\% \text{ C} \quad \text{and} \quad \text{Fe}_3\text{C} = 6.67\% \text{ C} \quad (\text{from the tie line at } 800^\circ\text{C})$$

$$\gamma = 0.96 = \frac{6.67 - x}{6.67 - 0.92} \quad x = 1.15\% \text{ C}$$

11.17 A steel is heated until 40% austenite, with a carbon content of 0.5%, forms. Estimate the temperature and the overall carbon content of the steel.

In order for γ to contain 0.5% C, the austenitising temperature must be about 760°C (from the tie line). At this temperature:

$$0.4 = \frac{x - 0.02}{0.5 - 0.02} \quad x = 0.212\% \text{ C}$$

11.18 A steel is heated until 85% austenite, with a carbon content of 1.05%, forms. Estimate the temperature and the overall carbon content of the steel.

In order for γ to contain 1.05% C, the austenitising temperature must be about 845°C (from the tie line). At this temperature:

$$0.85 = \frac{6.67 - x}{6.67 - 1.05} \quad x = 1.893\% \text{ C}$$

11.19 Determine the eutectoid temperature, the composition of each phase in the eutectoid reaction, and the amount of each phase present in the eutectoid microconstituent for the following systems. Comment on whether you expect the eutectoid microconstituent to be ductile or brittle.

- (a) $\text{ZrO}_2\text{-CaO}$ (See Figure 14.23)
- (b) Cu-Al at 11.8%Al (See Figure 13.10)
- (c) Cu-Zn at 47%Zn (See Figure 13.10)
- (d) Cu-Be (See Figure 13.10)

(a) @ 870°C : $\text{Tetragonal}_{13\% \text{CaO}} \rightarrow \text{Monoclinic}_{3\% \text{CaO}} + \text{Cubic}_{16\% \text{CaO}}$

$$\% \text{Monoclinic} = \frac{16 - 13}{16 - 3} \times 100 = 23\% \quad \% \text{Cubic} = 77\%$$

The eutectoid microconstituent (and the entire material, for that matter) will be brittle because the materials are ceramics)

(b) @ 565°C : $\beta_{11.8\% \text{Al}} \rightarrow \alpha_{9.4\% \text{Al}} + \gamma_{15.6\% \text{Al}}$

$$\% \alpha = \frac{15.6 - 11.8}{15.6 - 9.4} \times 100 = 61.3\% \quad \% \beta = 38.7\%$$

Most of the eutectoid microconstituent is α (solid solution strengthened copper) and is expected to be ductile.

(c) @ 250°C : $\beta'_{47\% \text{Zn}} \rightarrow \alpha_{36\% \text{Zn}} + \gamma_{59\% \text{Zn}}$

$$\% \alpha = \frac{59 - 47}{59 - 36} \times 100 = 52.2\% \quad \% \gamma = 47.8\%$$

Slightly more than half of the eutectoid is the copper solid solution; there is a good chance that the eutectoid would be ductile.

(d) @605°C: $\gamma_{16\% \text{ Be}} \rightarrow \alpha_{1.5\% \text{ Be}} + \gamma_{211\% \text{ Be}}$

$$\% \alpha = \frac{11 - 6}{11 - 1.5} \times 100 = 52.6\% \quad \% \beta = 47.4\%$$

Slightly more than half of the eutectoid is the copper solid solution; we might then expect the eutectoid to be ductile.

11.20 Compare the interlamellar spacing and the yield strength when an eutectoid steel is isothermally transformed to pearlite at (a) 700°C and (b) 600°C.

We can find the interlamellar spacing from Figure 11.18 and then use this spacing to find the strength from Figure 11.17.

(a) $\lambda = 7.5 \times 10^{-4} \text{ mm}$ $1/\lambda = 1333.3$ $YS = 200 \text{ MN.m}^{-2}$

(b) $\lambda = 1.5 \times 10^{-4} \text{ mm}$ $1/\lambda = 6666.7$ $YS = 460 \text{ MN.m}^{-2}$

11.21 An isothermally transformed eutectoid steel is found to have a yield strength of 410 MN.m⁻². Estimate (a) the transformation temperature and (b) the interlamellar spacing in the pearlite.

We can first find the interlamellar spacing from Figure 11.17; then using this interlamellar spacing, we can find the transformation temperature from Figure 11.18.

(a) transformation temperature = 610°C

(b) $1/\lambda = 5,610$ or $\lambda = 1.78 \times 10^{-4} \text{ mm}$

11.22 Determine the required transformation temperature and microconstituent if an eutectoid steel is to have the following hardnesses.

(a) HRC 38 (b) HRC 42 (c) HRC 48 (d) HRC 52

(a) 600°C (b) 400°C (c) 340°C (d) 300°C
pearlite bainite bainite bainite

11.23 Describe the hardness and microstructure in an eutectoid steel that has been heated to 800°C for 1 h, quenched to 350°C and held for 750 s, and finally quenched to room temperature.

HRC = 47 and the microstructure is all bainite.

11.24 Describe the hardness and microstructure in an eutectoid steel that has been heated to 800°C, quenched to 650°C and held for 500 s, and finally quenched to room temperature.

HRC = 25 and the microstructure is all pearlite.

11.25 Describe the hardness and microstructure in an eutectoid steel that has been heated to 800°C, quenched to 300°C and held for 10 s, and finally quenched to room temperature.

HRC = 66 and the microstructure is all martensite.

11.26 Describe the hardness and microstructure in an eutectoid steel that has been heated to 800°C, quenched to 300°C and held for 10 s, quenched to room temperature, and then reheated to 400°C before finally cooling to room temperature again.

From Figure 11.26:

HRC = 43 and the microstructure is all tempered martensite.

11.27 A steel containing 0.3% C is heated to various temperatures above the eutectoid temperature, held for 1 h, and then quenched to room temperature. Using Figure 12.2, determine the amount, composition, and hardness of any martensite that forms when the heating temperature is

- (a) 728°C (b) 750°C (c) 790°C (d) 850°C

Information superimposed on the eutectoid portion of the Fe-Fe₃C phase diagram of Figure 12.2 is inconsistent with Figure 11.13. It is recommended that the maximum solubility limit of carbon in ferrite is taken as 0.0218%C, the eutectoid composition as 0.77%C and the A₁ temperature as 727°C. Thus, the values superimposed on Figure 12.2 should be amended.

- (a) $\gamma: 0.77\% \text{ C}$ $\%M = \frac{0.3 - 0.0218}{0.77 - 0.0218} \times 100\% = 37.2\%$ HRC 65
 $= \alpha'$
- (b) $\gamma: 0.60\% \text{ C}$ $\%M = \frac{0.3 - 0.02}{0.6 - 0.02} \times 100\% = 48.3\%$ HRC 65
 $= \alpha'$
- (c) $\gamma: 0.35\% \text{ C}$ $\%M = \frac{0.3 - 0.02}{0.35 - 0.02} \times 100\% = 84.8\%$ HRC 58
- (d) $\gamma: 0.3\% \text{ C}$ $\%M = 100\%$ HRC 55

11.28 A steel containing 0.95% C is heated to various temperatures above the eutectoid temperature, held for 1 h, and then quenched to room temperature. Using Figure 12.2, determine the amount and composition of any martensite that forms when the heating temperature is

- (a) 728°C (b) 750°C (c) 780°C (d) 850°C
- (a) $\gamma = 0.77\% \text{ C}$ $\%M = \frac{6.67 - 0.95}{6.67 - 0.77} \times 100\% = 96.9\%$ HRC 65
- (b) $\gamma = 0.79\% \text{ C}$ $\%M = \frac{6.67 - 0.95}{6.67 - 0.79} \times 100\% = 97.3\%$ HRC 65
- (c) $\gamma = 0.88\% \text{ C}$ $\%M = \frac{6.67 - 0.95}{6.67 - 0.88} \times 100\% = 98.8\%$ HRC 65
- (d) $\gamma = 0.95\% \text{ C}$ $\%M = 100\%$ HRC 65

11.29 A steel microstructure contains 75% martensite and 25% ferrite; the composition of the martensite is 0.6% C. Using Figure 12.2, determine (a) the temperature from which the steel was quenched and (b) the carbon content of the steel.

In order for γ (and therefore martensite) to contain 0.6% C, the austenitising $T = 750^\circ\text{C}$. Then:

$$M = \gamma = 0.75 = \frac{x - 0.02}{0.6 - 0.02} \qquad x = 0.455\% \text{ C}$$

11.30 A steel microstructure contains 92% martensite and 8% Fe₃C; the composition of the martensite is 1.10% C. Using Figure 12.2, determine (a) the temperature from which the steel was quenched and (b) the carbon content of the steel.

In order for γ (and therefore martensite) to contain 1.10% C, the austenitising $T = 865^{\circ}\text{C}$. Then:

$$M = \gamma = 0.92 = \frac{6.67 - x}{6.67 - 1.10} \quad x = 1.55\% \text{ C}$$

11.31 A steel containing 0.8% C is quenched to produce all martensite. Estimate the volume change that occurs, assuming that the lattice parameter of the austenite is 0.36 nm. Does the steel expand or contract during quenching?

$$V_{\gamma} = (0.36 \text{ nm})^3 = (3.6 \times 10^{-10} \text{ m})^3 = 4.6656 \times 10^{-29} \text{ m}^3$$

From Figure 11.23(b), the lattice parameters and volume of the unit cell of the martensite are about:

$$a = 0.285 \text{ nm}, \quad c = 0.296 \text{ nm}, \quad V_m = a^2c = 2.40426 \times 10^{-29} \text{ m}^3$$

However, there are two atoms/cell in body centred tetragonal martensite and four atoms/cell in face centred cubic austenite. Thus the volume change is

$$\begin{aligned} \% \Delta V &= \frac{(2)(2.40426 \times 10^{-29}) - 4.6656 \times 10^{-29}}{4.6656 \times 10^{-29}} \times 100\% \\ &= 3.06\% \text{ expansion.} \end{aligned}$$

11.32 Describe the complete heat treatment required to produce a quenched and tempered eutectoid steel having a tensile strength of at least 860 MN.m⁻². Include appropriate temperatures.

*Austenitise at approximately 750°C,
Quench to below 130°C (the M_f temperature)
Temper at 620°C or less
Cool to room temperature.*

11.33 Describe the complete heat treatment required to produce a quenched and tempered eutectoid steel having a HRC hardness of less than 50. Include appropriate temperatures.

*Austenitise at approximately 750°C,
Quench to below the M_f (less than 130°C)
Temper at a temperature higher than 330°C, but less than 727°C
Cool to room temperature.*

11.34 In eutectic alloys, the eutectic microconstituent is generally the continuous one, but in the eutectoid structures, the primary microconstituent is normally continuous. By describing the changes that occur with decreasing temperature in each reaction, explain why this difference is expected.

In a eutectoid reaction, the original grain boundaries serve as nucleation sites; consequently the primary microconstituent outlines the original grain boundaries and isolates the eutectoid product as a discontinuous constituent.

In a eutectic reaction, the primary phase nucleates from the liquid and

grows. When the liquid composition approaches the eutectic composition, the eutectic constituent forms around the primary constituent, making the eutectic product the continuous constituent.

11.35 Describe how the memory metals might be useful as plates to be surgically placed around broken bones to provide more rapid healing.

We could make the plates a little longer than is required. The plates would then be fastened to the bone on either side of the fracture with screws. The heat of the body would cause the plate to attempt to become shorter, forcing the two halves of the bone securely together to give a good tight fit.

Chapter 12 Ferrous Alloys

12.1 Calculate the amounts of ferrite, cementite, primary microconstituent, and pearlite in the following plain carbon steels using Figure 12.2 noting comments made for solution 11.27.

(a) 0.15%C

$$\alpha = \frac{6.67 - 0.15}{6.67 - 0} \times 100 = 97.8\% \quad \text{Fe}_3\text{C} = 2.2\%$$

$$\text{primary } \alpha = \frac{0.77 - 0.15}{0.77 - 0.0218} \times 100 = 82.9\% \quad \text{pearlite} = 17.1\%$$

(b) 0.35%C

$$\alpha = \frac{6.67 - 0.35}{6.67 - 0} \times 100 = 94.8\% \quad \text{Fe}_3\text{C} = 5.2\%$$

$$\text{primary } \alpha = \frac{0.77 - 0.35}{0.77 - 0.0218} \times 100 = 56.1\% \quad \text{pearlite} = 43.9\%$$

(c) 0.95%C

$$\alpha = \frac{6.67 - 0.95}{6.67 - 0} \times 100 = 85.8\% \quad \text{Fe}_3\text{C} = 14.2\%$$

$$\text{primary } \text{Fe}_3\text{C} = \frac{0.95 - 0.77}{6.67 - 0.77} \times 100 = 3.1\% \quad \text{pearlite} = 96.9\%$$

(d) 1.30%C

$$\alpha = \frac{6.67 - 1.30}{6.67 - 0} \times 100 = 80.5\% \quad \text{Fe}_3\text{C} = 19.5\%$$

$$\text{primary } \text{Fe}_3\text{C} = \frac{1.30 - 0.77}{6.67 - 0.77} \times 100 = 9.0\% \quad \text{pearlite} = 91.0\%$$

12.2 Estimate the % carbon for steels having the following microconstituents: using Figure 12.2 noting comments made for solution 11.27.

(a) 38% pearlite - 62% primary ferrite

$$62\% = \frac{0.77 - x}{0.77 - 0.0218} \times 100 \quad x = 0.306\% \text{ C}$$

(b) 93% pearlite - 7% primary cementite

$$93\% = \frac{6.67 - x}{6.67 - 0.77} \times 100 \quad x = 1.183\% \text{ C}$$

(c) 97% ferrite - 3% cementite

$$97\% = \frac{6.67 - x}{6.67 - 0} \times 100 \quad x = 0.200\% \text{ C}$$

(d) 86% ferrite - 14% cementite

$$86\% = \frac{6.67 - x}{6.67 - 0} \times 100$$

$$x = 0.934\% \text{ C}$$

12.3 Complete the following table:

From Fig. 12.2 find A_1 , A_3 or A_{cm}
 A_{cm} for each steel

	<u>0.35% C steel</u>	<u>1.15% C steel</u>
A_1 temperature	727°C	727°C
A_3 or A_{cm} temperature	790°C	880°C
Full annealing temperature	820°C	757°C
Normalising temperature	845°C	935°C
Process annealing temperature	557-647°C	--
Spheroidising temperature	--	697°C

12.4 The pearlitic in a eutectoid steel consists of cementite platelets which are 4×10^{-4} mm thick, and the ferrite platelets are 14×10^{-4} mm thick. In a spheroidised 0.8% C steel, the cementite spheres are 4×10^{-2} mm in diameter. Estimate the total interface area between the ferrite and cementite in a cubic centimetre of each steel. Determine the percent reduction in surface area when the pearlitic steel is spheroidised. The density of ferrite is 7.87 Mg.m^{-3} and that of cementite is 7.66 Mg.m^{-3} .

First, we can determine the weight and volume percents of Fe_3C in the steel:

$$\text{wt\% Fe}_3\text{C} = \frac{0.80 - 0.0218}{6.67 - 0.0218} \times 100 = 11.705$$

$$\text{vol\% Fe}_3\text{C} = \frac{11.705 / 7.66}{(11.705 / 7.66) + (88.295 / 7.87)} \times 100 = 11.987$$

Pearlite: Based on the thicknesses of the ferrite and cementite platelets in pearlite, there are two interfaces per $(4 \times 10^{-4} \text{ mm} + 14 \times 10^{-4} \text{ mm}) = 18 \times 10^{-4} \text{ mm}$, or:

$$2 \text{ interfaces} / 18 \times 10^{-4} \text{ mm} = 1.1 \times 10^4 \text{ interfaces/cm}$$

If all of the platelets are parallel to one another, then in 1 cm^3 of pearlite, there is a total of

$$A = (1.1 \times 10^4 / \text{cm}) (1 \text{ cm}^3) = 11,000 \text{ cm}^2 \text{ of interface/cm}^3$$

Spheroidite: The volume of an Fe_3C sphere with $r = 2 \times 10^{-3} \text{ cm}$ is:

$$V = (4\pi/3) (2 \times 10^{-3} \text{ cm})^3 = 3.35 \times 10^{-8} \text{ cm}^3$$

The volume of Fe_3C in 1 cm^3 of spheroidite is given by the volume fraction of cementite, or 0.11987. The number of spheres in 1 cm^3 of spheroidite is:

$$\text{number} = 0.11987 \text{ cm}^3 / 3.35 \times 10^{-8} \text{ cm}^3 = 3.58 \times 10^6 \text{ spheres/cm}^3$$

The surface area of the spheres is therefore:

$$A = 4\pi (2 \times 10^{-3} \text{ cm})^2 (3.58 \times 10^6 \text{ spheres/cm}^3) \\ = 180 \text{ cm}^2 \text{ of interface/cm}^3$$

The percent reduction in surface area during spheroidising is then:

$$\% = \frac{(11,000 - 180) \text{ cm}^2}{11,000 \text{ cm}^2} \times 100 = 98.4\%$$

12.5 Describe the microstructure present in a 0.5%C steel after each step in the following heat treatments:

(a) Heat at 820°C, quench to 650°C and hold for 90 s, and quench to 25°C:

Austenite is present after heating to 820 °C; both ferrite and pearlite form during holding at 650°C; ferrite and pearlite remain after cooling to 25°C.

(b) Heat at 820°C, quench to 450°C and hold for 90 s, and quench to 25°C:

Austenite is present after heating to 820°C; bainite forms after holding at 450°C; and bainite remains after cooling.

(c) Heat at 820°C and quench to 25°C:

Austenite is present after heating to 820°C; martensite forms due to the quench.

(d) Heat at 820°C, quench to 720°C and hold for 100 s, and quench to 25°C:

Austenite is present after heating to 820°C; ferrite forms at 720°C, but some austenite still remains. During quenching, the remaining austenite forms martensite; the final structure is ferrite and martensite.

(e) Heat at 820°C, quench to 720°C and hold for 100 s, quench to 400°C and hold for 500 s, and quench to 25°C:

Austenite is present after heating to 820°C; ferrite begins to form at 720°C, but austenite still remains. At 400°C, the remaining austenite transforms to bainite; the final structure contains ferrite and bainite.

(f) Heat at 820°C, quench to 720°C and hold for 100 s, quench to 400°C and hold for 10 s, and quench to 25°C:

Austenite is present after heating to 820°C; ferrite begins to form at 720°C; some of the remaining austenite transforms to bainite at 400°C, but some austenite still remains after 10 s; the remaining austenite transforms to martensite during quenching. The final structure is ferrite, bainite, and martensite.

(g) Heat at 820°C, quench to 25°C, heat to 500°C and hold for 10³ s, and air cool to 25°C:

Austenite is present after heating to 820°C. The austenite transforms to martensite during quenching. During reheating to 500°C, the martensite tempers. The final structure is tempered martensite. Note that the TTT diagram isn't really needed for this part of the question.

12.6 Describe the microstructure present in a 1.1%C steel after each step in the following heat treatments:

(a) Heat to 900°C, quench to 400°C and hold for 10³ s, and quench to 25°C:

Austenite forms at 900°C. At 400°C, all of the austenite transforms to bainite. The final structure is all bainite.

- (b) Heat to 900 °C, quench to 600 °C and hold for 50 s, and quench to 25 °C:

Austenite forms at 900°C. At 600°C, all of the austenite transforms to cementite and pearlite, which gives the final structure.

- (c) Heat to 900°C and quench to 25°C:

Austenite forms at 900°C. All of the austenite transforms to martensite during quenching.

- (d) Heat to 900°C, quench to 300°C and hold for 200 s, and quench to 25°C:

Austenite forms at 900°C. None of the austenite transforms within 200 s at 300°C; consequently all of the austenite forms martensite during quenching. This is a martempering heat treatment.

- (e) Heat to 900°C, quench to 675°C and hold for 1 s, and quench to 25°C:

Austenite forms at 900°C. Cementite begins to form at 675°C; the remainder of the austenite transforms to martensite during quenching to 25°C. The final structure is cementite and martensite.

- (f) Heat to 900°C, quench to 675°C and hold for 1 s, quench to 400°C and hold for 900 s, and slowly cool to 25°C:

Austenite forms at 900°C. Cementite begins to form at 675°C. The remaining austenite transforms to bainite at 400°C. The final structure is cementite and bainite.

- (g) Heat to 900°C, quench to 675°C and hold for 1 s, quench to 300°C and hold for 10³ s, and air cool to 25°C.

Austenite forms at 900°C. Cementite begins to form at 675°C. At 300°C, some of the remaining austenite transforms to bainite, but the B_f line is not crossed. The remaining austenite forms martensite during air cooling. The final structure is cementite, bainite, and martensite.

- (h) Heat to 900°C, quench to 300°C and hold for 100 s, quench to 25°C, heat to 450°C for 3600 s, and cool to 25°C.

Austenite forms at 900°C. No transformation occurs at 300°C, since the time is too short. Consequently all of the austenite transforms to martensite during quenching. Reheating to 450°C for 3600 s (1 hour) tempers the martensite. The final structure is tempered martensite.

12.7 Recommend appropriate isothermal heat treatments to obtain the following, including appropriate temperatures and times:

- (a) an isothermally annealed 0.5%C steel with HRC 23:

*Austenitise at 820°C
Quench to 600°C and hold for more than 10 s
Cool to room temperature*

(b) an isothermally annealed 1.1%C steel with HRC 40:

*Austenitise at 900°C
quench to 640°C and hold for more than 10 s
Cool to room temperature*

(c) an isothermally annealed 0.8%C steel with HRC 38:

*Austenitise at 780°C
Quench to 600°C for more than 10 s
Cool to room temperature*

(d) an austempered 0.5%C steel with HRC 40:

*Austenitise at 820°C
quench to 390°C and hold for 100 s
Cool to room temperature*

(e) an austempered 1.1%C steel with HRC 55:

*Austenitise at 900°C
quench to 320°C and hold for 5000 s
Cool to room temperature*

(f) an austempered 0.8%C steel with HRC 50:

*Austenitise at 780°C
quench to 330°C and hold for 1000 s
Cool to room temperature*

12.8 Compare the minimum times required to isothermally anneal the following steels at 600°C. Discuss the effect of the carbon content of the steel on the kinetics of nucleation and growth during the heat treatment.

(a) 0.5%C: *The P_f time is about 5 s, the minimum time*

(b) 0.8%C: *The P_f time is about 10 s, the minimum time*

(c) 1.1%C: *The P_f time is about 3 s, the minimum time*

The carbon content has relatively little effect on the minimum annealing time (or the P_f time). The longest time is obtained for the 0.8%C, or eutectoid, steel.

12.9 We wish to produce a 0.5%C steel that has a Brinell hardness of at least 330 and an elongation of at least 15% (a) Recommend a heat treatment, including appropriate temperatures, that permits this to be achieved. Determine the yield strength and tensile strength that are obtained by this heat treatment. (b) What yield and tensile strength would be obtained in a 0.8%C steel by the same heat treatment? (c) What yield strength, tensile strength, and %Elongation would be obtained in the 0.5%C steel if it were normalised?

(a) Referring to Fig. 12.11, it is possible to obtain the required properties; the Brinell hardness is obtained if the steel is quenched and then tempered at a temperature below 480°C, and the %Elongation can be obtained if the tempering temperature is greater than 420°C. Therefore a possible heat treatment would be:

*Austenitise at 820°C
quench to room temperature
Temper between 420°C and 480°C
Cool to room temperature*

The quench and temper heat treatment will also give a yield strength between 945 MN.m^{-2} and 1100 MN.m^{-2} , while the tensile strength will be between 1044 MN.m^{-2} and 1240 MN.m^{-2} . The higher strengths are obtained for the lower tempering temperatures.

(b) If a 0.8%C steel is tempered in the same way (Figure 11.26), the yield strength would lie between 890 MN.m^{-2} and 940 MN.m^{-2} , and the tensile strength would be 1230 to 1280 MN.m^{-2} . The higher strengths are obtained for the lower tempering temperatures.

(c) If the 1050 steel were normalised rather than quench and tempered, the properties would be about (from Figure 12.5):

720 MN.m^{-2} tensile strength
448 MN.m^{-2} yield strength
20% Elongation

12.10 We wish to produce a 0.5%C steel that has a tensile strength of at least 1200 MN.m^{-2} and a %Reduction in area of at least 50%. (a) Recommend a heat treatment, including appropriate temperatures, that permits this to be achieved. Determine the Brinell hardness number, %Elongation, and yield strength that are obtained by this heat treatment. (b) What yield strength and tensile strength would be obtained in a 0.8%C steel by the same heat treatment? (c) What yield strength, tensile strength, and %Elongation would be obtained in the 0.5%C steel if it were annealed?

(a) Using a quench and temper heat treatment, we can obtain the minimum tensile strength by tempering below 430°C , and the minimum reduction in area by tempering above 400°C . Our heat treatment is then:

From Figure 12.11
Austenitise at 820°C
Quench to room temperature
Temper between 400°C and 430°C
Cool to room temperature

This heat treatment will also give:
390 to 410 BH
13 to 15% Elongation
1055 to 1150 MN.m^{-2} yield strength

(b) If the same treatment is used for a 0.8%C steel, the properties would be:

Figure 11.26
935 to 950 MN.m^{-2} yield strength
1267 to 1287 MN.m^{-2} tensile strength

(c) If the 0.5%C steel is annealed, the properties are (From Figure 12.5)

380 MN.m^{-2} yield strength
600 MN.m^{-2} tensile strength
25% Elongation

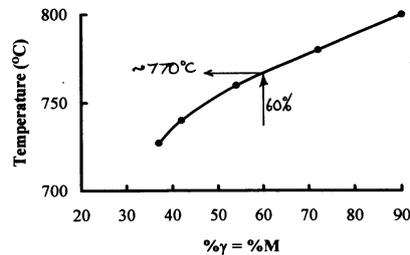
12.11 A 0.3%C steel is given an improper quench and temper heat treatment, producing a final structure composed of 60% martensite and 40% ferrite. Estimate the carbon content of the martensite and the austenitising temperature that was used. What austenitising temperature would you recommend?

We can work a lever law at several temperatures in the $\alpha + \gamma$ region of the iron-carbon phase diagram, finding the amount of austenite (and its composition) at each temperature. The composition of the ferrite at each of these temperatures is about 0.02% C. The amount and composition of the martensite that forms will be the same as that of the austenite:

at 800°C:	γ : 0.33%C	$\% \gamma = (0.30 - 0.02) / (0.33 - 0.02) = 90\%$
at 780°C:	γ : 0.41%C	$\% \gamma = (0.30 - 0.02) / (0.41 - 0.02) = 72\%$
at 760°C:	γ : 0.54%C	$\% \gamma = (0.30 - 0.02) / (0.54 - 0.02) = 54\%$
at 740°C:	γ : 0.68%C	$\% \gamma = (0.30 - 0.02) / (0.68 - 0.02) = 42\%$
at 727°C:	γ : 0.77%C	$\% \gamma = (0.30 - 0.02) / (0.77 - 0.02) = 37\%$

The amount of austenite (equal to that of the martensite) is plotted versus temperature in the graph. Based on this graph, 60% martensite forms when the austenitising temperature is about 770°C. The carbon content of the martensite that forms is about 0.48%C.

The A_3 temperature of the steel is about 805°C. A proper heat treatment might use an austenitising temperature of about 805°C + 55°C = 860°C.



12.12 A 0.5%C steel should be austenitised at 820°C, quenched in oil to 25°C, and tempered at 400°C for an appropriate time. (a) What yield strength, hardness, and % Elongation would you expect to obtain from this heat treatment? (b) Suppose the actual yield strength of the steel is found to be 860 MN.m⁻². What might have gone wrong in the heat treatment to cause this low strength? (c) Suppose the hardness is found to be HB 525. What might have gone wrong in the heat treatment to cause this high hardness?

(a) The properties expected for a proper heat treatment are:

1125 MPa yield strength
1685 MPa tensile strength
413 HB
13% Elongation

(b) If the yield strength is 860 MN.m⁻² (much lower than expected), then the tempering process might have been done at a tempering temperature greater than 400°C (perhaps as high as 500°C). Another possible problem could be an austenitising temperature that was lower than 820°C (even lower than about 770°C, the A_3), preventing complete austenitising and thus not all martensite during the quench.

(c) If the hardness is HB 525 (higher than expected), the tempering temperature may have been too low or, in fact, the steel probably was not tempered at all.

12.13 A part produced from a low alloy, 0.2% C steel (Figure 12.17) has a microstructure containing ferrite, pearlite, bainite, and martensite after quenching. What microstructure would be obtained if we used a 0.8%C steel? What microstructure would be obtained if we used a 4340 steel?

To produce ferrite, pearlite, bainite, and martensite in the same microstructure during continuous cooling, the cooling rate must have been between 10 and 20°C/s. If the same cooling rates are used for the other steels, the microstructures are:

0.8%C steel: fine pearlite (from Fig. 12.16)
4340 steel: martensite (from Fig. 12.18)

12.14 Fine pearlite and a small amount of martensite are found in a quenched 1080 steel. What microstructure would be expected if we used a low alloy, 0.2% C steel? What microstructure would be obtained if we used a 4340 steel?

A cooling rate of about 50°C/s will produce fine pearlite and a small amount of martensite in the 1080 steel. For the same cooling rate, the microstructure in the other steels will be:

low alloy, 0.2% C steel: ferrite, bainite, and martensite
4340 steel: martensite

12.15 We have found that a 1070 steel, when austenitised at 750°C, forms a structure containing pearlite and a small amount of grain boundary ferrite that gives acceptable strength and ductility. What changes in the microstructure, if any, would be expected if the 1070 steel contained an alloying element, such as Mo or Cr? Explain.

The alloying element may shift the eutectoid carbon content to below 0.7% C, making the steel hypereutectoid rather than hypoeutectoid. This in turn means that grain boundary Fe₃C will form rather than grain boundary ferrite. The grain boundary Fe₃C will embrittle the steel.

12.16 Using the TTT diagrams, compare the hardenabilities of 4340 and 1050 steels by determining the times required for isothermal transformation of ferrite and pearlite (F_s , P_s , and P_f) to occur at 650°C.

From the diagrams, we can find the appropriate times:

4340 steel:	$F_s = 165 \text{ s}$	$P_s = 3,675 \text{ s}$	$P_f = 12,220 \text{ s}$
1050 steel:	$F_s = 2 \text{ s}$	$P_s = 5 \text{ s}$	$P_f = 20 \text{ s}$

Because the transformation times are much longer for the 4340 steel, the 4340 steel has the higher hardenability.

12.17 We would like to obtain a hardness of HRC 38 to 40 in a quenched steel. What range of cooling rates would we have to obtain for the following steels? Are some steels inappropriate?

(a) 4340: not applicable; the hardnesses are always much higher than the desired range.

(b) 8640: a Jominy distance of about 28 to 32 mm is required to give the desired hardness; this corresponds to a cooling rate of about 4 to 5°C/s.

(c) 9310: a Jominy distance of 15 to 18 mm is required to give the desired hardness; this corresponds to a cooling rate of 9 to 12°C/s.

(d) 4320: a Jominy distance of about 9.5 to 12 mm is required to give the desired hardness; this corresponds to a cooling rate of 16 to 25°C/s.

(e) 1050: a Jominy distance of 6 to 6.5 mm is required to give the desired hardness; this corresponds to a cooling rate of 40 to 44°C/s.

(f) 1080: a Jominy distance of 8.5 to 11 mm is required to give the desired hardness; this corresponds to a cooling rate of 17.5 to 25°C/s.

12.18 A steel part must have an as-quenched hardness of HRC 35 in order to avoid excessive wear rates during use. When the part is made from 4320 steel, the hardness is only HRC 32. Determine the hardness if the part were made under identical conditions, but with the following steels. Which, if any, of these steels would be better choices than 4320?

The Jominy distance that gives a hardness of HRC 32 in the 4320 steel is 14 mm. The cooling rates, and hence Jominy distances, will be the same 14 mm for the other steels. From the hardenability curves, the hardnesses of the other steels are

- | | |
|------------------|------------------|
| (a) 4340: HRC 60 | (b) 8640: HRC 53 |
| (c) 9310: HRC 40 | (d) 1050: HRC 28 |
| (e) 1080: HRC 36 | |

All of the steels except the 1050 steel would develop an as-quenched hardness of at least HRC 35 and would be better choices than the 4320 steel. The 1080 steel might be the best choice, since it will likely be the least expensive (no alloying elements present).

12.19 A part produced from a 4320 steel has a hardness of HRC 35 at a critical location after quenching. Determine (a) the cooling rate at that location and (b) the microstructure and hardness that would be obtained if the part were made of a 1080 steel.

(a) To obtain the HRC 35 in a 4320 steel, the Jominy distance must be about 11 mm, corresponding to a cooling rate of 17.5°C/s.

(b) If the part is produced in a 1080 steel, the cooling rate will still be about 17.5°C/s. From the CCT diagram for the 1080 steel, the part will contain all pearlite, with a hardness of HRC 38.

12.20 A 1080 steel is cooled at the fastest possible rate that still permits all pearlite to form. What cooling rate, Jominy distance, and hardness are expected for this cooling rate?

The fastest possible cooling rate that still permits all pearlite is about 40°C/s. This cooling rate corresponds to a Jominy distance of about 6.5 mm. From the hardenability curve, the hardness will be HRC 44.

12.21 Determine the hardness and the microstructure at the centre of a 38 mm diameter 1080 steel bar produced by quenching in (a) unagitated oil, (b) unagitated water, and (c) agitated brine.

(a) unagitated oil: the H-factor for the 38 mm bar is 0.25. The Jominy distance will be about 17 mm, or a cooling rate of 10°C/s. From the CCT diagram, the steel is all pearlite. The hardness at centre of the bar is HRC 36.

(b) unagitated water: the H-factor for the bar is 1.0. The Jominy distance will be about 8 mm, or a cooling rate of 30°C/s. From the CCT diagram, the steel will contain pearlite. The hardness at the centre of the bar is HRC 42.

(c) agitated brine: the H-factor is now 5.0. The Jominy distance is about 5.5 mm, or a cooling rate of 50°C/s. The steel has a hardness of HRC 48 and the microstructure contains both pearlite and martensite.

12.22 A 50 mm diameter bar of 4320 steel is to have a hardness of at least HRC 35. What is the minimum severity of the quench (H coefficient)? What type of quenching medium would you recommend to produce the desired hardness with the least chance of quench cracking?

The hardness of HRC 35 is produced by a Jominy distance of 11 mm. In order to produce this Jominy distance in a 50 mm diameter bar, the H-coefficient must be greater or equal to 1.0. All of the quenching media described in Table 12.2 will provide this Jominy distance except unagitated oil. To prevent quench cracking, we would like to use the least severe quenchant; agitated oil and unagitated water, with $H = 1.0$, might be the best choices.

12.23 A steel bar is to be quenched in agitated water. Determine the maximum diameter of the bar that will produce a minimum hardness of HRC 40 if the bar is:

(a) 1050 steel: The H-coefficient for the agitated water is 4.0. For the 1050 steel, the Jominy distance must be at the most 6 mm to produce the desired hardness. Therefore the maximum diameter that will permit this Jominy distance (or cooling rate) is 39 mm.

(b) 1080 steel: Now the Jominy distance must be at the most 8 mm, with the same H-coefficient. The maximum diameter allowed is 48 mm.

(c) 4320 steel: The maximum Jominy distance is 8 mm, and the maximum diameter of the bar is 48 mm.

(d) 8640 steel: The minimum Jominy distance is 28 mm. Consequently bars with a maximum diameter of much greater than 60 mm will produce the desired cooling rate and hardness.

(e) 4340 steel: Bars with a maximum diameter of much greater than 60 mm produce the required cooling rate.

12.24 The centre of a 25 mm diameter bar of 4320 steel has a hardness of HRC 40. Determine the hardness and microstructure at the centre of a 50 mm bar of 1050 steel quenched in the same medium.

To obtain HRC 40 in the 4320 steel, we need a Jominy distance of 8 mm. For a 25 mm diameter bar, the quenching medium must have a minimum H-coefficient of 0.4. Therefore, if a 50 mm diameter bar is quenched in the same medium (i.e. $H = 0.4$), the Jominy distance will be about 6 mm; this Jominy distance produces a hardness of HRC 27 in a 1050 steel.

12.25 A 1010 steel is to be carburised using a gas atmosphere that produces 1.0% C at the surface of the steel. The case depth is defined as the distance below the surface that contains at least 0.5% C. If carburising is done at 1000°C, determine the time required to produce a case depth of 0.25 mm. (See Chapter 5 for review).

The diffusion coefficient for carbon in FCC iron at 1000°C is:

$$D = 2.3 \times 10^{-5} \exp[-137,700 / (8.314)(1273)] = 5.14 \times 10^{-11} \text{ m}^2 \cdot \text{s}^{-1}$$

The case depth "x" is to be 0.25 mm = 2.5×10^{-4} m

From Fick's law:

$$\frac{1.0 - 0.5}{1.0 - 0.10} = 0.555 = \text{erf}(x/2\sqrt{Dt})$$

$$x/2\sqrt{Dt} = 0.541 \quad (\text{from Table 5.3})$$

$$2.5 \times 10^{-4} / (2)\sqrt{(5.14 \times 10^{-11})t} = 0.541$$

$$t = 1039 \text{ s} = 0.29 \text{ h}$$

12.26 A 0.15%C steel is to be carburised at 1020°C for 2 h using a gas atmosphere that produces 1.2% C at the surface of the steel. Plot the percent carbon versus the distance from the surface of the steel. If the steel is slowly cooled after carburising, determine the amount of each phase and microconstituent at 0.05 mm intervals from the surface. (See Chapter 5.)

The diffusion coefficient for carbon in FCC iron at 1020°C is:

$$D = 2.3 \times 10^{-5} \exp[-137,700/(8.314)(1293)] \\ = 6.29 \times 10^{-11} \text{ m}^2 \cdot \text{s}^{-1}$$

As a calculation example, set the case depth at $x = 0.5 \text{ mm}$ with time t at 2 hrs = 7200 s.

From Fick's law:

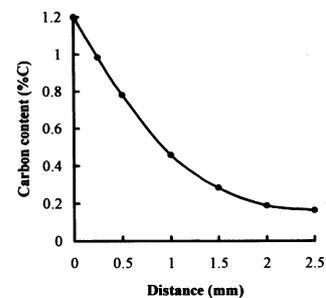
$$\frac{1.2 - C_x}{1.2 - 0.15} = \text{erf}[(0.5 \times 10^{-3})/2\sqrt{(6.29 \times 10^{-11})(7200)}]$$

$$\frac{1.2 - C_x}{1.2 - 0.15} = \text{erf}[0.3715] = 0.4000$$

$$C_x = 1.2 - (0.4000 \times 1.05) = 0.780\% \text{C}$$

This calculation can be repeated for other values of x , with the results shown below:

$x = 0.01 \text{ mm}$,	$C_x = 1.192\% \text{C}$
$x = 0.25 \text{ mm}$,	$C_x = 0.983\% \text{C}$
$x = 1.0 \text{ mm}$,	$C_x = 0.459\% \text{C}$
$x = 1.5 \text{ mm}$,	$C_x = 0.285\% \text{C}$
$x = 2.0 \text{ mm}$,	$C_x = 0.189\% \text{C}$
$x = 2.5 \text{ mm}$,	$C_x = 0.164\% \text{C}$



12.27 A 0.5%C steel is welded. After cooling, hardnesses in the heat affected zone are obtained at various locations from the edge of the fusion zone. Determine the hardnesses expected at each point if a 0.8%C steel were welded under the same conditions. Predict the microstructure at each location in the as-welded 0.8%C steel.

Distance from FZ	HRC in 0.5%C steel
0.05 mm	50
0.10 mm	40
0.15 mm	32
0.20 mm	28

We can take advantage of the fact that the cooling rate in the two steels will be virtually identical if the welding conditions are the

same. Thus at a distance of 0.05 mm from the edge of the fusion zone, the HRC 50 hardness of the 0.5%C steel is obtained with a Jominy distance of 4 mm, or a cooling rate of 80°C/s. At the same point in a 0.8%C steel weldment, the 4 mm Jominy distance gives a hardness of HRC 53 (from the hardenability curve) and the 80°C/s cooling rate gives a microstructure of pearlite and martensite (from the CCT curve). The table below shows the results for all four points in the weldment.

<u>distance</u>	<u>Jominy distance</u>	<u>Cooling rate</u>	<u>Hardness</u>	<u>Structure</u>
0.05 mm	4 mm	80°C/s	HRC 53	P + M
0.10 mm	6 mm	44°C/s	HRC 44	P + M
0.15 mm	10 mm	20°C/s	HRC 38	pearlite
0.20 mm	15 mm	12°C/s	HRC 36	pearlite

12.28 We wish to produce a martensitic stainless steel containing 17% Cr. Recommend a carbon content and austenitising temperature that would permit us to obtain 100% martensite during the quench. What microstructure would be produced if the martensite were then tempered until the equilibrium phases formed?

We must select a combination of a carbon content and austenitising temperature that puts us in the all-austenite region of the Fe-Cr phase diagram. One such combination is 1200°C and 0.5% C. If a 0.5% C steel is held at 1200°C to produce all austenite, and then is quenched, 100% martensite will form.

If the martensite is tempered until equilibrium is reached, the two phases will be ferrite and $M_{23}C_6$. The $M_{23}C_6$ is typically $Cr_{23}C_6$.

12.29 Occasionally, when an austenitic stainless steel is welded, the weld deposit may be slightly magnetic. Based on the Fe-Cr-Ni-C phase diagram [Figure 12.30(b)], what phase would you expect is causing the magnetic behavior? Why might this phase have formed? What could you do to restore the nonmagnetic behavior?

The magnetic behavior is caused by the formation of a BCC iron phase, in this case the high temperature δ -ferrite. The δ -ferrite forms during solidification, particularly when solidification does not follow equilibrium; subsequent cooling is too rapid for the δ -ferrite to transform to austenite, and the ferrite is trapped in the microstructure. If the steel is subsequently annealed at an elevated temperature, the δ -ferrite can transform to austenite and the steel is no longer magnetic.

12.30 A tensile bar of a grey iron casting is found to have a tensile strength of 345 MN.m⁻². Why is the tensile strength greater than that given by the class number? What do you think is the diameter of the test bar? (B.S. gives the strength as 275 MN.m⁻².)

The strength of grey iron depends on the cooling rate of the casting; faster cooling rates produce finer microstructures and more pearlite in the microstructure. Although the iron has a nominal strength of 275 MN.m⁻², rapid cooling can produce the fine graphite and pearlite that give the higher 345 MN.m⁻² strength.

The nominal 275 MN.m⁻² strength is expected for a casting with a diameter of about 30 mm; if the bar is only 16 mm in diameter, a tensile strength of 345 MN.m⁻² might be expected.

12.31 You would like to produce a grey iron casting that freezes with no primary austenite or graphite. If the carbon content in the iron is 3.5%, what percentage of silicon must you add?

We get neither primary phase when the carbon equivalent is 4.3%. Thus

$$CE = 4.3 = \%C + (1/3)\%Si$$

$$4.3 = 3.5 + (1/3)\%Si \quad \text{or} \quad \%Si = 2.4$$

12.32 We find that first stage graphitisation during the production of a 25 mm thick malleable iron casting can be accomplished in 6 h if the white iron casting is slowly heated to the FSG temperature. What will be the effect of the following changes on the FSG time? Explain.

(a) increasing the rate at which the casting is heated to the FSG temperature

Nucleation of graphite occurs during heating; if the heating rate is rapid, fewer nuclei for graphite are produced. During FSG, the diffusion distances for carbon to move to the graphite nodules will be longer, since fewer nodules are present. Longer times will be required to complete first stage graphitisation.

(b) producing malleable iron from a 12 mm thick white iron casting

The 12 mm thick casting will freeze more rapidly, producing a finer microstructure (such as a smaller secondary dendrite arm spacing). More interface area between phases is present for the finer structure; since graphite particles nucleate more easily at interfaces, the nodule count will be higher. The higher nodule count, in turn, reduces diffusion distances and speeds the rate at which first stage graphitisation takes place.

(c) increasing the silicon content of the white iron by 0.25%

Silicon is an effective graphite stabilising element. The higher percentage of silicon makes the Fe_3C less stable, permitting it to decompose more rapidly during FSG and thus reducing the time required for FSG.

12.33 When the thickness of an S.G. iron casting increases, the number of graphite nodules normally decreases.

a) What effect will this have on the amount of ferrite that is present in the matrix? Explain.

We would expect that, since the graphite nodule count is lower, the diffusion distances are longer, and a longer time will be required for austenite to transform to ferrite during cooling. Consequently we expect less ferrite for the thicker casting.

(b) Suppose you observed the opposite effect of thickness on the amount of ferrite. How would you explain this phenomenon?

The thicker castings also cool more slowly than thinner castings. Because of the slower cooling rate, more time is available for diffusion of carbon from the austenite to the graphite nodules, and consequently more ferrite can form. In some cases, this cooling rate can offset the nodule count effect.

12.34 We would like to produce a 420/12 grade of S.G. cast iron without heat treating. (a) What major phase should be present in the matrix? (b) Would increasing the number of graphite nodules produced during solidification help or hinder our efforts to produce this matrix? Explain. Suggest a method by which the number of graphite nodules might be changed. (c) What changes in the composition of the iron might help produce the desired matrix?

(a) *ferrite*

(b) *a larger number of graphite nodules reduces the diffusion distances and makes it easier for austenite to transform to ferrite, thus increasing the amount of ferrite. One method for increasing the number of graphite nodules is to improve the inoculation process. Faster cooling during solidification also increases nodule counts.*

(c) *Increasing the silicon content increases the amount of ferrite produced -- one of the mechanisms for this effect is that the eutectoid temperature is increased, causing austenite to transform at a higher temperature where diffusion of carbon is more rapid. Almost all other alloying elements, however, reduce the amount of ferrite.*

12.35 Compare the expected hardenabilities of a plain carbon steel, a malleable cast iron, and an S.G. cast iron. Explain why you expect different hardenabilities.

Plain carbon steels contain very little alloying elements and therefore are expected to have a low hardenability.

Malleable cast irons contain on the order of 1.5% Si; the silicon improves the hardenability of the austenite, making it easier to obtain martensite during quenching.

Ductile cast iron contains more silicon (often 2 to 3%); the higher silicon content gives the ductile iron higher hardenabilities than either plain carbon steels or malleable irons.

12.36 A B35/12 malleable cast iron is produced, requiring both FSG and SSG treatments. What went wrong with the treatment if:

(a) the final matrix includes ferrite with 10% pearlite

During SSG, we attempt to provide slow enough cooling so there is sufficient time for austenite to transform to ferrite. If the matrix contains pearlite, the SSG treatment must have been incorrect. Perhaps the cooling rate during SSG was too fast.

(b) the final structure includes ferrite with 15% massive Fe₃C

During FSG, we attempt to dissolve all of the Fe₃C that formed during solidification. If 15% Fe₃C is still present in the structure, FSG must have been done improperly. Perhaps the casting was heated too rapidly, so that the nodule count was too small. Perhaps the FSG temperature was too low, or the FSG time was too short.

Chapter 13 Nonferrous Alloys

13.1 In some cases, we may be more interested in cost per unit volume than in cost per unit weight. Rework Table 13.1 to show the cost of each metal in terms of U\$/m³. Does this change the relationship between the different metals?

We can find the density (in Mg.m⁻³) of each metal from Appendix A. Multiplying by 10³ gives density in kg. m⁻³. We obtain cost/volume by multiplying cost/kg by density in kg.m⁻³ to give units of US \$/m³. The left hand side of the table shows the results of these conversions, with the metals ranked in order of cost per volume. The right hand side of the table shows the cost per pound.

	cost/volume	rank (low to high)		cost/kg	rank (low to high)
Fe	\$ 1,731 per m ³	1	Fe	\$0.22	1
Al	\$ 3,509 per m ³	2	Pb	\$0.80	2
Mg	\$ 5,214 per m ³	3	Zn	\$1.25	3
Zn	\$ 8,916 per m ³	4	Al	\$1.30	4
Pb	\$ 9,088 per m ³	5	Cu	\$2.45	5
Cu	\$ 21,878 per m ³	6	Mg	\$3.00	6
Ti	\$ 54,760 per m ³	7	Ni	\$9.00	7
Ni	\$ 80,118 per m ³	8	Ti	\$12.15	8
W	\$ 423,588 per m ³	9	W	\$22.00	9
Be	\$1,219,680 per m ³	10	Be	\$660.00	10

The relationship is changed; for example, aluminium is fourth based on weight, but second on the basis of volume. Titanium is more expensive than nickel on a weight basis, but less expensive than nickel on a volume basis.

13.2 Assuming that the density remains unchanged, compare the specific strength of the 2090-T6 aluminium alloy to that of a die cast 443-F aluminium alloy (Table 13.5). If you considered the actual density, do you think the difference between the specific strengths would increase or become smaller? Explain.

$$2090-T6: \quad TS = 550 \text{ MN.m}^{-2}$$

$$\text{Spec. strength} = \frac{(550 \text{ MN.m}^{-2})}{(2.7 \text{ Mg.m}^{-3})} = 203703 \text{ m}^2.\text{s}^{-2}$$

$$443-F: \quad TS = 230 \text{ MN.m}^{-2}$$

$$\text{Spec. strength} = \frac{(230 \text{ MN.m}^{-2})}{(2.7 \text{ Mg.m}^{-3})} = 85185 \text{ m}^2.\text{s}^{-2}$$

Both should increase since both Li and Si (the major alloying elements) are less dense than Al.

13.3 Explain why aluminium alloys containing more than about 15% Mg are not used.

When more than 15% Mg is added to Al, a eutectic microconstituent is produced during solidification. This eutectic contains

$$\% \beta_{\text{Eut}} = \frac{35 - 14.9}{35.5 - 14.9} = 97.6\%$$

Most of the eutectic is the brittle intermetallic compound β , and it will likely embrittle the eutectic. The brittle eutectic, which is the continuous microconstituent, will then make the entire alloy brittle.

13.4 Calculate the modulus of elasticity-to-density ratio (also called the specific modulus) of an Al-3% Li alloy and compare with the ratio for pure aluminium.

The density of pure Al is 2.699 Mg.m^{-3} and the modulus of elasticity is 70 GN.m^{-2} . From Figure 13.5, the density of Al-3%Li is 2.47 Mg.m^{-3} and the modulus of elasticity is 79 GN.m^{-2} .

$$\text{Al-3\% Li: } \frac{\text{Modulus}}{\text{density}} = \frac{(79 \text{ GN.m}^{-2})}{(2.47 \text{ Mg.m}^{-3})} = 320 \times 10^6 \text{ m}^2 \cdot \text{s}^{-2}.$$

$$\text{Pure Al: } \frac{\text{Modulus}}{\text{density}} = \frac{(70 \text{ GN.m}^{-2})}{(2.699 \text{ Mg.m}^{-3})} = 25.9 \times 10^6 \text{ m}^2 \cdot \text{s}^{-2}.$$

Note that the Al-Li alloy has a specific modulus that is nearly 25% greater than that of pure aluminium and most other conventional aluminium alloys.

13.5 Estimate the secondary dendrite arm spacing for each structure in Figure 13.4 and, from Figure 8.7, estimate the solidification time obtained by each of the three casting processes. Do you expect higher strengths for die casting, permanent mould casting, or sand casting? Explain.

The secondary dendrite arm spacing is related to the local solidification time (from Example 8-3) by the equation:

$$\text{SDAS} = (8 \times 10^{-3}) (\text{LST})^{0.42} \quad \text{or} \quad \text{LST} = (\text{SDAS} / 8 \times 10^{-3})^{2.38}$$

We can measure the distance between the centre of one dendrite and the centre of another dendrite, divide by the number of dendrite spacings measured, and then divide by the magnification of 350; this will give us the SDAS. From the equation above, we can calculate the solidification time, (or read the LST value from Figure 8.7).

$$\text{sand cast: } \begin{aligned} \text{SDAS} &= (25 \text{ mm}/1 \text{ dendrite spacing})/350 = 0.071 \text{ mm} \\ \text{LST} &= 180 \text{ s} \end{aligned}$$

$$\text{permanent mould: } \begin{aligned} \text{SDAS} &= (28 \text{ mm}/4 \text{ dendrite spacing})/350 = 0.02 \text{ mm} \\ \text{LST} &= 9 \text{ s} \end{aligned}$$

$$\text{Die cast: } \begin{aligned} \text{SDAS} &= (18 \text{ mm}/5 \text{ dendrite spacing})/350 = 0.01 \text{ mm} \\ \text{LST} &= 1.7 \text{ s} \end{aligned}$$

We expect the highest strength from the die casting.

13.6 Would you expect a 2024-T9 aluminium alloy to be stronger or weaker than a 2024-T6 alloy? Explain.

The T9 treatment will give the higher strength; in this temper cold working and age hardening are combined, while in T6, only age hardening is done.

13.7 Based on the data in Figure 7.22, estimate the mechanical properties if the 3105 aluminium alloy is in the H18 condition.

The H18 treatment refers to %CW = 75%. Therefore:

$$\begin{aligned} \text{Tensile Strength} &= 192 \text{ MN.m}^{-2} \\ \text{Yield Strength} &= 177 \text{ MN.m}^{-2} \\ \% \text{ Elongation} &= 3\% \end{aligned}$$

13.8 Estimate the tensile strength expected for the following aluminium alloys.

(a) 1100-H14 (b) 5182-H12 (c) 3004-H16

(a) The tensile strength for 1100-H14 is the average of the 0 and H18 treatments.

$$\begin{aligned} 1100-0 (0\% \text{ CW}): & \quad TS = 90 \text{ MN.m}^{-2} \\ 1100-H18(75\% \text{ CW}): & \quad TS = 165 \text{ MN.m}^{-2} \end{aligned}$$

$$TS_{H14} = \frac{90 + 165}{2} = 127 \text{ MN.m}^{-2}$$

(b) The tensile strength for 5182-H12 is the average of the 0 and H14 treatments, and H14 is the average of the 0 and H18 treatments. We do not have data in Table 13.5 for 5182-H18. However, 5182-H19 has a tensile strength of 420 MN.m⁻² and H18 should be 14 MN.m⁻² less, or 406 MN.m⁻².

$$\begin{aligned} 5182-0 (0\% \text{ CW}): & \quad TS = 290 \text{ MN.m}^{-2} \\ 5182-H18(75\% \text{ CW}): & \quad TS = 406 \text{ MN.m}^{-2} \end{aligned}$$

$$TS_{H14} = \frac{290 + 406}{2} = 348 \text{ MN.m}^{-2}$$

$$TS_{H12} = \frac{290 + 348}{2} = 319 \text{ MN.m}^{-2}$$

(c) The tensile strength for 3004-H16 is the average of the H14 and H18 treatments, and H14 is the average of the 0 and H18 treatments.

$$\begin{aligned} 3004-H0: & \quad TS = 180 \text{ MN.m}^{-2} \\ 3004-H18: & \quad TS = 285 \text{ MN.m}^{-2} \end{aligned}$$

$$TS_{H14} = \frac{180 + 285}{2} = 232 \text{ MN.m}^{-2}$$

$$TS_{H16} = \frac{285 + 232}{2} = 258 \text{ MN.m}^{-2}$$

13.9 Suppose, by rapid solidification from the liquid state, that a super-saturated Al-7% Li alloy can be produced and subsequently aged. Compare the amount of β that will form in this alloy with that formed in a 2090 alloy.

The 2090 alloy contains 2.4% Li; from the Al-Li phase diagram, the composition of the β is about 20.4% Li and that of the α is approximately 2% Li at a typical ageing temperature or at room temperature:

$$\text{Al-7\% Li: } \% \beta = \frac{7 - 2}{20.4 - 2} \times 100\% = 27\%$$

$$2090: \quad \% \beta = \frac{2.4 - 2}{20.4 - 2} \times 100\% = 2.2\%$$

13.10 Determine the amount of Mg_2Al_3 (β) expected to form in a 5182-0 aluminium alloy.

The 5182 alloy contains 4.5% Mg. Thus from the Mg-Al phase diagram, which shows the α contains about 0% Mg and β contains about 35% Mg:

$$\% \beta = \frac{4.5 - 0}{35 - 0} \times 100\% = 12.9\%$$

13.11 Based on the phase diagrams, which of the following alloys would be most suited for thixocasting? Explain your answer. (See Figures 10.22, 11.5, and 13.3.)

- (a) Al-12% Si (b) Al-1% Cu (c) Al-10% Mg

Alloys best suited for thixocasting are those with a large freezing range. Of the alloys listed, Al-10% Mg has a freezing range of 110°C, which is the largest freezing range of the three and is therefore most desirable. Al-12% Si is a eutectic alloy (approximately 0°C freezing range), and Al-1% Cu has a freezing range of only 10°C.

13.12 From the data in Table 13.6, estimate the ratio by which the yield strength of magnesium can be increased by alloying and heat treatment and compare with that of aluminium alloys.

The exact values will differ depending on the alloys we select for comparison. The table below provides an example. Strengthening of Mg is only about 1/10 as effective as in Al.

	Magnesium			Aluminium	
	YS	YS/YS _{Mg}		YS	YS/YS _{Al}
Pure Mg:	90 MN.m ⁻²	--	Pure Al	17 MN.m ⁻²	--
Cold Worked	115 MN.m ⁻²	1.3	CW 1100-H18	150 MN.m ⁻²	8.8
Casting & T6 (ZK61A-T6)	195 MN.m ⁻²	2.2	5182-0 Alloy	130 MN.m ⁻²	7.6
Wrought & T5 (AZ80A-T5)	275 MN.m ⁻²	3.1	2090-T6	517 MN.m ⁻²	30.4

13.13 Suppose a 600 mm long round bar is to support a load of 1.8 kN without any permanent deformation. Calculate the minimum diameter of the bar if it is made of (a) AZ80A-T5 magnesium alloy and (b) 6061-T6 aluminium alloy. Calculate the weight of the bar and the approximate cost (based on pure Al and Mg) in each case.

$$A = F/\text{Yield Strength}$$

$$(a) \quad AZ80A-T5: \quad YS = 275 \text{ MN.m}^{-2}$$

$$A = 1800N/275 \text{ MN.m}^{-2} = 6.5454 \times 10^{-6} \text{ m}^2$$

$$d = \sqrt{4A/\pi} = 2.89 \times 10^{-3} \text{ m}$$

$$\begin{aligned} \text{Weight} &= (600 \times 10^{-3} \text{ m}) (6.5454 \times 10^{-6} \text{ m}^2) (1.74 \text{ MN.m}^{-2}) \\ &= 6.83 \times 10^{-6} \text{ Mg} \end{aligned}$$

$$\text{cost} = (\$3.0/\text{kg}) (6.83 \times 10^{-3} \text{kg}) = \$0.021$$

b) 6061-T6: $YS = 275 \text{ MN.m}^{-2}$ therefore;

$$A = 6.5454 \times 10^{-6} \text{m}^2 \quad d = 2.89 \times 10^{-3} \text{m} \quad \text{as in part (a), but:}$$

$$\begin{aligned} \text{Weight} &= (600 \times 10^{-3} \text{m}) (6.5454 \times 10^{-6} \text{m}^2) (2.70 \text{ Mg.m}^{-3}) \\ &= 1.06 \times 10^{-2} \text{Mg} \end{aligned}$$

$$\text{cost} = (\$1.30/\text{kg}) (1.06 \times 10^{-2} \text{kg}) = \$0.014$$

Al is less costly than Mg, even though Mg is lighter.

13.14 A 10 m rod 5 mm in diameter must elongate no more than 2 mm under load. What is the maximum force that can be applied if the rod is made of (a) aluminium, (b) magnesium, and (c) beryllium?

$$E = \sigma/\epsilon = F/A\epsilon \quad \therefore F = EA\epsilon \quad \text{diameter} = 5 \text{ mm} = 5 \times 10^{-3} \text{m}$$

$$\epsilon = \frac{10.002 \text{ m} - 10.000 \text{ m}}{10.000 \text{ m}} = 0.0002 \text{ m.m}^{-1}$$

$$F_{\text{Al}} = (70 \text{ GN.m}^{-2}) (\pi/4) (5 \times 10^{-3} \text{ m})^2 (0.0002 \text{ m.m}^{-1}) = 275 \text{ N}$$

$$F_{\text{Mg}} = (45 \text{ GN.m}^{-2}) (\pi/4) (5 \times 10^{-3} \text{ m})^2 (0.0002 \text{ m.m}^{-1}) = 177 \text{ N}$$

$$F_{\text{Be}} = (290 \text{ GN.m}^{-2}) (\pi/4) (5 \times 10^{-3} \text{ m})^2 (0.0002 \text{ m.m}^{-1}) = 1139 \text{ N}$$

13.15 A Cu-20% Sn alloy is found, after cooling from the liquid to room temperature, to have a microstructure containing 50% α , 30% β , and 20% γ . What microstructure would you predict under equilibrium conditions? Explain why the observed microstructure is not unexpected.

equilibrium: $\alpha + \epsilon$

non-equilibrium: α forms first, with β forming next by the peritectic reaction. Some of the β transforms to γ during a eutectic reaction, but some β remains as a metastable phase. Kinetics are so slow that γ and β do not transform to either δ or ϵ during subsequent cooling.

13.16 We say that copper can contain up to 40% Zn or 9% Al and still be single phase. How do we explain this statement in view of the phase diagrams in Figure 13.10?

This is possible due to slow kinetics of transformation at low temperatures.

13.17 Compare the percentage increase in the yield strength of commercially pure annealed aluminium, magnesium, and copper by strain hardening. Explain the differences observed.

$$\text{Al: } \frac{1100 - H18}{1100 - 0} = \frac{150}{35} \times 100\% = 429\%$$

$$\text{Mg: } \frac{\text{C.W.}}{\text{Annealed}} = \frac{115}{90} \times 100\% = 128\%$$

$$\text{Cu: } \frac{70\% \text{ C.W.}}{\text{Annealed}} = \frac{365}{70} \times 100\% = 521\%$$

Both Al and Cu (with an FCC structure) have high strain hardening coefficients and can be cold worked a large amount (due to their good ductility). Mg has the HCP structure, a low strain hardening coefficient, and a limited ability to be cold worked.

13.18 We would like to produce a quenched and tempered aluminium bronze containing 13% Al. Recommend a heat treatment, including appropriate temperatures. Calculate the amount of each phase after each step of the treatment.

Heat to above about 710°C to get all β ; 100% β , β : 13% Al

Quench; still all β containing 13% Al.

Reheat; temper at 400°C to allow γ_2 to form.

$$\% \gamma_2 = \frac{13 - 9.4}{16 - 9.4} = 54.5\% \quad \gamma_2: 16\% \text{ Al}, \alpha: 9.4\% \text{ Al}$$

We want to be sure to temper above 400°C so we obtain γ_2 in a matrix of α rather than a structure containing $\gamma + \gamma_2$.

13.19 A number of casting alloys have very high lead contents; however the Pb content in wrought alloys is comparatively low. Why isn't more lead added to the wrought alloys? What precautions must be taken when a leaded wrought alloy is hot worked or heat treated?

The lead phase may melt during hot working or may form stringers during cold working.

We must be sure that the temperature is low enough to avoid melting of the lead phase.

13.20 Would you expect the fracture toughness of quenched and tempered aluminium bronze to be high or low? Would there be a difference in the resistance of the alloy to crack nucleation compared to crack growth? Explain.

The fracture toughness should be relatively good. The acicular, or Widmanstätten, microstructure forces a crack to follow a very tortuous path, which consumes a large amount of energy.

This microstructure is less resistant to crack nucleation. The acicular structure may concentrate stresses that lead to easier formation of a crack.

13.21 Based on the photomicrograph in Figure 13.13(a), would you expect the γ' precipitate or the carbides to provide a greater strengthening effect in superalloys at low temperatures? Explain.

The γ' phase is more numerous and also more uniformly and closely spaced; consequently the γ' should be more effective than the smaller number of coarse carbides at blocking slip at low temperatures.

13.22 The density of Ni_3Al is $6.56 \text{ Mg}\cdot\text{m}^{-3}$. Suppose a Ni-5 wt% Al alloy is heat treated so that all of the aluminium reacts with nickel to produce Ni_3Al . Determine the volume percent of the Ni_3Al precipitate in the nickel matrix.

Let's assume that the density of the Ni-5 wt% Al alloy is the same as that of pure Ni ($8.902 \text{ Mg}\cdot\text{m}^{-3}$).

In 100 g of the alloy, the total atoms present are:

$$\begin{aligned} \text{atoms} &= \frac{(95 \text{ g/Ni})N_A}{58.71 \text{ g.mol}} + \frac{(5 \text{ g Al})N_A}{26.981 \text{ g.mol}} \\ &= 1.6181 N_A + 0.1853 N_A = 1.803 N_A \end{aligned}$$

If all of the Al reacts to form Ni_3Al , then the number of atoms in the compound is $0.1853 N_A$ of Al and $(3)(0.1853 N_A) = 0.5559 N_A$ of Ni.

The weight of the Ni_3Al is then:

$$\begin{aligned} \text{wt} &= \frac{(0.1853 N_A \text{ of Al})(26.981 \text{ g.mol})}{N_A} + \frac{(0.5559 N_A \text{ of Ni})(58.71 \text{ g.mol})}{N_A} \\ &= 37.64 \text{ g of Ni}_3\text{Al} \end{aligned}$$

The wt of the Ni matrix is thus 62.36 g. The vol% Ni_3Al is thus:

$$\text{vol\% Ni}_3\text{Al} = \frac{37.64 \text{ g}/6.56 \text{ g.cm}^3}{(37.64 \text{ g}/6.56 \text{ g.cm}^3) + (62.36 \text{ g}/8.902 \text{ g.cm}^3)} \times 100\% = 45\%$$

(Note: density in Mg.m^{-3} equals that in g.cm^{-3} .)

Even a small amount (5 wt%) aluminium produces a very large volume percent of precipitate in the microstructure.

13.23 Figure 13.13(b) shows a nickel superalloy containing two sizes of γ' precipitates. Which precipitate likely formed first? Which precipitate formed at the higher temperature? What does our ability to perform this treatment suggest concerning the effect of temperature on the solubility of Al and Ti in nickel? Explain.

The larger precipitate forms first and at the higher temperatures

The solubility of Al and Ti in Ni decreases as temperature decreases; at a high temperature, the Al and Ti form the γ' , but some Al and Ti still remain in solution in the matrix. As the temperature decreases, the solubility decreases as well and more of the γ' can form.

13.24 When steel is joined using arc welding, only the liquid fusion zone must be protected by a gas or flux. However, when titanium is welded, both the front and back sides of the welded metal must be protected. Why must these extra precautions be taken when joining titanium?

The titanium may be contaminated or embrittled anytime the temperature is above about 535°C . Therefore the titanium must be protected until the metal cools below this critical temperature. Since both sides of the titanium plate will be heated by the welding process, special provisions must be made to shield all sides of the titanium until the metal cools sufficiently.

13.25 Both a Ti-15% V alloy and a Ti-35% V alloy are heated to a temperature at which all β just forms. They are then quenched and reheated to 300°C . Describe the changes in microstructures during the heat treatment for each alloy, including the amount of each phase. What is the matrix and what is the precipitate in each case? Which is an age hardening process? Which is a quench and temper process? [See Figure 13.19(a)]

Ti-15% V: $100\% \beta \rightarrow 100\% \alpha' \rightarrow \beta$ precipitates in α matrix.

$$\% \alpha_{300C} = \frac{46 - 15}{46 - 5} \times 100\% = 76\% \quad \beta = 24\%$$

This is a quench and temper process.

Ti-35% V: 100% $\beta \rightarrow 100\% \beta_{ss} \rightarrow \alpha$ precipitates in β matrix.

$$\% \alpha_{300C} = \frac{46 - 35}{46 - 5} \times 100\% = 27\% \quad \beta = 73\%$$

This is an age hardening process.

13.26 The θ phase in the Ti-Mn phase diagram has the formula MnTi. Calculate the amount of α and θ in the eutectoid microconstituent. [See Figure 13.15(d)]

$$\text{wt\% Mn in } \theta = \frac{(1 \text{ atom of Mn}) (54.938 \text{ g.mol})}{(1 \text{ atom of Mn}) (54.938) + (1 \text{ atom of Ti}) (47.9)} = 53.4\%$$

$$\% \alpha_{\text{eutectoid}} = \frac{53.4 - 20}{53.4 - 1} \times 100\% = 63.7\% \quad \% \theta = 36.3\%$$

13.27 The temperature of a coated tungsten part is increased. What happens when the protective coating on a tungsten part expands more than the tungsten? What happens when the protective coating on a tungsten part expands less than the tungsten?

If the protective coating expands more than tungsten, compressive stresses will build up in the coating and the coating will flake.

If the protective coating expands less than tungsten, tensile stresses will build up in the coating and the coating will crack and become porous.

13.28 Determine the ratio of the yield strengths of the strongest Al, Mg, Cu, Ti, and Ni alloy to the yield strength of the pure metal. Compare the alloy systems and rank them in order of their response to strengthening mechanisms. Try to explain their order.

<u>Metal</u>	<u>YS (MN.m⁻²)</u>	<u>alloy</u>	<u>alloy YS (MN.m⁻²)</u>	<u>YS/YS</u>	<u>structure</u>
Cu	35	Cu-2% Be	1205	34.4	FCC
Al	17	2090-T6	517	30.4	FCC
Ti	170	Ti-13V-11Cr-3Al	1210	7.1	HCP
Ni	110	Monel K-500	760	6.9	HCP
Mg	90	AZ80A-T5	275	3.1	HCP

The FCC metals have the greater response to strengthening, but they also the lowest yield strength for the pure metal. The HCP metals have less response, but they have the highest yield strength for the pure metal.

13.29 Determine the specific yield strength of the strongest Al, Mg, Cu, Ti, and Ni alloys. Use the densities of the pure metals, in Mg.m⁻³, in your calculations. Try to explain their order.

	Strength (MN.m ⁻²)	Density	Strength-to-weight ratio (m ² .s ⁻²)
Ti	1210	4.507 Mg.m ⁻³	268471
Al	517	2.699 Mg.m ⁻³	191552
Mg	275	1.738 Mg.m ⁻³	158228
Cu	1205	8.93 Mg.m ⁻³	134938
Monel	760	8.93 Mg.m ⁻³	85106

Titanium is both strong and relatively low density. Cu, Ni are strong but dense. Al and Mg have modest strength but light weight.

13.30 Based on the phase diagrams, estimate the solubilities of Ni, Zn, Al, Sn, and Be in copper at room temperature. Are these solubilities expected in view of Hume-Rothery's conditions for solid solubility? Explain.

	Solubility	Structure	Valence	Atom size difference
Cu-Ni	100% Ni	FCC	2	$\frac{0.1278 - 0.1243}{0.1278} \times 100 = 2.7\%$
Cu-Zn	30% Zn	HCP	2	$\frac{0.1278 - 0.1332}{0.1278} \times 100 = -4.2\%$
Cu-Al	8% Al	FCC	3	$\frac{0.1278 - 0.1432}{0.1278} \times 100 = -12.1\%$
Cu-Be	0.2% Be	hex	2	$\frac{0.1278 - 0.1143}{0.1278} \times 100 = 10.6\%$
Cu-Sn	0% Sn	DC	4	$\frac{0.1278 - 0.1405}{0.1278} \times 100 = -9.9\%$

Hume-Rothery's conditions do help to explain the differences in solubility. Solubilities tend to decrease as atom size difference increases.

Chapter 14 Ceramic Materials

14.1 Calculate the lattice parameter, packing factor, and density expected for BaTiO₃ [Figure 14.1(a)], using the data in the appendices.

First, in order to determine the lattice parameter, we need to determine the direction in which the ions touch. The direction that gives us the largest value for the lattice parameter is the close packed direction. Let's try this for three directions in the unit cell, using ionic radii:

$$\langle 100 \rangle: a_0 = 2r_{\text{O}} + 2r_{\text{Ti}} = 2(0.132) + 2(0.068) = 0.4 \text{ nm}$$

$$\langle 110 \rangle: a_0 = (2r_{\text{Ba}} + 2r_{\text{O}}) / \sqrt{2} = [2(0.134) + 2(0.132)] / \sqrt{2} = 0.378 \text{ nm}$$

$$\langle 111 \rangle: a_0 = (2r_{\text{Ba}} + 2r_{\text{Ti}}) / \sqrt{3} = [2(0.134) + 2(0.068)] / \sqrt{3} = 0.233 \text{ nm}$$

Therefore the ions must touch along the $\langle 100 \rangle$ directions and the lattice parameter is $a_0 = 0.40 \text{ nm}$.

$$\begin{aligned} \text{Packing factor} &= \frac{[(1 \text{ Ti})(0.068)^3 + (1 \text{ Ba})(0.134)^3 + (3 \text{ O})(0.132)^3](4\pi/3)}{(0.40)^3} \\ &= 0.63 \end{aligned}$$

$$\begin{aligned} \text{Density} &= \frac{(1 \text{ Ti})(47.9 \text{ g.mol}) + (1 \text{ Ba})(137.3 \text{ g.mol}) + (3 \text{ O})(16 \text{ g.mol})}{(4.00 \times 10^{-8} \text{ cm})^3 (6.02 \times 10^{23} \text{ ions/mol})} \\ &= 6.053 \text{ g.cm}^3 = 6.053 \text{ Mg.m}^{-3} \end{aligned}$$

14.2 Calculate the packing factor and density expected for MgAl₂O₄ [Figure 14.1(c)] if the lattice parameter is 0.808 nm.

$$\begin{aligned} \text{Packing factor} &= \frac{[(8 \text{ Mg})(0.066)^3 + (16 \text{ Al})(0.051)^3 + (32 \text{ O})(0.132)^3](4\pi/3)}{(0.808)^3} \\ &= 0.62 \end{aligned}$$

$$\begin{aligned} \rho &= \frac{(8 \text{ Mg})(24.312 \text{ g.mol}) + (16 \text{ Al})(26.981 \text{ g.mol}) + (32 \text{ O})(16 \text{ g.mol})}{(8.08 \times 10^{-8} \text{ cm})^3 (6.02 \times 10^{23} \text{ ions/mol})} \\ &= 3.584 \text{ g.cm}^3 = 3.584 \text{ Mg.m}^{-3} \end{aligned}$$

14.3 Quartz (SiO₂) has a hexagonal crystal structure with lattice parameters of $a_0 = 0.4913 \text{ nm}$ and $c_0 = 0.5405 \text{ nm}$ and a density of 2.65 Mg.m^{-3} . Determine (a) the number of SiO₂ groups in quartz and (b) the packing factor of the quartz unit cell.

(a) From the density and lattice parameters, we can determine the number of SiO₂ groups "x":

$$\begin{aligned} 2.65 \text{ Mg.m}^{-3} &= \frac{(x)[(1 \text{ Si})(28.08 \text{ g.mol}) + (2 \text{ O})(16 \text{ g.mol})]}{(4.913 \times 10^{-10} \text{ m})^2 (5.405 \times 10^{-10} \text{ m})(\cos 30)(6.02 \times 10^{23} \text{ per mol})} \\ x &= 3 \text{ SiO}_2 \text{ groups} \end{aligned}$$

(b) There are 3 Si and 6 O ions per unit cell. Using the ionic radii:

$$\text{packing factor} = \frac{[3r_{\text{Si}}^3 + 6r_{\text{O}}^3](4\pi/3)}{\text{Volume of unit cell}}$$

$$= \frac{[(3)(0.042)^3 + (6)(0.132)^3](4\pi/3)}{(0.4913)^2(0.5405)\cos 30} = 0.52$$

14.4 Tungsten carbide (WC) has a hexagonal structure, with lattice parameters of $a_0 = 0.291$ nm and $c_0 = 0.284$ nm. If the density of WC is 15.77 Mg.m⁻³, determine the number of tungsten and carbon atoms per cell.

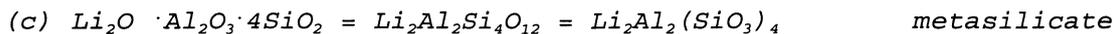
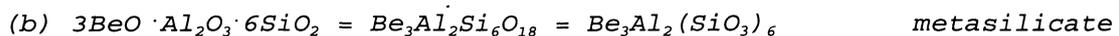
From the density equation,

$$15.77 \text{ Mg.m}^{-3} = \frac{(x)(183.85 \text{ g.mol} + 12 \text{ g.mol})}{(2.91 \times 10^{-10} \text{ m})^2(2.84 \times 10^{-10} \text{ m})(\cos 30)(6.02 \times 10^{23} / \text{mol})}$$

$$x = 1 \text{ WC group per unit cell}$$

14.5 Determine whether the following are orthosilicate, pyrosilicate, metasilicate, or sheet types of ceramics.

We can rearrange each equation to group the Si and all of the oxygen ions together, then note the new formula:



14.6 The density of orthorhombic forsterite (Mg_2SiO_4) is 3.21 Mg.m⁻³ and the lattice parameters are $a_0 = 0.476$ nm, $b_0 = 1.020$ nm, and $c_0 = 0.599$ nm. Calculate the number of SiO_4^{4-} ionic groups in each unit cell.

Let "x" be the number of SiO_4^{4-} groups. From the density equation:

$$3.21 \text{ Mg.m}^{-3} = \frac{2x(24.312 \text{ g.mol}) + x(28.08 \text{ g.mol}) + (4x)(16 \text{ g.mol})}{(4.76 \times 10^{-10} \text{ m})(10.20 \times 10^{-10} \text{ m})(5.99 \times 10^{-10} \text{ m})(6.02 \times 10^{23})}$$

$$x = 4 \text{ SiO}_4^{4-} \text{ groups}$$

14.7 Suppose 10% of the Al^{3+} ions in montmorillonite are replaced by Mg^{2+} ions. How many grams of Na^+ ions will be attracted to the clay per kg of clay?

The normal atomic % Al^{3+} is:

$$\text{at \%Al} = \frac{2 \text{ Al}}{(2 \text{ Al} + 4 \text{ Si} + 12 \text{ O} + 2 \text{ H})} \times 100 = 10\%$$

After the ion exchange, 10% or 0.2 Al^{3+} ion is missing and 0.2 Mg^{2+} is introduced. To satisfy the charge, 0.2 Na^+ will be attracted. The number of ions is now 1.8 Al^{3+} , 0.2 Mg^{2+} , 0.2 Na^+ , 4 Si, 12 O and 2 H. The atomic % Na is:

$$\text{at \% Na} = \frac{0.2}{(1.8 + 0.2 + 0.2 + 4 + 12 + 2)} \times 100 = 0.9901\%$$

The mass % of each ion in the clay:

(at % Na) (MW_{Na})	= (0.9901) (22.99)	= 22.7624
(at % Mg) (MW_{Mg})	= (0.9901) (24.312)	= 24.0713
(at % Al) (MW_{Al})	= (8.9109) (26.981)	= 240.4250
(at % Si) (MW_{Si})	= (19.8020) (28.08)	= 556.0402
(at % O) (MW_O)	= (59.4059) (16)	= 950.4944
(at % H) (MW_H)	= (9.9010) (1)	= 9.9010
		<u>1803.6943</u>

$$\text{mass \% Na} = (\text{at \% Na}) (MW_{Na}) / \Sigma (\text{at \% ion}) (MW_{ion})$$

$$\frac{22.762}{1803.6943} \times 100 = 1.262\%$$

1.262% = x gNa/100 g clay x 100
So, g of Na per kg clay is 12.62 g clay.

14.8 Show that Mg_2SiO_4 and Fe_2SiO_4 are expected to display complete solid solubility.

We can check the Hume-Rothery conditions as they apply to the ceramic materials:

$$r_{Mg} = 0.066 \text{ nm} \quad r_{Fe} = 0.074 \text{ nm} \quad \Delta r = (0.074 - 0.066) / 0.066 \times 100 = 12.1\%$$

Thus the ionic radii are less than 15% different. Both ceramics have the same crystal structure; and Mg^{2+} and Fe^{2+} have the same valence. Therefore we expect the two minerals to display complete solid solubility, as they in fact do.

14.9 A typical composition for FeO (wustite) is 52 at% oxygen. Calculate the number of Fe^{3+} ions and the number of vacancies per cm^3 expected in this typical composition. FeO has the sodium chloride crystal structure.

For FeO of sodium chloride structure we would expect to have 4 Fe^{2+} ions and 4 O^{2-} ions per unit cell. In say, 25 unit cells, there would ordinarily be 100 Fe^{2+} ions and 100 O^{2-} ions.

For a composition of 52 at % O, then Fe^{3+} ions have replaced some Fe^{2+} ions by the reaction



Thus, if x is the number of Fe^{2+} ions replaced by Fe^{3+} ions then the following equation applies for a composition of 52 at % O, based on 25 unit cells:

$$52\% = \frac{(100)(O \text{ ions})}{(100)(O \text{ ions}) + (100 - x)(Fe^{2+} \text{ ions}) + (2x/3)(Fe^{3+} \text{ ions})} \times 100$$

so, x = 23.1 Fe^{2+} ions replaced per 100 O ions.

\therefore there are $2x/3 = 15.4$ Fe^{3+} ions introduced per 100 O ions

\therefore No of vacancies created per 100 O ions

= No of Fe^{2+} ions replaced - No. of Fe^{3+} ions introduced per 100 O ions

= 23.1 - 15.4

= 7.7

In one unit cell,

$$\text{Fe}^{3+} = 15.4 / 25 \text{ cells} = 0.616 \text{ ions/cell}$$

$$\text{vacancies} = 7.7 / 25 = 0.308 \text{ vacancies/cell}$$

In FeO, which has the sodium chloride structure, the close packed directions are $\langle 100 \rangle$, where oxygen and iron ions touch. The lattice parameter is:

$$a_o = 2r_o + 2r_{\text{Fe}} = 2(0.132) + 2(0.074) = 0.412 \text{ nm} = 4.12 \times 10^{-8} \text{ cm.}$$

Therefore:

$$\text{Fe}^{3+}/\text{cm}^3 = 0.616 \text{ ions/cell} / (4.12 \times 10^{-8} \text{ cm})^3 = 8.808 \times 10^{21} / \text{cm}^3$$

$$\text{vacancies}/\text{cm}^3 = 0.308 \text{ per cell} / (4.12 \times 10^{-8} \text{ cm})^3 = 4.404 \times 10^{21} / \text{cm}^3$$

The formula for the oxide is therefore $\text{Fe}_{0.923}\text{O}$.

($\text{Fe}^{2+} = 3.076 \text{ ions/cell}$, $\text{O} = 4 \text{ ions/cell}$, $\text{Fe}^{3+} = 0.616 \text{ ions/cell}$, $\text{vacancies} = 0.308 \text{ ions/cell}$).

14.10 Each vacancy in FeO provides one charge carrier that will contribute to electrical conductivity in a ceramic. If the ratio between Fe^{3+} ions to Fe^{2+} ions is 1 to 25, calculate the number of charge carriers per cubic centimetre. This is one way of producing a semiconducting ceramic material.

Let's call "x" the number of Fe^{2+} ions replaced in the FeO structure. Remember that two Fe^{3+} ions replace three Fe^{2+} ions, creating a vacancy. If we consider 25 unit cells (or 100 Fe^{2+} sites) as our basis, we can calculate the number of Fe^{2+} ions "x" that are replaced:

$$\text{Fe}^{3+} / \text{Fe}^{2+} = (2/3)x / (100 - x) = 1 / 25$$

$$x = 5.660 \text{ Fe}^{2+} \text{ ions replaced in 25 unit cells}$$

$$(2/3)x = 3.773 \text{ Fe}^{3+} \text{ ions are introduced in 25 unit cells}$$

$$5.660 - 3.773 = 1.887 \text{ vacancies are introduced in 25 unit cells}$$

As shown in Problem 14.9, the lattice parameter of the FeO is $4.12 \times 10^{-8} \text{ cm}$. One charge carrier for electrical conduction is introduced for each vacancy. The number of charge carriers, or vacancies, per cm^3 , is then:

$$\text{charge carriers} = \frac{1.887 \text{ per 25 cells}}{(25 \text{ cells})(4.12 \times 10^{-8} \text{ cm})^3} = 1.079 \times 10^{21} / \text{cm}^3$$

14.11 Using the $\text{MgO} \cdot \text{Al}_2\text{O}_3$ phase diagram, determine the weight percent Al_2O_3 if the spinel had the stoichiometric composition. (a) Is the spinel nonstoichiometric on the MgO-rich side of the phase diagram? If so, what type of lattice imperfections might be present? (b) Is the spinel nonstoichiometric on the Al_2O_3 side of the phase diagram? If so, what type of lattice imperfections might be present?

$$MW_{\text{MgO}} = 24.312 + 16 = 40.312 \text{ g.mol}$$

$$MW_{\text{Alumina}} = 2(26.981) + 3(16) = 101.962 \text{ g.mol}$$

For the stoichiometric composition $\text{MgO} \cdot \text{Al}_2\text{O}_3$, the wt% Al_2O_3 must be:

$$\text{wt\% Al}_2\text{O}_3 = \frac{(50 \text{ mol\% Al}_2\text{O}_3)(101.962 \text{ g.mol})}{(50)(101.962 \text{ g.mol}) + (50)(40.312 \text{ g.mol})} \times 100 = 71.7\%$$

On the MgO-rich side of the phase diagram: two Al³⁺ ions are replaced by three Mg²⁺ ions. This may create a Mg²⁺ interstitial ion.

On the Al₂O₃-rich side of the phase diagram: two Al³⁺ ions replace three Mg²⁺ ions. This may create a vacancy at a Mg²⁺ site.

14.12 The specific gravity of Al₂O₃ is 3.96 Mg.m⁻³. A ceramic part is produced by sintering alumina powder. It weighs 80 g when dry, 92 g after it has soaked in water, and 58 g when suspended in water. Calculate the apparent porosity, the true porosity, and the closed pores.

From the problem statement, $\rho = 3.96$, $W_d = 80$ g, $W_w = 92$, and $W_s = 58$.
From the equations,

$$\text{apparent porosity} = \frac{W_w - W_d}{W_w - W_s} \times 100 = \frac{92 - 80}{92 - 58} \times 100 = 35.29\%$$

The bulk density is $B = W_d / (W_w - W_s) = 80 / (92 - 58) = 2.3529$ g.cm³.
Therefore:

$$\text{true porosity} = \frac{\rho - B}{\rho} \times 100 = \frac{3.96 - 2.3529}{3.96} \times 100 = 40.58\%$$

$$\text{closed porosity} = 40.58 - 35.29 = 5.29\%$$

14.13 Silicon carbide (SiC) has a specific gravity of 3.1 Mg.m⁻³. A sintered SiC part is produced, occupying a volume of 500 cm³ and weighing 1200 g. After soaking in water, the part weighs 1250 g. Calculate the bulk density, the true porosity, and the volume fraction of the total porosity that consists of closed pores.

The appropriate constants required for the equations are:

$$\rho = 3.1 \text{ Mg.m}^{-3} \quad B = 1200 \text{ g} / 500 \text{ cm}^3 = 2.4 \text{ g.cm}^3 = 2.4 \text{ Mg.m}^{-3}$$

$$W_w = 1250 \text{ g} \quad W_d = 1200 \text{ g}$$

Therefore:

$$B = 2.4 = W_d / (W_w - W_s) = 1200 / (1250 - W_s) \quad \text{or} \quad W_s = 750 \text{ g}$$

$$\text{apparent porosity} = \frac{W_w - W_d}{W_w - W_s} \times 100 = \frac{1250 - 1200}{1250 - 750} \times 100 = 10\%$$

$$\text{true porosity} = \frac{(\rho - B)}{\rho} = \frac{(3.1 - 2.4)}{3.1} \times 100 = 22.58\%$$

$$\text{closed porosity} = 22.58 - 10 = 12.58\%$$

$$f_{\text{closed}} = 12.58 / 22.58 = 0.56$$

14.14 Calculate the O:Si ratio when 20 wt% Na₂O is added to SiO₂. Explain whether this material will provide good glass forming tendencies. Above what temperature must the ceramic be heated to be all liquid?

$$MW_{\text{soda}} = 2(22.99) + 16 = 61.98 \text{ g.mol}$$

$$MW_{\text{silica}} = 28.08 + 2(16) = 60.08 \text{ g.mol}$$

$$\text{mole fraction Na}_2\text{O} = \frac{20 \text{ g} / 61.98 \text{ g.mol}}{20 / 61.98 + 80 / 60.08} = 0.1951$$

$$O/Si = \frac{(1 O/Na_2O)(0.1951) + (2 O/SiO_2)(0.8049)}{(1 Si/SiO_2)(0.8049)} = 2.24$$

Since the O/Si ratio is less than 2.5, it should be possible to produce a glass.

From the Na₂O-SiO₂ phase diagram (Figure 14.18), we find that, for 20 wt% Na₂O, the liquidus temperature is about 1000°C. We must heat the material above 1000°C to begin the glass-making operation.

14.15 How many grams of BaO can be added to 1 kg of SiO₂ before the O:Si ratio exceeds 2.5 and glass-forming tendencies are poor? Compare this to the case when Li₂O is added to SiO₂.

We can first calculate the required mole fraction of BaO required to produce an O:Si ratio of 2.5:

$$O/Si = 2.5 = \frac{(1 O/BaO)f_{BaO} + (2 O/SiO_2)(1 - f_{BaO})}{(1 Si/SiO_2)(1 - f_{BaO})}$$

$$f_{BaO} = 0.33 \quad \text{and} \quad f_{silica} = 0.67$$

The molecular weight of BaO is 137.3 + 16 = 153.3 g.mol, and that of silica is 60.08 g.mol. The weight percent BaO is therefore:

$$wt\% BaO = \frac{(0.33 \text{ mol})(153.3 \text{ g.mol})}{(0.33)(153.3) + (0.67)(60.08)} \times 100 = 55.69\%$$

For 1 kg of SiO₂, the amount of BaO is:

$$0.5569 = \frac{x \text{ g BaO}}{x \text{ g BaO} + 1000 \text{ g SiO}_2} \quad \text{or} \quad x = 1257 \text{ g BaO}$$

The mole fraction of Li₂O required is:

$$O/Si = 2.5 = \frac{(1 O/Li_2O)f_{Li_2O} + (2 O/SiO_2)(1 - f_{Li_2O})}{(1 Si/SiO_2)(1 - f_{Li_2O})}$$

$$f_{Li_2O} = 0.33 \quad \text{and} \quad f_{silica} = 0.67$$

The molecular weight of Li₂O is 2(6.94) + 16 = 29.88 g.mol, and that of silica is 60.08 g.mol. The weight percent Li₂O is therefore:

$$wt\% Li_2O = \frac{(0.33 \text{ mol})(29.88 \text{ g.mol})}{(0.33)(29.88) + (0.67)(60.08)} \times 100 = 19.7\%$$

For 1 kg of SiO₂, the amount of Li₂O is:

$$0.197 = \frac{x \text{ g Li}_2\text{O}}{x \text{ g Li}_2\text{O} + 1000 \text{ g SiO}_2} \quad \text{or} \quad x = 245 \text{ g Li}_2\text{O}$$

Much larger amounts of BaO can be added compared to Li₂O and still retain the ability to form a glass.

14.16 Calculate the O:Si ratio when 30 wt% Y₂O₃ is added to SiO₂. Will this material provide good glass-forming tendencies?

$$MW_{yttria} = 2(88.91) + 3(16) = 225.82 \text{ g.mol}$$

$$MW_{silica} = 60.08 \text{ g.mol}$$

The mole fraction of yttria is (assuming a base of 100 g of ceramic):

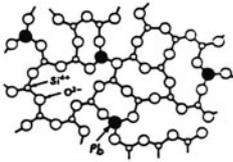
$$f_{yttria} = \frac{30 \text{ g} / 225.82 \text{ g.mol}}{30 / 225.82 + 70 / 60.08} = 0.102$$

The O/Si ratio is then:

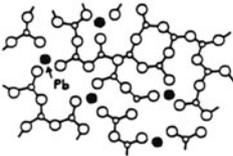
$$O/Si = \frac{(3 \text{ O/Y}_2\text{O}_3)(0.102) + (2 \text{ O/SiO}_2)(0.898)}{(1 \text{ Si/SiO}_2)(0.898)} = 2.34$$

The material will produce a glass.

14.17 Lead can be introduced into a glass either as PbO (where the Pb has a valence of +2) or as PbO₂ (where the Pb has a valence of +4). Draw a sketch (similar to Figure 14.17) showing the effect of each of these oxides on the silicate network. Which oxide is a modifier and which is an intermediate?



PbO₂ provides the same number of metal and oxygen atoms to the network as does silica; the PbO₂ does not disrupt the silicate network; therefore the PbO₂ is a intermediate.



PbO does not provide enough oxygen to keep the network intact; consequently PbO is an modifier.

14.18 A glass composed of 65 mol% SiO₂, 20 mol% CaO, and 15 mol% Na₂O is prepared. Calculate the O:Si ratio and determine whether the material has good glass-forming tendencies. Estimate the liquidus temperature of the material using Figure 14.31.

Based on the mole fractions, we can determine the O:Si ratio:

$$O/Si = \frac{(2 \text{ O/SiO}_2)(0.65) + (1 \text{ O/CaO})(0.20) + (1 \text{ O/Na}_2\text{O})(0.15)}{(1 \text{ Si/SiO}_2)(0.65)}$$

$$O/Si = 2.54$$

The glass-forming tendencies are relatively poor and special attention to the cooling rate may be required.

To determine the liquidus, we must find the weight percentages of each constituent. The molecular weights are:

$$\begin{aligned} MW_{silica} &= 60.08 \text{ g.mol} \\ MW_{CaO} &= 40.08 + 16 = 56.08 \text{ g.mol} \\ MW_{soda} &= 2(22.99) + 16 = 61.98 \text{ g.mol} \end{aligned}$$

$$wt\% \text{ SiO}_2 = \frac{(0.65)(60.08)}{(0.65)(60.08) + (0.20)(56.08) + (0.15)(61.98)} \times 100 = 65.56\%$$

$$wt\% \text{ CaO} = \frac{(0.20)(56.08)}{(0.65)(60.08) + (0.20)(56.08) + (0.15)(61.98)} \times 100 = 18.83\%$$

$$wt\% \text{ Na}_2\text{O} = \frac{(0.15)(61.98)}{(0.65)(60.08) + (0.20)(56.08) + (0.15)(61.98)} \times 100 = 15.61\%$$

From the ternary phase diagram, this overall composition gives a liquidus temperature of about 1140°C.

14.19 Hot pressed Si_3N_4 has a tensile strength of 550 MN.m^{-2} . Flaws are present due to porosity remaining in the part; the radius of curvature of these flaws is determined to be 0.005 cm. The part should be capable of withstanding an applied stress of 200 MN.m^{-2} . What is the maximum length of flaws that can be tolerated?

The maximum allowable stress σ_{actual} acting on a flaw must be no more than 550 MPa. The applied stress σ is 200 MPa, while $r = 0.005 \text{ cm}$. The length "a" of the maximum sized crack is:

$$\sigma_{\text{actual}} = 2\sigma\sqrt{a/r} \quad \text{or} \quad 550 = 2(200)\sqrt{a/0.005}$$

$$a = 0.0095 \text{ cm} \\ = 0.095 \text{ mm}$$

14.20 A sialon ceramic typically has a flexural strength of 825 MN.m^{-2} . In a three-point bend test (see Chapter 6), a sialon bar 12 mm thick and 25 mm wide is supported at two points 230 mm apart. The part is known to contain flaws 0.025 mm long with a radius at the tip of 50 nm. At what load during the test is the bar expected to fail?

$$a = 0.025 \text{ mm} \quad r = 50 \text{ nm} = 50 \times 10^{-9} \text{ m}$$

$$\sigma_{\text{actual}} = 825 \text{ MN.m}^{-2} \text{ (i.e. the flexural strength)} = 825 \text{ N.mm}^{-2}$$

The maximum applied stress σ is:

$$825 \text{ N.mm}^{-2} = 2\sigma\sqrt{0.025 \text{ mm} / 50 \times 10^{-6} \text{ mm}}$$

$$\sigma = 18.45 \text{ N.mm}^{-2}$$

The maximum allowable force is then:

$$\sigma = 18.45 \text{ N.mm}^{-2} = 3FL / 2wh^2 = (3)F(230 \text{ mm}) / (2)(25 \text{ mm})(12 \text{ mm})^2$$

$$F = 192.5 \text{ N.mm}^{-2}$$

14.21 A large ceramic part is produced from partially stabilized ZrO_2 has a fracture toughness of $11 \text{ MN.m}^{-3/2}$ and an expected yield strength of 450 MN.m^{-2} . If the part is to withstand an applied stress of half of its expected yield strength, determine the maximum size of flaws that can be present in the structure. Assume that $f = 1$.

The stress σ must be no more than $450 / 2 = 225 \text{ MN.m}^{-2}$. The maximum flaw size is then:

$$\sigma = K_{Ic} / f \sqrt{\pi a} \\ \sqrt{a} = 11 \text{ MN.m}^{-3/2} / (1) (\sqrt{\pi}) (225) \\ a = 7.6 \times 10^{-4} \text{ m} \\ = 0.076 \text{ mm.}$$

14.22 An alumina part with a square cross-section has a fracture toughness of $5.5 \text{ MN.m}^{-3/2}$ and is to be subjected to a tensile force of 89 kN. Nondestructive testing has shown that there are no flaws greater than 0.25 mm in length. Determine the minimum size of the part. Assume that $f = 1.1$.

The applied stress is:

$$\sigma = F/A = F/w^2 \quad \text{where } w \text{ is the width of the part}$$

From the fracture toughness equation:

$$\sigma = F/w^2 = K/f \sqrt{(\pi a)} \quad \text{or} \quad w^2 = Ff \sqrt{(\pi a)} / K$$

$$w^2 = \frac{(0.089 \text{ MN})(1.1)\sqrt{(\pi(0.25 \times 10^{-3}))\text{m}}}{5.5 \text{ MN.m}^{-3/2}} = 4.99 \times 10^{-4} \text{m}^2 \quad \text{or} \quad w = 0.022 \text{ m}$$

14.23 You would like a transformation toughened ZrO₂ part 2 cm thick and 3 cm wide to withstand a force of 20,000 N. The part is supported by blocks 10 cm apart. The fracture toughness of the ceramic is 9 MN.m^{-3/2} and flaws 0.05 cm long are known to be present at the surface of the part. Can this load be sustained without the part failing? Assume that f = 1.

We can calculate the maximum applied stress that will prevent a flaw from propagating:

$$K = f\sigma\sqrt{(\pi a)} \quad \text{or} \quad 9 \text{ MN.m}^{-3/2} = (1)\sigma\sqrt{(\pi(0.0005 \text{ m}))} \quad \text{or} \quad \sigma = 227 \text{ MN.m}^{-2}$$

The maximum allowable force applied to the part is therefore:

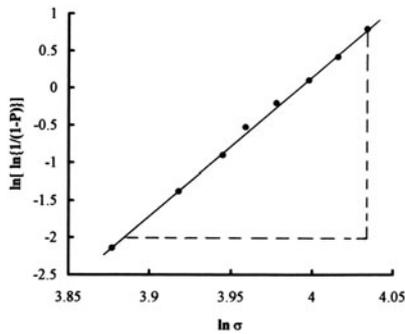
$$F = 2wh^2\sigma / 3L = \frac{(2)(0.03 \text{ m})(0.02 \text{ m})^2(227 \text{ MN.m}^{-2})}{(3)(0.1 \text{ m})} = 0.01816 \text{ MN} = 18160 \text{ N}$$

The actual stress may be as high as 20,000 N; therefore the flaws may propagate during loading.

14.24 A set of ceramic parts is subjected to bend tests, and the stress required for failure is measured; the results are as follows: 55.5 MN.m⁻², 54.5 MN.m⁻², 48.3 MN.m⁻², 52.4 MN.m⁻², 56.5 MN.m⁻², 50.3 MN.m⁻², 53.4 MN.m⁻², 51.7 MN.m⁻². Calculate the Weibull modulus for the material. Discuss whether the material would be a good choice if we wished to design a reasonably reliable part.

First we can arrange the data in order of increasing failure stress, assigning a number to each of the eight parts. Since there are eight parts, we assign a probability P that is the part number divided by nine. We can then determine the values for ln(ln[1/(1-P)]) and for ln(σ).

sample	P	ln(ln[1/(1-P)])	σ	ln(σ)
1	1/9 = 0.1111	- 2.1390	48.3	3.877
2	2/9 = 0.2222	- 1.3812	50.3	3.918
3	3/9 = 0.3333	- 0.9028	51.7	3.945
4	4/9 = 0.4444	- 0.5315	52.4	3.959
5	5/9 = 0.5555	- 0.2097	53.4	3.978
6	6/9 = 0.6666	+ 0.0939	54.5	3.998
7	7/9 = 0.7777	+ 0.4079	55.5	4.016
8	8/9 = 0.8888	+ 0.7868	56.5	4.034



This data is plotted in the graph. The slope of the straight line gives the Weibull modulus:

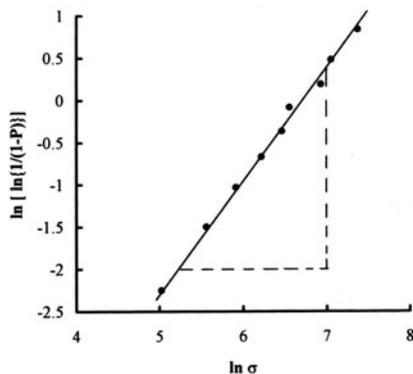
$$m = \frac{\Delta \ln(\ln[1/(1-P)])}{\Delta \ln(\sigma)} = \frac{0.8 - (-2.0)}{4.034 - 3.884} = 18.67$$

This high value for the Weibull modulus suggests that reliable parts can be made using the ceramic; the distribution of failure stress is relatively narrow and predictable.

14.25 A set of ceramic parts are tested and the stress required for failure is measured; the results are as follows: 152 MN.m⁻², 260 MN.m⁻², 500 MN.m⁻², 1150 MN.m⁻², 700 MN.m⁻², 640 MN.m⁻², 370 MN.m⁻², 1020 MN.m⁻², and 1590 MN.m⁻². Calculate the Weibull modulus for the material. Discuss whether the material would be a good choice if we wished to design a reasonably reliable part.

First we can arrange the data in order of increasing failure stress, assigning a number to each of the nine parts. Since there are nine parts, we assign a probability P that is the part number divided by ten. We can then determine the values for $\ln(\ln[1/(1-P)])$ and for $\ln(\sigma)$.

sample	P	$\ln(\ln[1/(1-P)])$	σ	$\ln(\sigma)$
1	1/10 = 0.1	- 2.250	152 MN.m ⁻²	5.024
2	2/10 = 0.2	- 1.500	260 MN.m ⁻²	5.561
3	3/10 = 0.3	- 1.031	370 MN.m ⁻²	5.914
4	4/10 = 0.4	- 0.672	500 MN.m ⁻²	6.215
5	5/10 = 0.5	- 0.367	640 MN.m ⁻²	6.461
6	6/10 = 0.6	- 0.087	700 MN.m ⁻²	6.551
7	7/10 = 0.7	+ 0.186	1020 MN.m ⁻²	6.928
8	8/10 = 0.8	+ 0.476	1150 MN.m ⁻²	7.050
9	9/10 = 0.9	+ 0.834	1590 MN.m ⁻²	7.371



This data is plotted in the graph. The slope of the straight line gives the Weibull modulus:

$$m = \frac{\Delta \ln(\ln[1/(1-P)])}{\Delta \ln(\sigma)} = \frac{0.40 - (-2.0)}{7.00 - 5.20} = 1.33$$

This low value for the Weibull modulus suggests that reliable parts cannot be made using the ceramic; the distribution of failure stress is relatively wide and unpredictable.

14.26 Ceramic moulds used in producing castings contain alcohol which is burned prior to casting. As a result, a network of microcracks is produced that allow gases to escape through the mould when the liquid metal is poured into the mould. What effect might the microcracks have on the properties of the mould?

If the cracks are very fine, they may be too small to propagate on their own. However, they may block the movement of larger cracks whose length exceeds the critical length. By blunting the larger cracks, the mould may actually be more resistant to catastrophic cracking. This process is often used in the "ceramic moulding" casting process.

14.27 A shear stress of 20 MN.m^{-2} is to be used to deform the surface of a 1 cm thick plate of soda-lime glass, providing a surface velocity of 1 cm.s^{-1} . The velocity at the opposite surface is to be zero. What viscosity is required, assuming a linear velocity gradient in the glass? To what temperature must the glass be heated to do this?

$$\text{The velocity gradient} = \frac{10^{-2} \text{ m.s}^{-1} - 0 \text{ m.s}^{-1}}{10^{-2} \text{ m}} = 1 \text{ s}^{-1}$$

The viscosity is:

$$\eta = \tau / dv/dz = 20 \times 10^6 \text{ N.m}^{-2} / 1 \text{ s}^{-1} = 20 \times 10^6 \text{ Pas} = \text{or } \text{N.s.m}^{-2}$$

From the graph in Figure 14.25, we find that, to obtain $2 \times 10^6 \text{ N.s.m}^{-2}$, we need a value of $10,000/T = 11$ or $\frac{10,000}{T} = 11$

$$\therefore T = 909 \text{ K} = 636^\circ\text{C}$$

14.28 Calculate and compare the activation energies for viscous flow in fused silica and soda-lime glass. Explain how this influences the processing of the glass.

We can pick two points from the graphs in Figure 14.25 in order to determine the activation energy for each glass.

For fused silica: $\eta = 10^{13} \text{ N.s.m}^{-2}$ at $10,000/T = 7.5$ or $T = 1333 \text{ K}$
 $\eta = 10^7 \text{ N.s.m}^{-2}$ at $10,000/T = 5.3$ or $T = 1887 \text{ K}$

$$\frac{10^{13}}{10^7} = \frac{\eta_0 \exp[Q/R(1333)]}{\eta_0 \exp[Q/R(1887)]}$$

$$10^6 = \exp[(9.023 \times 10^{-5} - 6.374 \times 10^{-5})Q] = \exp[(2.649 \times 10^{-5})Q]$$

$$\ln(10^6) = 13.816 = 2.649 \times 10^{-5}Q \quad \text{or} \quad Q = 521556 \text{ J.mol}^{-1}$$

For soda-lime: $\eta = 10^5 \text{ poise}$ at $10,000/T = 9.9$ or $T = 1010 \text{ K}$
 $\eta = 10 \text{ poise}$ at $10,000/T = 6.1$ or $T = 1639 \text{ K}$

$$\frac{10^5}{10} = \frac{\eta_0 \exp[Q/(8.314)(1010)]}{\eta_0 \exp[Q/(8.314)(1639)]}$$

$$10^4 = \exp[(1.1909 \times 10^{-4} - 7.3385 \times 10^{-5})Q] = \exp(4.5702 \times 10^{-5}Q)$$

$$\ln(10^4) = 9.210 = 4.5702 \times 10^{-5}Q \quad \text{or} \quad Q = 201523 \text{ J.mol}^{-1}$$

The activation energy for the viscosity of soda-lime glass is much lower than that for the fused silica. This low activation energy helps make the soda-lime glass much more easily worked into useful products.

14.29 A stress of 25 MN.m^{-2} is applied to a 150 mm long rod of borosilicate glass held at a constant temperature. If the rod is to elongate no more than 1 mm in one year, what is the maximum temperature that can be used? The viscosity of the glass is 10^7 N.s.m^{-2} at 1042 K and 10^3 N.s.m^{-2} at 1471 K.

First we can determine the constants Q and η_0 by using Figure 14.25.

$$\begin{aligned}\eta &= 10^7 \text{ N.s.m}^{-2} \text{ at } 1042 \text{ K} \\ \eta &= 10^3 \text{ N.s.m}^{-2} \text{ at } 1471 \text{ K}\end{aligned}$$

$$\frac{10^7}{10^3} = \frac{\eta_0 \exp[Q/(8.314)(1042)]}{\eta_0 \exp[Q/(8.314)(1471)]}$$

$$10^4 = \exp[(1.1543 \times 10^{-4} - 8.1766 \times 10^{-5})Q] = \exp(3.3664 \times 10^{-5}Q)$$

$$\ln(10^4) = 9.2103 = 3.3664 \times 10^{-5}Q \text{ or } Q = 273595 \text{ J.mol}^{-1}$$

$$\eta_0 = 10^7 / \exp[273595 / (8.314)(1042)] = 1.913 \times 10^{-7} \text{ N.s.m}^{-2}$$

The maximum allowable strain is $\Delta\epsilon = 10^{-3} \text{ m} / 150 \times 10^{-3} \text{ m} = 0.00667 \text{ m.m}^{-1}$

The time is 1 year = $31.536 \times 10^6 \text{ s}$

The maximum allowable strain rate is therefore:

$$d\epsilon/dt = 0.00667 / 31.536 \times 10^6 = 2.115 \times 10^{-10} \text{ m.m}^{-1}.\text{s}^{-1}$$

The stress acting on the part is $25 \text{ MN.m}^{-2} = 25 \times 10^6 \text{ N.m}^{-2}$ (the units for the viscosity). The strain rate can be approximated using Equation 14.10:

$$d\epsilon/dt = \sigma / \eta \quad \text{or}$$

$$2.115 \times 10^{-10} \text{ m.m}^{-1}.\text{s}^{-1} = 25 \times 10^6 \text{ N.m}^{-2} / 1.913 \times 10^{-7} \exp(273595/8.314T)$$

$$\exp(32,908/T) = 6.179 \times 10^{23}$$

$$32,908/T = \ln(6.179 \times 10^{23}) = 54.7806$$

$$T = 601 \text{ K} = 328^\circ\text{C}$$

14.30 At 70 MN.m^{-2} , the creep rate ($d\epsilon/dt$) of a silicon nitride ceramic is $0.0635 \text{ mm.mm}^{-1}.\text{h}^{-1}$ at 1400°C ; at 1250°C , the creep rate is $0.0002 \text{ mm.mm}^{-1}.\text{h}^{-1}$. Estimate the activation energy for viscous flow, using these data.

From Equation 14.10,

$$\frac{d\epsilon}{dt} = \frac{\sigma}{n}$$

$$\frac{d\epsilon}{dt} = 70 \times 10^6 / n_0 \exp(Q/RT)$$

$$\text{or } n_0 \exp(Q/RT) = 70 \times 10^6 / d\epsilon/dt$$

Solving for Q :

$$\frac{\exp[Q/(8.314)(1400+273)]}{\exp[Q/(8.314)(1250+273)]} = \frac{70 \times 10^6 / 0.0635}{70 \times 10^6 / 0.0002}$$

$$\exp[Q(7.1894 \times 10^{-5} - 7.8975 \times 10^{-5})] = 3.1496 \times 10^{-3}$$

$$\exp[-7.0808 \times 10^{-6}Q] = 3.1496 \times 10^{-3}$$

$$- 7.0808 \times 10^{-6} Q = \ln(3.1496 \times 10^{-3}) = -5.7605$$

$$\therefore Q = 813535 \text{ J.mol}^{-1}$$

14.31 From Figure 14.27, determine the relationship between creep rate and grain size for MgO. Explain why the grain size influences creep rate. If you were designing an MgO refractory for optimum resistance to creep, would you use a large or a small grain size?

The graph is a straight line in a log-log plot; therefore the relationship must be of the form:

$$d\epsilon/dt = a(d)^n$$

where d is the grain diameter. One method for finding the constants is (since the "x" and "y" logarithmic scales are the same) to simply measure the slope with a ruler. By doing so, the slope $n = -2$. The slope could also be found using simultaneous equations from two points on the graph. If $n = -2$, then we can pick a point on the graph, such as strain rate = $10^{-5} \text{ mm.mm}^{-1}.\text{h}^{-1}$ when $d = 70$ microns, and find "a":

$$10^{-5} = a(70)^{-2} \quad \text{or} \quad a = 0.049$$

Our equation is therefore $d\epsilon/dt = 0.049 / d^2$

The equation shows that, as the grain size increases, the creep rate decreases. For best creep resistance, we prefer a large grain size, which helps to minimize grain boundary sliding.

14.32 Suppose you combine 6 mol of SiO_2 with 1 mol of Na_2O and 1 mol of CaO . Determine the liquidus temperature for the ceramic.

$$MW_{\text{silica}} = 60.08 \text{ g.mol} \quad MW_{\text{CaO}} = 56.08 \text{ g.mol} \quad MW_{\text{soda}} = 61.98 \text{ g.mol}$$

We can then calculate the weight percentages of each ceramic material:

$$\text{wt\% SiO}_2 = \frac{(6 \text{ mol SiO}_2)(60.08 \text{ g.mol})}{(6)(60.08) + (1)(61.98) + (1)(56.08)} \times 100 = 75.33\%$$

$$\text{wt\% Na}_2\text{O} = \frac{(1 \text{ mol Na}_2\text{O})(61.98 \text{ g.mol})}{(6)(60.08) + (1)(61.98) + (1)(56.08)} \times 100 = 12.95\%$$

$$\text{wt\% CaO} = \frac{(1 \text{ mol CaO})(56.08 \text{ g.mol})}{(6)(60.08) + (1)(61.98) + (1)(56.08)} \times 100 = 11.72\%$$

From the ternary phase diagram, the liquidus temperature is about 1200°C .

14.33 A SiO_2 - Al_2O_3 fireclay brick can perform satisfactorily at 1700°C if no more than 20% liquid surrounds the mullite present in the microstructure. What is the minimum percent alumina that must be in the refractory?

We can draw a tie line in the phase diagram, Figure 14.38, at 1700°C . The ends of the tie line are 72% and 16%. If our unknown composition is "x", then

$$20\% L = \frac{71 - x}{71 - 16} \times 100 \quad \text{or} \quad x = 60.0\% \text{ Al}_2\text{O}_3$$

14.34 How much kaolinite clay must be added to 100 g of quartz to produce a SiO₂-30% Al₂O₃ fireclay brick after firing?

The formula for kaolinite is Al₂Si₂O₅(OH)₄ or Al₂O₃·2SiO₂·2H₂O. The molecular weight of the clay is therefore 101.962 g.mol of alumina + 2(60.08 g.mol of silica) + 2(18 g.mol of water) = 258.122 g.mol. But after firing, the water is lost, leaving behind Al₂O₃·2SiO₂ with a molecular weight of 258.122 - 36 = 222.122 g.mol. The weight percent Al₂O₃ in the dried clay is:

$$\text{wt\% Al}_2\text{O}_3 \text{ in dried clay} = \frac{(1 \text{ mol})(101.962 \text{ g.mol})}{(1)(101.962) + (2)(60.08)} \times 100 = 45.9\%$$

The weight percent of alumina in the final dried product is 30%, given by:

$$\text{wt\% Al}_2\text{O}_3 = \frac{(0.459)(x \text{ g clay})}{x \text{ g clay} + 100 \text{ g quartz}} \times 100 = 30\%$$

$$x = 188.68 \text{ g of dried clay}$$

The amount of clay that must be added to the quartz prior to firing is:

$$\frac{188.68 \text{ g dried clay}}{222.122 \text{ g.mol}} = \frac{x}{258.122 \text{ g.mol}}$$

$$x = 219.26 \text{ g clay (before firing)}$$

14.35 Suppose we combine 60 kg of Al₂O₃·4SiO₂·H₂O with 120 kg of 2CaO·Al₂O₃·SiO₂ to produce a clay body. The ceramic is dried and fired at 1600°C. Determine the weight and composition of the clay body after firing.

$$\text{MW (Al}_2\text{O}_3 \cdot 4\text{SiO}_2 \cdot \text{H}_2\text{O)} = 360.282 \text{ g.mol} \quad \text{MW (H}_2\text{O)} = 18 \text{ g.mol}$$

After the water is driven off, the clay's composition is Al₂O₃·4SiO₂ and its molecular weight is 360.282 - 18 = 342.282 g.mol. The amount of clay left after drying is therefore:

$$60 \text{ kg} / 360.282 = x / 342.282 \quad \text{or} \quad x = 57 \text{ kg}$$

We combine this 57 kg with 120 kg of the mineral, which has a molecular weight of MW (2CaO·Al₂O₃·SiO₂) = 274.202 g.mol. The total weight of the clay body will be 57 + 120 = 177 kg. The weight percentage of Al₂O₃ in the clay body is found by using the mole fraction of alumina in each material, where the molecular weight of alumina is 101.962 g.mol:

$$\text{wt\% Al}_2\text{O}_3 = \frac{(57 \text{ kg})(101.962 / 342.282) + (120 \text{ kg})(101.962 / 274.202)}{177 \text{ kg}} \times 100$$

$$= 34.8\%$$

Similarly we can find the percentages of CaO (MW = 56.08 g.mol) and SiO₂ (MW = 60.08 g.mol):

$$\text{wt\% SiO}_2 = \frac{(57 \text{ kg})(4)(60.08 / 342.282) + (120 \text{ kg})(60.08 / 274.202)}{177 \text{ kg}} \times 100$$

$$= 37.5\%$$

$$\text{wt\% CaO} = \frac{(120 \text{ kg})(2)(56.08 / 274.202)}{177 \text{ kg}} \times 100 = 27.7\%$$

The clay body therefore weighs 177 kg and contains 34.8% Al₂O₃, 37.5% SiO₂, and 27.7% CaO.

Chapter 15 Polymers

15.1 The molecular weight of polymethyl methacrylate (see Table 15.5) is $250,000 \text{ g.mol}^{-1}$. If all of the polymer chains are the same length, calculate (a) the degree of polymerization and (b) the number of chains in 1 g of the polymer.

The molecular weight of methyl methacrylate is:

$$MW = 5 C + 2 O + 8 H = 100 \text{ g.mol}^{-1}$$

(a) Degree of polymerization = $250,000 / 100 = 2,500$

(b) In 1 g of the polymer:

$$\frac{(1 \text{ g})(6.02 \times 10^{23} \text{ chains.mol}^{-1})}{250,000 \text{ g.mol}^{-1}} = 2.408 \times 10^{18} \text{ chains}$$

15.2 The degree of polymerization of polytetrafluoroethylene (see Table 15.5) is 7500. If all of the polymer chains are the same length, calculate (a) the molecular weight of the chains and (b) the total number of chains in 1000 g of the polymer.

The molecular weight of tetrafluoroethylene is:

$$MW = 2 C + 4 F = 100 \text{ g.mol}^{-1}$$

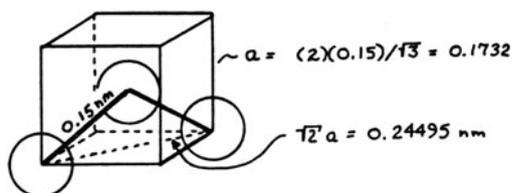
(a) $MW_{\text{chains}} = (7500)(100) = 750,000 \text{ g.mol}^{-1}$

(b) In 1000 g of the polymer:

$$\frac{(1000 \text{ g})(6.02 \times 10^{23} \text{ chains.mol}^{-1})}{750,000 \text{ g.mol}^{-1}} = 8.03 \times 10^{20} \text{ chains}$$

15.3 The distance between the centers of two adjacent carbon atoms in linear polymers is approximately 0.15 nm. Calculate the length of an ultrahigh molecular weight polyethylene chain that has a molecular weight of $1,000,000 \text{ g.mol}^{-1}$.

The carbon atoms in the chains form a tetrahedron. The distance from the corner carbon atom to the center carbon atom is about 0.15 nm. The edge "a" of the tetrahedral cube is $\sqrt{3}a/2 = 0.15$, or $a = 0.1732 \text{ nm}$. The length of one repeat unit is then the face diagonal of the tetrahedral cube, or $\sqrt{2}a = 0.24495 \text{ nm}$. This distance is the length of one repeat unit.



The molecular weight of ethylene is $2 C + 4 H = 28 \text{ g.mol}^{-1}$. The number of monomers, or repeat units, in a chain of the ultrahigh molecular weight polymer is:

$$\text{repeat units} = 1,000,000 / 28 = 35,714$$

The length of one repeat unit is 0.24495 nm. Therefore the length of a chain is:

$$\text{length} = (0.24495 \text{ nm})(35,714 \text{ units}) = 8,748 \text{ nm.chain}^{-1}$$

15.4 A polyethylene rope weighs 370 g per metre. If each chain contains 7000 repeat units, calculate (a) the number of polyethylene chains in a 3 m length of rope and (b) the total length of chains in the rope, assuming that carbon atoms in each chain are approximately 0.15 nm apart.

The molecular weight of ethylene is 28 g.mol^{-1} , so the molecular weight of the polyethylene is $7000 \times 28 = 196,000 \text{ g.mol}^{-1}$. The weight of the 3 m length of rope is $(370 \text{ g m}^{-1})(3 \text{ m}) = 1110 \text{ g}$.

(a) The number of chains is:

$$\frac{(1110 \text{ g})(6.02 \times 10^{23} \text{ chain./mol}^{-1})}{196,000 \text{ g.mol}^{-1}} = 3.41 \times 10^{21} \text{ chains}$$

(b) The length of one repeat unit is 0.24495 nm (from Problem 15.3). Therefore the length of each chain, which contain 7000 repeat units, is

$$\text{one chain} = (7000)(0.24495 \text{ nm}) = 1715 \text{ nm} = 1.715 \times 10^{-6} \text{ m}$$

$$\text{all chains} = (1.715 \times 10^{-6} \text{ m.chain}^{-1})(3.41 \times 10^{21} \text{ chains})$$

$$= 5.847 \times 10^{15} \text{ m}$$

15.5 Suppose 20 g of benzoyl peroxide are introduced to 5 kg of propylene monomer. If 30% of the initiator groups are effective, calculate the expected degree of polymerization and the molecular weight of the polypropylene polymer if (a) all of the termination of the chains occurs by combination and (b) all of the termination occurs by disproportionation.

$$MW_{\text{propylene}} = 3 C + 6 H = 42 \text{ g.mol}^{-1}$$

$$5000 \text{ g} / 42 \text{ g.mol}^{-1} = 119 \text{ mol of propylene}$$

$$MW_{\text{benzoyl peroxide}} = 14 C + 4 O + 10 H = 242 \text{ g.mol}^{-1}$$

$$20 \text{ g} / 242 \text{ g.mol}^{-1} = 0.0826 \text{ mol of benzoyl peroxide}$$

If only 30% of the initiator is effective, the actual number of moles of benzoyl peroxide involved in the polymerization process is

$$(0.3)(0.0826 \text{ mol}) = 0.0248 \text{ mol of benzoyl peroxide}$$

(a) For combination, 1 mol of benzoyl peroxide produces 1 chain:

$$\text{degree of polymerization} = 119 \text{ mol} / 0.0248 \text{ mol} = 4798$$

(b) For disproportionation, 1 mol of benzoyl peroxide produces 2 chains:

$$\text{degree of polymerization} = (2)(119 \text{ mol}) / 0.0248 \text{ mol} = 9597$$

15.6 Suppose hydrogen peroxide (H_2O_2) is used as the initiator for 10 kg of vinyl chloride monomer. Show schematically how the hydrogen peroxide will initiate the polymer chains. Calculate the required amount of hydrogen peroxide (assuming that it is 10% effective) required to produce a degree of polymerization of 4000 if (a) termination of the chains occurs by combination and (b) termination occurs by disproportionation.

$$MW_{\text{vinyl chloride}} = 2 C + 3 H + 1 Cl = 62.453 \text{ g.mol}^{-1}$$

$$10,000 \text{ g} / 62.453 \text{ g.mol}^{-1} = 160.12 \text{ mol of vinyl chloride}$$

$$MW_{\text{hydrogen peroxide}} = 2 H + 2 O = 34 \text{ g.mol}^{-1}$$

(a) One molecule of hydrogen peroxide decomposes to produce two OH groups; one could initiate a chain and the other could terminate the chain. In order to obtain a degree of polymerization of 4000 by the combination mechanism, the number of mols of hydrogen peroxide (x) required for each chain is:

$$4000 = \frac{160.12 \text{ mol of v.c.}}{x \text{ mol } H_2O_2}$$

or $x = 0.04 \text{ mol of } H_2O_2$

The peroxide is only 10% effective; the amount of peroxide required is therefore:

$$(0.04 \text{ mol}) (34 \text{ g.mol}^{-1}) / 0.1 = 13.6 \text{ g}$$

(b) For disproportionation, 1 mol of peroxide is sufficient for polymerizing two chains of polymer. Only 0.02 mol is required per chain. Therefore the amount required is:

$$(0.02 \text{ mol}) (34 \text{ g.mol}^{-1}) / 0.1 = 6.8 \text{ g}$$

15.7 A common copolymer is produced by including both ethylene and propylene monomers in the same chain. Calculate the molecular weight of the polymer produced using 1 kg of ethylene and 3 kg of propylene, giving a degree of polymerization of 5000.

We will consider that each repeat unit -- whether ethylene or propylene -- counts towards the degree of polymerization. We can calculate the number of moles of each monomer in the polymer:

$$MW_{\text{ethylene}} = 28 \text{ g.mol}^{-1} \quad 1000 \text{ g} / 28 \text{ g.mol}^{-1} = 35.71 \text{ mol of ethylene}$$

$$MW_{\text{propylene}} = 42 \text{ g.mol}^{-1} \quad 3000 \text{ g} / 42 \text{ g.mol}^{-1} = 71.43 \text{ mol of propylene}$$

The mole fraction of each monomer is:

$$f_{\text{ethylene}} = 35.71 / (35.71 + 71.43) = 0.333$$

$$f_{\text{propylene}} = 71.43 / (35.71 + 71.43) = 0.667$$

The molecular weight of the polymer is then:

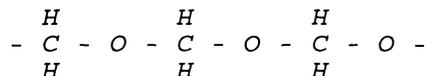
$$MW_{\text{polymer}} = 5000 [(0.333)(28) + (0.667)(42)] = 186,690 \text{ g.mol}^{-1}$$

15.8 The formula for formaldehyde is HCHO. (a) Draw the structure of the formaldehyde molecule and repeat unit. (b) Does formaldehyde polymerize to produce an acetal polymer (see Table 15.8) by the addition mechanism or the condensation mechanism? Try to draw a sketch of the reaction and the acetal polymer by both mechanisms.

(a) The structure of the monomer is:

$$\begin{array}{c} \text{H} \\ | \\ \text{C} = \text{O} \\ | \\ \text{H} \end{array}$$

(b) In the addition mechanism, the double (unsaturated) bond between the carbon and oxygen is replaced by a single bond, permitting repeat units to be joined:



Polymerization by the condensation mechanism cannot occur with only the formaldehyde monomer.

15.9 You would like to combine 5 kg of dimethyl terephthalate with ethylene glycol to produce polyester (PET). Calculate (a) the amount of ethylene glycol required, (b) the amount of byproduct evolved, and (c) the amount of polyester produced.

$$\begin{aligned} MW_{\text{ethylene glycol}} &= 2 \text{ C} + 2 \text{ O} + 6 \text{ H} = 62 \text{ g.mol}^{-1} \\ MW_{\text{terephthalate}} &= 10 \text{ C} + 4 \text{ O} + 10 \text{ H} = 194 \text{ g.mol}^{-1} \end{aligned}$$

$$5000 \text{ g} / 194 \text{ g mol} = 25.773 \text{ mol of dimethyl terephthalate}$$

(a) Equal numbers of moles of the two monomers are required for polymerization to occur:

$$5 \text{ kg} / 194 \text{ g.mol}^{-1} = x \text{ kg} / 62 \text{ g.mol}^{-1}$$

$$x = 1.598 \text{ kg of ethylene glycol}$$

(b) The byproduct of the condensation reaction between the two monomers is methyl alcohol, COH_4 . One molecule of the alcohol is produced for each monomer that is attached to the growing polymer chain. Thus the amount of byproduct produced is $2 \times 25.773 \text{ mol}$, or 51.546 mol :

$$MW_{\text{alcohol}} = 1 \text{ C} + 1 \text{ O} + 4 \text{ H} = 32 \text{ g.mol}^{-1}$$

$$\text{weight} = (51.546 \text{ mol})(32 \text{ g.mol}^{-1}) = 1649 \text{ g of methyl alcohol}$$

(c) The total weight of the PET produced is the sum of the two monomers minus the weight of the byproduct:

$$\text{weight} = 5 \text{ kg} + 1.598 \text{ kg} - 1.649 \text{ kg} = 4.949 \text{ kg}$$

15.10 Would you expect polyethylene to polymerize at a faster or slower rate than polymethyl methacrylate? Explain. Would you expect polyethylene to polymerize at a faster or slower rate than a polyester? Explain.

In both cases, we would expect polyethylene to polymerize at a faster rate. The ethylene monomer is smaller than the methyl methacrylate monomer and therefore should diffuse more quickly to the active ends of the growing chains. In polymethyl methacrylate, two different monomers must diffuse to the active end of the chain in order for polymerization to continue; this would also be expected to occur at a slower rate than diffusion of only ethylene monomer.

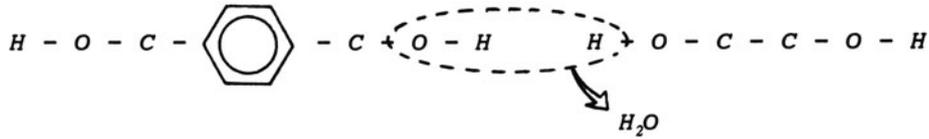
15.11 You would like to combine 10 kg of ethylene glycol with terephthalic acid to produce a polyester. The monomer for terephthalic acid is shown below. (a) Determine the byproduct of the condensation reaction and (b)

calculate the amount of terephthalic acid required, the amount of byproduct evolved, and the amount of polyester produced.

$$MW_{\text{glycol}} = 2 C + 2 O + 6 H = 62 \text{ g.mol}^{-1}$$

$$MW_{\text{acid}} = 8 C + 4 O + 6 H = 166 \text{ g.mol}^{-1}$$

(a) The O-H group from the terephthalic acid combines with H from ethylene glycol, producing water as the byproduct. One molecule of water is produced for each monomer that is attached to the polymer chain.



(b) The number of moles of ethylene glycol present is

$$10,000 \text{ g} / 62 \text{ g.mol}^{-1} = 161.29 \text{ mol of ethylene glycol}$$

The amount of terephthalic acid must also be 161.29 mol. Its mass is:

$$x \text{ g of acid} / 166 \text{ g.mol}^{-1} = 161.29 \text{ mol}$$

$$x = 26,770 \text{ g} = 26.77 \text{ kg of terephthalic acid}$$

Two moles of water are produced for each mole of ethylene glycol. The amount of water evolved is then:

$$(2) (161.29 \text{ mol of glycol}) (18 \text{ g.mol}^{-1}) = 5,810 \text{ g} = 5.81 \text{ kg}$$

The total weight of the polymer is:

$$\text{weight} = 10 \text{ kg} + 26.77 \text{ kg} - 5.81 \text{ kg} = 30.96 \text{ kg}$$

15.12 The data below were obtained for polyethylene. Determine (a) the weight average molecular weight and degree of polymerization and (b) the number average molecular weight and degree of polymerization.

Molecular weight range (g.mol^{-1})	x_i	f_i	M_i	$f_i M_i$	$x_i M_i$
0-3,000	0.03	0.01	1,500	15	45
3,000-6,000	0.10	0.08	4,500	360	450
6,000-9,000	0.22	0.19	7,500	1425	1650
9,000-12,000	0.36	0.27	10,500	2835	3780
12,000-15,000	0.19	0.23	13,500	3105	2565
15,000-18,000	0.07	0.11	16,500	1815	1155
18,000-21,000	0.02	0.06	19,500	1170	390
21,000-24,000	0.01	0.05	22,500	1125	225
		sum =		11,850	10,260

(a) The molecular weight of the ethylene monomer is 28 g.mol. Therefore The weight average molecular weight and degree of polymerization are:

$$M_x = 11,850 \text{ g.mol}^{-1} \quad DP_x = 11,850 / 28 \text{ g.mol}^{-1} = 423$$

(b) The number average molecular weight and degree of polymerization are:

$$M_n = 10,260 \text{ g.mol}^{-1} \quad DP_n = 10,260 / 28 \text{ g.mol}^{-1} = 366$$

15.13 Analysis of a sample of polyacrylonitrile (PAN) (see Table 15.6) shows that there are six lengths of chains, with the following number of chains of each length. Determine (a) the weight average molecular weight and degree of polymerization and (b) the number average molecular weight and degree of polymerization.

Number of chains	Mean Molecular weight of chains (g.mol)	x_i	$x_i M_i$	weight	f_i	$f_i M_i$
10,000	3,000	0.137	411	30×10^6	0.044	132
18,000	6,000	0.247	1482	108×10^6	0.159	954
17,000	9,000	0.233	2097	153×10^6	0.226	2034
15,000	12,000	0.205	2460	180×10^6	0.265	3180
9,000	15,000	0.123	1845	135×10^6	0.199	2985
4,000	18,000	0.055	990	72×10^6	0.106	1908
73,000		sum =	9,285	678×10^6		11,193

The molecular weight of the acrylonitrile monomer is:

$$MW_{\text{acrylonitrile}} = 3 \text{ C} + 1 \text{ N} + 3 \text{ H} = 53 \text{ g.mol}^{-1}$$

(a) The weight average molecular weight and degree of polymerization are:

$$MW_w = 11,193 \text{ g.mol}^{-1} \quad DP_w = 11,193 / 53 = 211$$

(b) The number average molecular weight and degree of polymerization are:

$$MW_n = 9,285 \text{ g.mol}^{-1} \quad DP_n = 9285 / 53 \text{ g.mol}^{-1} = 175$$

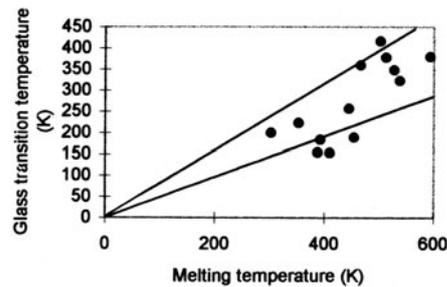
15.14 Explain why you would prefer that the number average molecular weight of a polymer be as close as possible to the weight average molecular weight.

We do not want a large number of small chains in the polymer; the small chains, due to less entanglement, will reduce the mechanical properties.

15.15 Using Table 15.2, plot the relationship between the glass transition temperatures and the melting temperatures of the addition thermoplastic polymers. What is the approximate relationship between these two critical temperatures? Do the condensation thermoplastic polymers and the elastomers also follow the same relationship?

Converting temperature to Kelvin:

	T_m	T_g
LD polyethylene	388	153
HD polyethylene	410	153
PVC	448-485	360
Polypropylene	441-449	257
Polystyrene	513	358-398
PAN	593	380
Acetal	454	188
6,6 Nylon	538	323
Polycarbonate	503	416
Polyester	528	348
Polybutadiene	393	183
Polychloroprene	353	223
Polyisoprene	303	200



Most of the polymers fall between the lines constructed with the relationships $T_g = 0.5T_m$ and $T_g = 0.75T_m$. The condensation thermoplastics and elastomers also follow this relationship.

15.16 List the addition polymers in Table 15.2 that might be good candidates for making the bracket that holds the rear view mirror onto the outside of an automobile, assuming that temperatures frequently fall below zero degrees Celsius. Explain your choices.

Because of the mounting of the rear view mirror, it is often subject to being bumped; we would like the mounting material to have reasonable ductility and impact resistance so that the mirror does not break off the automobile. Therefore we might want to select a material that has a glass transition temperature below 0°C . Both polyethylene and polypropylene have low glass transition temperatures and might be acceptable choices. In addition, acetal (polyoxymethylene) has a low glass transition temperature and (from Table 15.6) is twice as strong as polyethylene and polypropylene. Finally all of the elastomers listed in Table 15.2 might be appropriate choices.

15.17 Based on Table 15.2, which of the elastomers might be suited for use as a gasket in a pump for liquid CO_2 at -78°C ? Explain.

We wish to select a material that will not be brittle at very low temperatures, that is, the elastomer should have a glass transition temperature below -78°C . Of the materials listed in Table 15.2, only polybutadiene and silicone have glass transition temperatures below -78°C .

15.18 How do the glass transition temperatures of polyethylene, polypropylene, and polymethyl methacrylate compare? Explain their differences, based on the structure of the monomer.

From Table 15.2: polyethylene $T_g = -120^\circ\text{C}$
 polypropylene $T_g = -16^\circ\text{C}$
 polymethyl methacrylate .. $T_g = +90$ to 105°C

The side groups in polyethylene are small hydrogen atoms; in polypropylene, more complex side groups are present; in PMMA, the side groups are even more extensive (see Table 15.5). As the complexity of the monomers increases, the glass transition temperature increases.

15.19 Which of the addition polymers in Table 15.2 are used in their leathery condition at room temperature? How is this condition expected to affect their mechanical properties compared with those of other addition polymers?

Both polyethylene and polypropylene have glass transition temperatures below room temperature and therefore are presumably in the leathery condition. As a consequence, they are expected to have relatively low strengths compared to most other thermoplastic polymers.

15.20 The density of polypropylene is approximately 0.89 Mg.m^{-3} . Determine the number of propylene repeat units in each unit cell of crystalline polypropylene.

From Table 15.3, we find the lattice parameters for orthorhombic polypropylene. The volume of the unit cell is:

$$V_{\text{cell}} = (1.45 \times 10^{-9} \text{ m})(0.569 \times 10^{-9} \text{ m})(0.740 \times 10^{-9} \text{ m}) = 6.10537 \times 10^{-28} \text{ m}^3$$

The molecular weight of propylene is $3 \text{ C} + 6 \text{ H} = 42 \text{ g.mol}^{-1}$. The number of repeat units "x" is:

$$0.89 \text{ Mg.m}^{-3} = \frac{(x)(42 \text{ g.mol}^{-1})}{(6.10537 \times 10^{-28} \text{ m}^3)(6.02 \times 10^{23} \text{ units/mol})}$$

$$x = 7.8$$

Therefore there are 8 propylene repeat units in one unit cell of crystalline polypropylene.

15.21 The density of polyvinyl chloride is approximately 1.4 Mg.m^{-3} . Determine the number of vinyl chloride repeat units, hydrogen atoms, chlorine atoms, and carbon atoms in each unit cell of crystalline PVC.

From Table 15.3, we can find the lattice parameters for orthorhombic polyvinyl chloride. The volume of the unit cell is

$$V_{\text{cell}} = (1.040 \times 10^{-9} \text{ m})(0.530 \times 10^{-9} \text{ m})(0.51 \times 10^{-9} \text{ m}) = 2.811 \times 10^{-28} \text{ m}^3$$

The molecular weight of vinyl chloride is $2 \text{ C} + 3 \text{ H} + 1 \text{ Cl} = 62.453 \text{ g.mol}^{-1}$. The number of repeat units "x" is therefore:

$$1.4 \text{ Mg.m}^{-3} = \frac{x(62.453 \text{ g.mol}^{-1})}{(2.811 \times 10^{-28} \text{ m}^3)(6.02 \times 10^{23} \text{ units.mol}^{-1})}$$

$$x = 3.8$$

Therefore in each unit cell, there are:

*4 vinyl chloride repeat units
8 carbon atoms
12 hydrogen atoms
4 chlorine atoms*

15.22 A polyethylene sample is reported to have a density of 0.97 Mg.m^{-3} . Calculate the percent crystallinity in the sample. Would you expect that the structure of this sample has a large or small amount of branching? Explain.

From Example 15.8, we find that the density of completely crystalline polyethylene is 0.9932 Mg.m^{-3} . The density of completely amorphous polyethylene was also given in the example as 0.87 Mg.m^{-3} . Therefore:

$$\% \text{crystallinity} = \frac{(0.9932)(0.97 - 0.87)}{(0.97)(0.9932 - 0.87)} \times 100 = 83.1\%$$

Because the %crystallinity is very high, it is likely that the sample has a very small amount of branching; increasing the branching decreases crystallinity.

15.23 Amorphous polyvinyl chloride is expected to have a density of 1.38 Mg.m⁻³. Calculate the %crystallization in PVC that has a density of 1.45 Mg.m⁻³. (Hint -- find the density of completely crystalline PVC from its lattice parameters, assuming four repeat units per unit cell.

The molecular weight of vinyl chloride is 2 C + 3 H + 1 Cl = 62.453 g.mol⁻¹. The lattice parameters are given in Table 15.3. The density of completely crystalline PVC is therefore:

$$\rho_{\text{PVC}} = \frac{(4 \text{ units/cell})(62.453 \text{ g.mol}^{-1})}{(10.4 \times 10^{-9} \text{ m})(5.3 \times 10^{-9} \text{ m})(5.1 \times 10^{-9} \text{ m})(6.02 \times 10^{23} \text{ units.mol}^{-1})}$$

$$= 1.476 \text{ Mg.m}^{-3}$$

The percent crystallization is therefore:

$$\% \text{crystallization} = \frac{(1.476)(1.45 - 1.38)}{(1.45)(1.476 - 1.38)} \times 100 = 74.2\%$$

15.24 Describe the relative tendencies of the following polymers to crystallize. Explain your answer.

(a) Linear polyethylene is more likely to crystallize than branched polyethylene. The branching prevents close packing of the polymer chains into the crystalline structure.

(b) Polyethylene is more likely to crystallize than the polyethylene-propylene copolymer. The propylene monomers have larger side groups than polyethylene and, of course, different repeat units are present in the polymer chains. These factors make close packing of the chains more difficult, reducing the ease with which crystallization occurs.

(c) Isotactic polypropylene is more likely to crystallize than atactic polypropylene. In isotactic polypropylene, the side groups are aligned, making the polymer chain less random, and permitting the chains to pack more closely in a crystalline manner.

(d) Acetal, or polyoxymethylene, is more symmetrical and has smaller side groups than polymethyl methacrylate; consequently acetal polymers are more likely to crystallize.

15.25 At room temperature, a polymer is found to have a creep rate of 0.007 mm.mm⁻¹.h⁻¹ when the stress is 18 MN.m⁻²; a creep rate of 0.002 mm.mm⁻¹.h⁻¹ when the applied stress is 15.5 MN.m⁻²; and a creep rate of 0.0009 mm.mm⁻¹.h⁻¹ when the applied stress is 14 MN.m⁻². The creep rate is found to depend on $a\sigma^n$, where a and n are constants. Find the constants a and n and determine the maximum stress that will assure that the polymer will deform no more than 2% in one year.

The data is shown in the sketch below; the plot is linear in this log-log plot. If we select two points from the graph, we can solve for the constants in the expression $d\epsilon/dt = a(\sigma)^n$

$$\ln(0.007) = \ln(a) + n \ln(18)$$

$$\ln(0.0009) = \ln(a) + n \ln(14)$$

$$\ln(0.007) - \ln(0.0009) = n[\ln(18) - \ln(14)]$$

$$n = 2.0513 / 0.2513 = 8.16$$

$$\ln(0.007) = \ln(a) + 8.16 \ln(18)$$

$$-4.9618 = \ln(a) + (8.16)(2.8904)$$

$$\ln(a) = -28.547$$

$$a = 4 \times 10^{-13} \text{ MN.m}^{-2}$$

Thus the equation describing the effect of applied stress on creep rate is:

$$d\epsilon/dt = 4 \times 10^{-13} \sigma^{8.16}$$

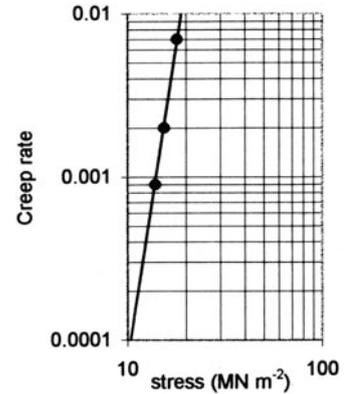
To assure that no more than 2% (or 0.02 m.m⁻¹) of strain occurs in one year (and assuming that the creep rate is uniform), we need to find the applied stress that produces a creep rate of no more than

$$(0.02 \text{ m.m}^{-1}) / (1 \text{ yr})(365 \text{ d/yr})(24 \text{ h/d}) = 2.283 \times 10^{-6} \text{ m.m}^{-1}.\text{h}^{-1}$$

$$2.283 \times 10^{-6} = 4 \times 10^{-13} \sigma^{8.16}$$

$$\sigma^{8.16} = 5.7 \times 10^6$$

$$\sigma = 6.73 \text{ MN.m}^{-2}$$



15.26 A stress of 17 MN.m⁻² is applied to a polymer serving as a fastener in a complex assembly. At a constant strain, the stress drops to 16.5 MN.m⁻² after 100 h. If the stress on the part must remain above 14.5 MN.m⁻² in order for the part to function properly, determine is the life of the assembly.

First we can determine the relaxation constant λ :

$$\sigma = \sigma_0 \exp(-t/\lambda)$$

$$16.5 = 17 \exp(-100 / \lambda) \quad \text{or} \quad \ln(16.5/17) = -100 / \lambda$$

$$-0.0299 = -100 / \lambda \quad \text{or} \quad \lambda = 3350 \text{ h}$$

Then we can determine the time required before the stress relaxes to 14.5 MN.m⁻²:

$$14.5 = 17 \exp(-t/3350) \quad \text{or} \quad \ln(14.5/17) = -t / 3350$$

$$-0.1591 = -t / 3350 \quad \text{or} \quad t = 533 \text{ h}$$

15.27 A stress of 7 MN.m⁻² is applied to a polymer that operates at a constant strain; after six months, the stress drops to 5.9 MN.m⁻². For a particular application, a part must maintain a stress of 6.2 MN.m⁻² after 12 months. What should be the original stress applied to the polymer for this application?

First we can determine the relaxation constant λ , using a time of 6 months = 4380 h;

$$\sigma = \sigma_0 \exp(-t/\lambda)$$

$$5.9 = 7 \exp(-4380/\lambda) \quad \text{or} \quad \ln(5.9/7) = -4380/\lambda$$

$$- 0.1710 = - 4380/\lambda \quad \text{or} \quad \lambda = 25,620 \text{ h}$$

Then we can determine the initial required stress that will assure a stress of only 6.2 MN.m^{-2} after 12 months = 8760 h:

$$6.2 = \sigma_o \exp(-8760 / 25,620) = \sigma_o (0.710)$$

$$\sigma_o = 8.7 \text{ MN.m}^{-2}$$

15.28 Data for the rupture time of polyethylene are shown in Figure 15.20. At an applied stress of 4.8 MN.m^{-2} , the figure indicates that the polymer ruptures in 0.2 h at 90°C but survives for 10,000 h at 65°C . Assuming that the rupture time is related to the viscosity, calculate the activation energy for the viscosity of polyethylene and estimate the rupture time at 23°C .

We expect the rupture time to follow the expression:

$$t_r = a \exp(Q_\eta/RT)$$

For $T = 90^\circ\text{C} = 363 \text{ K}$, $t_r = 0.2 \text{ h}$, while for $T = 65^\circ\text{C} = 338 \text{ K}$, $t_r = 10,000 \text{ h}$. By solving simultaneous equations, we can find the constant "a" and the activation energy Q:

$$0.2 = a \exp[Q_\eta/(8.314)(363)] = a \exp(0.0003313Q_\eta)$$

$$10,000 = a \exp[Q_\eta/(8.314)(338)] = a \exp(0.0003559Q_\eta)$$

$$0.00002 = \exp[(0.0003313 - 0.0003559)Q_\eta] = \exp(-0.00002451Q_\eta)$$

$$\ln(0.00002) = -10.8198 = -0.00002451Q_\eta$$

$$Q_\eta = 441400 \text{ J.mol}^{-1}$$

$$0.2 = a \exp[441,400 / (8.314)(363)] = a \exp(146.27)$$

$$a = 0.2 / 3.347 \times 10^{63} = 6.0 \times 10^{-65}$$

The rupture time at $23^\circ\text{C} = 296 \text{ K}$ is therefore:

$$t_r = 6.0 \times 10^{-65} \exp[441,400 / (8.314)(296)]$$

$$t_r = 4.40 \times 10^{13} \text{ h}$$

The polyethylene will essentially not rupture at 23°C .

15.29 Figure 15.21 shows the effect of stress and time on the strain in polypropylene at 20°C . (a) Determine from the data the constants "a" and "n" in Equation 15.7 for each applied stress. (b) Determine the percent strain in the polypropylene if a stress of 8.6 MN.m^{-2} is applied at 20°C for 1 year.

(a) From the data in Figure 15.21:

@ 4 MN.m^{-2} : 1 h gives 0.40% = 0.004 mm.mm^{-1} strain
 10 h gives 0.48% = $0.0048 \text{ mm.mm}^{-1}$ strain
 100 h gives 0.60% = 0.006 mm.mm^{-1} strain

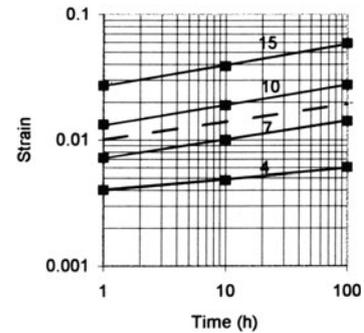
@ 7 MN.m^{-2} : 1 h gives 0.72% = $0.0072 \text{ mm.mm}^{-1}$ strain
 10 h gives 1.00% = $0.0100 \text{ mm.mm}^{-1}$ strain
 100 h gives 1.40% = $0.0140 \text{ mm.mm}^{-1}$ strain

@ 10 MN.m^{-2} : 1 h gives 1.32% = $0.0132 \text{ mm.mm}^{-1}$ strain
 10 h gives 1.88% = $0.0188 \text{ mm.mm}^{-1}$ strain
 100 h gives 2.72% = $0.0272 \text{ mm.mm}^{-1}$ strain

@15 MN.m⁻²: 1 h gives 2.68% = 0.0268 mm.mm⁻¹ strain
 10 h gives 3.84% = 0.0384 mm.mm⁻¹ strain
 100 h gives 5.88% = 0.0588 mm.mm⁻¹ strain

The relationship between strain and time is given by a power law, $\epsilon = at^n$. The relationship will be linear in a log-log graph, as shown in the sketch. The constant "a" is the value of the "y" intercept when $t = 1$; the slope is simply the rise over the run in the plot, as measured with a normal ruler. The values of "a" and "n" found in this manner are:

Stress (MN.m ⁻²)	a	n
4	0.0040	0.09
7	0.0072	0.14
10	0.0132	0.16
15	0.0268	0.18

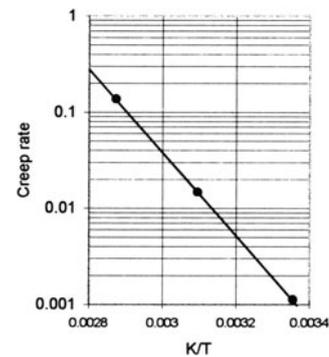


(b) We have no data at 8.6 MN.m⁻²; however by interpolating the given data, we can estimate that $a = 0.010$ and $n = 0.15$. In one year, there are 8760 h. Therefore the strain expected at 8.6 MN.m⁻² in one year is:

$$\epsilon = 0.010(8760)^{0.15} = 0.039 \text{ mm.mm}^{-1} \text{ or } 3.9\%$$

15.30 A polymer in the shape of a 100 mm long rod is placed into service under a constant tensile stress. The creep rate, which is measured as a function of temperature, is shown below. Determine the time required for the rod to stretch to 130 mm at 85°C.

T (°C)	dε/dt (mm.mm ⁻¹ .h ⁻¹)	1/T (K ⁻¹)
25	0.0011	0.003356
50	0.0147	0.003096
75	0.1375	0.002874



The creep rate is given by the expression $d\epsilon/dt = a \exp(-Q/RT)$; the data should be linear on a semilog plot when the strain rate is plotted versus reciprocal temperature, as shown in the sketch. From the graph:

$$Q/R = \frac{\ln(0.1375) - \ln(0.0011)}{0.002874 - 0.003356} = 10,017$$

$$Q = 83,280 \text{ J.mol}^{-1}$$

$a = 0.1375 \text{ mm.mm}^{-1}.\text{h}^{-1} / \exp[-83280/(8.314)(348)] = 4.359 \times 10^{11}$
 Therefore, at $T = 85^\circ\text{C} = 358 \text{ K}$, the creep rate is expected to be:

$$d\epsilon/dt = 4.359 \times 10^{11} \exp[-83,280 / (8.314)(358)] = 0.3073 \text{ mm.mm}^{-1}.\text{h}^{-1}$$

The total strain is to be $\epsilon = (130 \text{ mm} - 100 \text{ mm}) / 100 \text{ mm} = 0.3 \text{ mm.mm}^{-1}$.
 The time required to reach this total strain is:

$$0.3 \text{ mm.mm}^{-1} / 0.3073 \text{ mm.mm}^{-1}.\text{h}^{-1} = 0.976 \text{ h}$$

15.31 For each of the following pairs, recommend the one that will most likely have the better impact properties at 25°C. Explain each of your choices.

(a) Polyethylene is expected to have better impact properties than polystyrene. The polyethylene chains are symmetrical, with small hydrogen side groups, and consequently will deform rapidly when an impact load is applied.

(b) Low density polyethylene, which contains substantial branching is expected to have better impact properties than high density polyethylene. The more loosely packed chains in LD polyethylene can more easily move when an impact load is applied.

(c) Polytetrafluoroethylene is expected to have better impact properties than polymethyl methacrylate (PMMA). PTFE has symmetrical chains with relatively small (F) side groups compared to the chains in PMMA. Consequently chain sliding will be more quickly accomplished in PTFE.

15.32 The polymer ABS can be produced with varying amounts of styrene, butadiene, and acrylonitrile monomers, which are present in the form of two copolymers -- BS rubber and SAN. (a) How would you adjust the composition of ABS if you wanted to obtain good impact properties? (b) How would you adjust the composition if you wanted to obtain good ductility at room temperature? (c) How would you adjust the composition if you wanted to obtain good strength at room temperature?

(a) Improved impact properties are obtained by increasing the amount of butadiene monomer; the elastomer provides large amounts of elastic strain, which helps to absorb an impact blow.

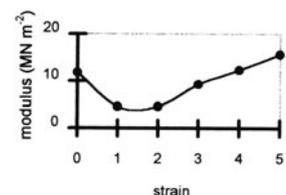
(b) The styrene helps to provide good ductility; the butadiene provides good "elastic" strain but not "plastic" strain. Acrylonitrile has poor ductility when polymerized.

(c) Higher acrylonitrile will help produce higher strengths.

15.33 Figure 15.34 shows the stress-strain curve for an elastomer. From the curve, calculate and plot the modulus of elasticity versus strain and explain the results.

We obtain the modulus of elasticity by finding the slope of the tangent drawn to the stress-strain curve at different values of strain. Examples of such calculations are shown below:

$$\begin{aligned}
 \epsilon = 0 \quad \Delta\sigma/\Delta\epsilon &= (58-0)/(5-0) = 11.8 \text{ MN.m}^{-2} \\
 = 1 &= (31-4)/(6-0) = 4.5 \text{ MN.m}^{-2} \\
 = 2 &= (30-3)/(6-0) = 4.5 \text{ MN.m}^{-2} \\
 = 3 &= (47-0)/(6-0) = 9.2 \text{ MN.m}^{-2} \\
 = 4 &= (56-7)/(6-2) = 12.25 \text{ MN.m}^{-2} \\
 = 5 &= (61.4-0)/(5-2.07) = 15.62 \text{ MN.m}^{-2}
 \end{aligned}$$



The modulus of elasticity is plotted versus strain in the sketch. Initially the modulus decreases as some of the chains become untwisted. However eventually the modulus increases again as the chains become straight and higher stresses are required to stretch the bonds within the chains.

15.34 The maximum number of cross-linking sites in polyisoprene is the number of unsaturated bonds in the polymer chain. If three sulfur atoms are in each cross-linking sulfur strand, calculate the amount of sulfur required to provide cross-links at every available site in 5 kg of polymer and the wt% S that would be present in the elastomer. Is this typical?

$$MW_{\text{isoprene}} = 5 C + 8 H = 68 \text{ g.mol}^{-1} \quad MW_{\text{sulfur}} = 32 \text{ g.mol}^{-1}$$

After the original chains polymerize, there remains one unsaturated bond per repeat unit within the chain. Each time one double bond is broken, two active sites are created and sulfur atoms then join two repeat units. Therefore, on the average, there is one set of cross-linking sulfur groups per each repeat unit. In other words, the number of moles of isoprene is equal to the number of sulfur atom groups if every cross-linking site is utilized. If just one sulfur atom provided cross-linking at every site, then the amount of sulfur required would be:

$$S / 32 \text{ g.mol}^{-1} = 5000 \text{ g isoprene} / 68 \text{ g.mol}^{-1}$$

$$S = 2,353 \text{ g} = 2.353 \text{ kg}$$

But if there are three sulfur atoms in each cross-linking strand, the total amount of sulfur for complete cross-linking is

$$\text{weight of sulfur} = (3)(2.353 \text{ kg}) = 7.059 \text{ kg}$$

The weight percent sulfur in the polymerized and cross-linked elastomer is:

$$\text{wt\% S} = \frac{7.059}{7.059 + 5} \times 100 = 58.5\%$$

This is far higher than typical elastomers, for which the %S is less than about 5%.

15.35 Suppose we vulcanize polychloroprene, obtaining the desired properties by adding 1.5% sulfur by weight to the polymer. If each cross-linking strand contains an average of four sulfur atoms, calculate the fraction of the unsaturated bonds that must be broken.

$$MW_{\text{chloroprene}} = 4 C + 5 H + 1 Cl = 88.453 \text{ g.mol}^{-1} \quad MW_{\text{sulfur}} = 32 \text{ g.mol}^{-1}$$

As in Problem 15.34, one mole of sulfur would be required per each mole of chloroprene if all cross-linking sites were satisfied by one sulfur atom. In 1000 g of chloroprene, the number of moles of chloroprene (and also of sulfur, assuming one sulfur at each site) is:

$$1000 \text{ g} / 88.453 \text{ g.mol}^{-1} = 11.305 \text{ mol of chloroprene}$$

But we have an average of 4 sulfur atoms per strand; therefore for cross-linking at every site, we need (4)(11.305) = 45.22 mol of sulfur. The total weight of sulfur that must be added to 1000 g of the monomer to produce cross-linking at every site with four sulfur atom strands is:

$$\text{maximum sulfur} = (45.22 \text{ mol})(32 \text{ g.mol}^{-1}) = 1447 \text{ g}$$

But only 1.5% sulfur is present. If the amount of chloroprene is 1000 g, then the amount of sulfur is:

$$\frac{\text{g of S}}{\text{g of S} + 1000 \text{ g}} \times 100 = 1.5\% \quad \text{or} \quad S = 15.228 \text{ g}$$

The fraction of the unsaturated bonds that are actually broken is therefore:

$$\text{fraction} = 15.228 \text{ g} / 1447 \text{ g} = 0.0105$$

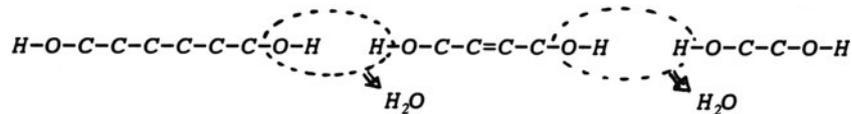
Only a small fraction, about 1%, of the available cross-linking sites

are actually used.

15.36 The monomers for adipic acid, ethylene glycol, and maleic acid are shown below. These monomers can be joined into chains by condensation reactions, then cross-linked by breaking unsaturated bonds and inserting a styrene molecule as the cross-linking agent.

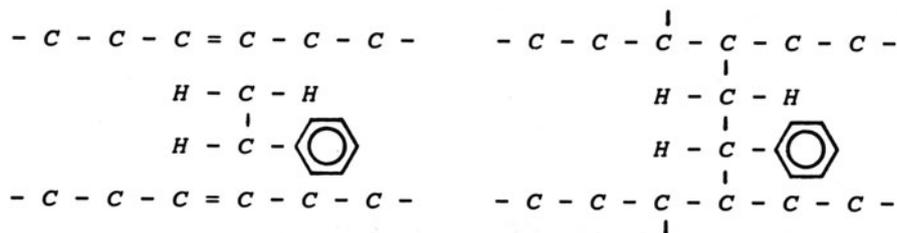
- Show how a linear chain composed of these three monomers can be produced.
- Explain why a thermosetting polymer cannot be produced using just adipic acid and ethylene glycol.
- show how styrene provides cross-linking between the linear chains.
- If 50 g of adipic acid, 100 g of maleic acid, and 50 g of ethylene glycol are combined, calculate the amount of styrene required to completely cross-link the polymer.

(a) The original chains are produced by condensation reactions involving the H at the ends of the maleic acid monomer and OH groups at the ends of the other two monomers, producing water as a byproduct:



(b) Unsaturated bonds are introduced into the linear polymer chain through the maleic acid. If the maleic acid were not present, cross-linking could not occur.

(c) During cross-linking, the unsaturated bonds in the chains, provided by the maleic acid, are broken. This frees up active sites at the two carbon atoms in the maleic acid monomer. When styrene is introduced, the unsaturated bond in styrene is also broken, providing two active sites on it. The active sites on both the chain and the styrene can be satisfied by inserting the styrene as a cross-linking agent:



(d) For complete cross-linking, we need to introduce one styrene monomer for each maleic acid monomer (assuming only one styrene provides the cross-link). The molecular weights are:

$$MW_{\text{maleic acid}} = 4 \text{ C} + 4 \text{ O} + 4 \text{ H} = 116 \text{ g.mol}^{-1}$$

$$MW_{\text{styrene}} = 8 \text{ C} + 8 \text{ H} = 104 \text{ g.mol}^{-1}$$

The amount of styrene is then:

$$g \text{ of styrene} / 104 \text{ g.mol}^{-1} = 100 \text{ g maleic acid} / 116 \text{ g.mol}^{-1}$$

$$g \text{ of styrene} = 89.655 \text{ g}$$

15.37 How much formaldehyde is required to completely cross-link 10 kg of phenol to produce a thermosetting phenolic polymer? How much byproduct is evolved?

To make the chain, we must add 1 mole of formaldehyde per mole of phenol. Then, to completely cross-link the chains (remembering that phenol is really trifunctional), we need an additional mole of

formaldehyde for each mole of phenol. The number of moles of phenol added is:

$$MW_{\text{phenol}} = 6 \text{ C} + 6 \text{ H} + 1 \text{ O} = 94 \text{ g.mol}^{-1}$$

$$\text{moles of phenol} = 10,000 \text{ g} / 94 \text{ g.mol}^{-1} = 106.383 \text{ mol}$$

But we need twice as many moles of formaldehyde, or 212.766 mol. The amount of formaldehyde is therefore:

$$MW_{\text{formaldehyde}} = 1 \text{ C} + 1 \text{ O} + 2\text{H} = 30 \text{ g.mol}^{-1}$$

$$\text{weight of formaldehyde} = (212.766 \text{ mol})(30 \text{ g.mol}^{-1}) = 6383 \text{ g}$$

The byproduct formed during polymerization is water. For complete polymerization (both chain formation and cross-linking), two moles of water are produced for each mole of phenol. The amount of water is then:

$$\text{weight of water} = (212.766 \text{ mol})(18 \text{ g.mol}^{-1}) = 3830 \text{ g}$$

15.38 Explain why the degree of polymerization is not usually used to characterize thermosetting polymers.

Individual chains are no longer present after the polymer is completely cross-linked and polymerized; instead the entire polymer should be considered continuous.

15.39 Defend or contradict the choice to use the following materials as hot-melt adhesives for an application in which the assembled part is subjected to impact-type blows.

(a) Polyethylene is expected to have relatively good impact resistance due to the ease with which chains can move; the polyethylene is well above its glass transition temperature.

(b) Polystyrene is expected to have relatively poor impact resistance due to the resistance to chain sliding by the large benzene ring side groups.

(c) Styrene-butadiene thermoplastic elastomers are expected to have good impact resistance; although the styrene portion may be rather brittle, the high energy absorbing capability of the butadiene component provides good impact properties.

(d) Polyacrylonitrile will have relatively poor impact properties due to the presence of the side groups.

(e) Polybutadiene, an elastomer, will provide good impact properties.

15.40 Many paints are polymeric materials. Explain why plasticizers are added to paints. What must happen to the plasticizers after the paint is applied?

The plasticizers lower the viscosity and make the paint flow more easily, providing better coverage. The plasticizer must evaporate, however, for the polymer material to harden.

15.41 You want to extrude a complex component from an elastomer. Should you vulcanize the rubber before or after the extrusion operation? Explain.

The elastomer must be extruded before vulcanization, while it still behaves much like a thermoplastic polymer. After extrusion, vulcanization can occur. Now the polymer develops its high elastic strain, although it can no longer be plastically deformed.

15.42 Suppose a thermoplastic polymer can be produced in sheet form either by rolling (deformation) or by continuous casting (with a rapid cooling rate). In which case would you expect to obtain the higher strength? Explain.

During rolling, the chains become aligned in the direction of rolling, perhaps even assuming a high degree of crystallinity. The rolled sheet will have a high tensile strength, particularly in the direction of rolling.

During solidification, particularly at a high rate of cooling, crystallization will be suppressed and a relatively low strength, amorphous polymer structure is expected.

Chapter 16 Composite Materials

16.1 Nickel containing 2 wt% thorium is produced in powder form, consolidated into a part, and sintered in the presence of oxygen, causing all of the thorium to produce ThO_2 spheres 80 nm in diameter. Calculate the number of spheres per mm^3 . The density of ThO_2 is 9.86 Mg.m^{-3} .

In 100 g of material, there are $98 \text{ g} / 8.902 \times 10^6 \text{ g.m}^{-3} = 1.1009 \times 10^{-5} \text{ m}^3$ of nickel. From the reaction $\text{Th} + \text{O}_2 = \text{ThO}_2$,

$$2 \text{ g Th} / 232 \text{ g.mol} = x \text{ g ThO}_2 / 264 \text{ g.mol}$$

$$x = 2.2759 \text{ g ThO}_2$$

The total volume of the oxide is:

$$V_{\text{oxide}} = 2.2759 \text{ g} / 9.86 \times 10^6 \text{ g.m}^{-3} = 2.3082 \times 10^{-7} \text{ m}^3$$

The volume fraction of the oxide is

$$f_{\text{oxide}} = \frac{2.3082 \times 10^{-7}}{2.3082 \times 10^{-7} + 1.1009 \times 10^{-5}} = 0.0205$$

The volume of each oxide sphere is:

$$V_{\text{sphere}} = (4\pi/3)r^3 = (4\pi/3)(40 \times 10^{-6} \text{ mm})^3 = 2.681 \times 10^{-13} \text{ mm}^3$$

In one cubic millimetre there is 0.0205 mm^3 of oxide. The number of oxide particles is
number = $0.0205 / 2.681 \times 10^{-13} = 7.65 \times 10^{10}$ per mm^3 .

16.2 Spherical aluminium powder 0.002 mm in diameter is treated to create a thin oxide layer and is then used to produce a SAP dispersion strengthened material containing 10 vol% Al_2O_3 . Calculate the average thickness of the oxide film prior to compaction and sintering of the powders into the part.

The volume of an aluminium powder particle is:

$$V_{\text{Al}} = (4\pi/3)(0.002 \text{ mm} / 2)^3 = 4.19 \times 10^{-9} \text{ mm}^3$$

The volume fraction Al_2O_3 is

$$0.10 = \frac{V_{\text{oxide}}}{V_{\text{oxide}} + V_{\text{Al}}} = \frac{V_{\text{oxide}}}{V_{\text{oxide}} + 4.19 \times 10^{-9}}$$

$$V_{\text{oxide}} = 4.654 \times 10^{-10} \text{ mm}^3$$

We can then calculate the radius of the particle after oxidation has occurred:

$$V_{\text{oxide}} = (4\pi/3)r^3 - 4.19 \times 10^{-9} = 4.65 \times 10^{-10}$$

$$r = 1.0358 \times 10^{-3} \text{ mm} = 0.0010358 \text{ mm}$$

The thickness of the oxide layer must therefore be:

$$\text{thickness} = 0.0010358 - 0.001 = 0.0000358 \text{ mm} = 3.58 \times 10^{-5} \text{ mm}$$

16.3 Yttria (Y_2O_3) particles 75 nm in diameter are introduced into tungsten by internal oxidation. Measurements using an electron microscope show that there are 5×10^{11} oxide particles per mm^3 . Calculate the wt% Y originally in the alloy. The density of Y_2O_3 is 5.01 Mg.m^{-3} .

The volume of each particle is:

$$V_{\text{oxide}} = (4\pi/3) (75 \times 10^{-6} / 2 \text{ mm})^3 = 2.209 \times 10^{-13} \text{ mm}^3$$

The total volume of oxide particles per mm^3 is given by:

$$V_{\text{yttria}} = (2.209 \times 10^{-13} \text{ mm}^3) (5 \times 10^{11} \text{ particles}) = 0.11 \text{ mm}^3$$

The volume fraction of yttria is therefore

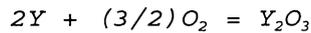
$$f_{\text{oxide}} = 0.11$$

The weight percentages of oxide and tungsten are

$$\text{wt}\% \text{ Y}_2\text{O}_3 = \frac{(0.11) (5.01 \text{ Mg.m}^{-3})}{(0.11) (5.01) + (0.89) (19.254)} \times 100 = 3.116\%$$

$$\text{wt}\% \text{ W} = 96.884\%$$

In 100 g of material, there are 3.116 g of oxide. From the equation



$$x \text{ g of Y} / 2(88.91 \text{ g.mol}) = 3.116 \text{ g of Y}_2\text{O}_3 / 225.82 \text{ g.mol}$$

$$x = 2.45 \text{ g of Y}$$

The weight percent Y in the original alloy was therefore:

$$\text{wt}\% \text{ Y} = \frac{2.45 \text{ g Y}}{2.45 \text{ g Y} + 96.884 \text{ g W}} \times 100 = 2.47\%$$

16.4 With no special treatment, aluminium is typically found to have an Al_2O_3 layer that is 3 nm thick. If spherical aluminium powder prepared with a total diameter of 0.01 mm is used to produce the SAP dispersion strengthened material, calculate the volume percent Al_2O_3 in the material and the number of oxide particles per mm^3 . Assume that the oxide breaks into disc-shaped flakes 3 nm thick and 3×10^{-4} mm in diameter. Compare the number of oxide particles per mm^3 to the number of solid solution atoms per mm^3 when 3 at% of an alloying element is added to aluminium.

The total volume of the powder particle is:

$$V_{\text{total}} = (4\pi/3) (0.01 / 2)^3 = 5.235988 \times 10^{-7} \text{ mm}^3$$

The volume of just the oxide layer is:

$$V_{\text{oxide}} = 5.235988 \times 10^{-7} - (4\pi/3) (0.005 - 3 \times 10^{-6})^3 = 0.009419 \times 10^{-7} \text{ mm}^3$$

The volume fraction of the oxide is:

$$f_{\text{oxide}} = 0.009419 \times 10^{-7} / 5.235988 \times 10^{-7} = 0.001799$$

The volume of one disc-shaped oxide flake is:

$$V_{flake} = (\pi/4) (3 \times 10^{-4} \text{ mm})^2 (3 \times 10^{-6} \text{ mm}) = 2.12 \times 10^{-13} \text{ mm}^3$$

In one mm^3 of SAP, there is 0.001799 mm^3 of oxide. The number of oxide particles per mm^3 is therefore

$$\text{number} = 0.001799 \text{ mm}^3 / 2.12 \times 10^{-13} \text{ mm}^3/\text{particle} = 8.49 \times 10^9 \text{ flakes/mm}^3$$

The number of solid solution atoms per mm^3 in an Al-3 at% alloying element alloy is calculated by first determining the volume of the unit cell:

$$V_{cell} = (4.04958 \times 10^{-7} \text{ mm})^3 = 6.641 \times 10^{-20} \text{ mm}^3$$

In 25 unit cells of FCC aluminium, there are 100 atom sites. In the alloy, 3 of these sites are filled with substitutional atoms, the other 97 sites by aluminium atoms. The number of solid solution atoms per mm^3 is therefore:

$$\begin{aligned} \text{number} &= 3 \text{ atoms in 25 cells} / (25 \text{ cells}) (6.641 \times 10^{-20} \text{ mm}^3/\text{cell}) \\ &= 1.81 \times 10^{18} \text{ substitutional atoms/mm}^3 \end{aligned}$$

The number of substitutional point defects is eight orders of magnitude larger than the number of oxide flakes.

16.5 Calculate the density of a cemented carbide, or cermet, based on a titanium matrix if the composite contains 50 wt% WC, 22 wt% TaC, and 14 wt% TiC. (See Example 16.2 for densities of the carbides.)

We must find the volume fractions from the weight percentages. Using a basis of 100 g of the cemented carbide:

$$f_{WC} = \frac{50 \text{ g WC} / 15.77 \text{ Mg.m}^{-3}}{(50/15.77) + (22/14.5) + (14/4.94) + (14/4.507)} = 0.298$$

$$f_{TaC} = \frac{22 \text{ g TaC} / 14.5 \text{ Mg.m}^{-3}}{(50/15.77) + (22/14.5) + (14/4.94) + (14/4.507)} = 0.143$$

$$f_{TiC} = \frac{14 \text{ g TiC} / 4.94 \text{ Mg.m}^{-3}}{(50/15.77) + (22/14.5) + (14/4.94) + (14/4.507)} = 0.267$$

$$f_{Ti} = \frac{14 \text{ g Ti} / 4.507 \text{ Mg.m}^{-3}}{(50/15.77) + (22/14.5) + (14/4.94) + (14/4.507)} = 0.292$$

The density is then found from the rule of mixtures:

$$\begin{aligned} \rho_c &= (0.298)(15.77) + (0.143)(14.5) + (0.267)(4.94) + (0.292)(4.507) \\ &= 9.408 \text{ Mg.m}^{-3} \end{aligned}$$

16.6 A typical grinding wheel is 230 mm in diameter, 25 mm thick, and weighs 2.7 kg. The wheel contains SiC (density of 3.2 Mg.m^{-3}) bonded by a silica glass (density of 2.5 Mg.m^{-3}); 5 vol% of the wheel is porosity. The SiC is in the form of 0.4 mm cubes. Calculate (a) the volume fraction of SiC particles in the wheel and (b) the number of SiC particles lost from the wheel after it is worn to a diameter of 200 mm.

(a) To find the volume fraction of SiC:

$$V_{wheel} = (\pi/4)D^2h = (\pi/4)(230 \times 10^{-3}\text{m})^2(25 \times 10^{-3}\text{m}) = 1.0387 \times 10^{-3}\text{m}^3$$

$$W_{\text{wheel}} = 2.7 \times 10^{-3} \text{Mg}$$

$$\rho_{\text{wheel}} = 2.7 \times 10^{-3} \text{Mg} / 1.0387 \times 10^{-3} \text{m}^3 = 2.599 \text{Mg.m}^{-3}$$

From the rule of mixtures:

$$2.599 = f_{\text{pore}}\rho_{\text{pore}} + f_{\text{SiC}}\rho_{\text{SiC}} + f_{\text{glass}}\rho_{\text{glass}}$$

$$2.599 = (0.05)(0) + f_{\text{SiC}}(3.2) + (1 - 0.05 - f_{\text{SiC}})(2.5)$$

$$f_{\text{SiC}} = 0.32$$

(b) First we can determine the volume of the wheel that is lost; then we can find the number of particles of SiC per m^3 .

$$V_{\text{lost}} = (\pi/4)(230 \times 10^{-3})^2(25 \times 10^{-3}) - (\pi/4)(200 \times 10^{-3})^2(25 \times 10^{-3}) \\ = 2.533 \times 10^{-4} \text{m}^3$$

$$V_{\text{particles}} = (0.4 \times 10^{-3} \text{m})^3 = 6.4 \times 10^{-11} \text{m}^3$$

In 1m^3 of the wheel, there are $(0.32)(1 \text{m}^3) = 0.32 \text{m}^3$ of SiC. The number of SiC particles per m^3 of the wheel is:

$$0.32 \text{m}^3 / 6.4 \times 10^{-11} \text{m}^3/\text{particle} = 5 \times 10^9 \text{particles}/\text{m}^3$$

The number of particles lost during use of the wheel is:

$$\text{particles lost} = (5 \times 10^9/\text{m}^3)(2.533 \times 10^{-4} \text{m}^3) = 1.27 \times 10^6 \text{particles}$$

16.7 An electrical contact material is produced by infiltrating copper into a porous tungsten carbide (WC) compact. The density of the final composite is 12.3Mg.m^{-3} . Assuming that all of the pores are filled with copper, calculate (a) the volume fraction of copper in the composite, (b) the volume fraction of pores in the WC compact prior to infiltration, and (c) the original density of the WC compact before infiltration.

$$(a) \rho_c = 12.3 \text{Mg.m}^{-3} = f_{\text{Cu}}\rho_{\text{Cu}} + f_{\text{WC}}\rho_{\text{WC}} = f_{\text{Cu}}(8.93) + (1 - f_{\text{Cu}})(15.77)$$

$$f_{\text{Cu}} = 0.507$$

(b) The copper fills the pores. Therefore the volume fraction of the pores prior to infiltration is equal to that of the copper, or $f_{\text{pores}} = 0.507$.

(c) Before infiltration, the composite contains tungsten carbide and pores (which have zero density):

$$\rho_{\text{compact}} = f_{\text{WC}}\rho_{\text{WC}} + f_{\text{pore}}\rho_{\text{pore}} = (0.493)(15.77) + (0.507)(0) = 7.775 \text{Mg.m}^{-3}$$

16.8 An electrical contact material is produced by first making a porous tungsten compact that weighs 125 g. Liquid silver is introduced into the compact; careful measurement indicates that 105 g of silver is infiltrated. The final density of the composite is 13.8Mg.m^{-3} . Calculate the volume fraction of the original compact that is interconnected porosity and the volume fraction that is closed porosity (no silver infiltration).

First we can find the volume of the tungsten and silver:

$$V_W = 125 \text{g} / 19.254 \text{Mg.m}^{-3} = 6.492 \times 10^{-6} \text{m}^3 = 6.492 \text{cm}^3$$

$$V_{\text{Ag}} = 105 \text{g} / 10.49 \text{Mg.m}^{-3} = 10.010 \times 10^{-6} \text{m}^3 = 10.010 \text{cm}^3$$

The volume fractions of each constituent are:

$$f_w = \frac{6.492}{6.492 + 10.010 + V_{pore}}$$

$$f_{Ag} = \frac{10.010}{6.492 + 10.010 + V_{pore}}$$

$$f_{pore} = \frac{V_{pore}}{6.492 + 10.010 + V_{pore}}$$

From the rule of mixtures:

$$13.8 = [6.492/(16.502+V_{pore})](19.254) + [10.010/(16.502+V_{pore})](10.49) + 0$$

$$V_{pore} = 0.165 \text{ cm}^3$$

The total volume is $6.492 + 10.010 + 0.165 = 16.667 \text{ cm}^3$. The fraction of the contact material that is interconnected porosity prior to silver infiltration is equal to the volume fraction of silver; the volume fraction of closed porosity is obtained from V_{pore} .

$$f_{interconnected} = 10.010 / 16.667 = 0.6006$$

$$f_{closed} = 0.165 / 16.667 = 0.0100$$

16.9 How much clay must be added to 10 kg of polyethylene to produce a low cost composite having a modulus of elasticity greater than 0.8 GN.m^{-2} and a tensile strength greater than 14 MN.m^{-2} ? The density of the clay is 2.4 Mg.m^{-3} and that of polyethylene is 0.92 Mg.m^{-3} .

From Figure 16.6, we find that f_{clay} must be greater than 0.29 if the modulus is to exceed 0.8 GN.m^{-2} ; however the f_{clay} must be less than 0.45 to assure that the tensile strength exceeds 14 MN.m^{-2} . Therefore any clay fraction between 0.29 and 0.45 should be satisfactory.

$$f_{clay} = \frac{W_{clay} / 2.4 \text{ Mg.m}^{-3}}{(W_{clay} / 2.4) + (0.01 \text{ Mg} / 0.92)}$$

If $f_{clay} = 0.29$, then $W_{clay} = 0.0106 \text{ Mg} = 10.6 \text{ kg}$

If $f_{clay} = 0.45$, then $W_{clay} = 0.0213 \text{ Mg} = 21.3 \text{ kg}$

The overall cost of the composite will be reduced as the amount of clay added to the composite increases.

16.10 We would like to produce a lightweight epoxy part to provide thermal insulation. We have available hollow glass beads for which the outside diameter is 1.6 mm and the wall thickness is 0.025 mm. Determine the weight and number of beads that must be added to the epoxy to produce a one-kilogram composite with a density of 0.65 Mg.m^{-3} . The density of the glass is 2.5 Mg.m^{-3} and that of the epoxy is 1.25 Mg.m^{-3} .

Note that the wall thickness given in the book question is incorrect and should read 0.025 mm not 0.25 mm. The following solution uses wall thickness of 0.025 mm.

First we can find the total volume of a glass bead, the volume of the glass portion of the bead, the weight of the glass in the bead, and finally the overall density (weight of glass divided by the total volume) of the bead. The air in the hollow bead is assumed to be weightless.

$$V_{\text{bead}} = (4\pi/3) (0.8 \times 10^{-3})^3 = 2.1447 \times 10^{-9} \text{m}^3$$

$$V_{\text{glass}} = 2.1447 \times 10^{-9} - (4\pi/3) [0.8 \times 10^{-3} - 0.025 \times 10^{-3}]^3 \\ = 1.949 \times 10^{-10} \text{m}^3.$$

$$W_{\text{glass}} = (1.949 \times 10^{-10} \text{m}^3) (2.5 \text{Mg.m}^{-3}) = 4.8721 \times 10^{-10} \text{Mg/bead}$$

$$\rho_{\text{bead}} = 4.8721 \times 10^{-10} \text{Mg} / 2.1447 \times 10^{-9} \text{m}^3 = 0.227 \text{Mg.m}^{-3}$$

Now we can use the rule of mixtures to determine the volume fraction of beads that must be introduced into the epoxy.

$$\rho_c = 0.65 = f_{\text{bead}} \rho_{\text{bead}} + (1 - f_{\text{bead}}) \rho_{\text{epoxy}} = f_{\text{bead}} (0.227) + (1 - f_{\text{bead}}) (1.25)$$

$$f_{\text{bead}} = 0.59$$

We want to produce 1 kg of composite = $10^{-3} \text{Mg} / 0.65 \text{Mg.m}^{-3} = 1.538 \times 10^{-3} \text{m}^3$ of composite material. The volume of beads required is

$$(1.538 \times 10^{-3} \text{m}^3) (0.59) = 9.074 \times 10^{-4} \text{m}^3 \text{ of beads}$$

$$\text{wt of beads} = (9.074 \times 10^{-4} \text{m}^3) (0.227 \text{Mg.m}^{-3}) = 2.06 \times 10^{-4} \text{Mg of beads}$$

The number of beads needed is

$$\text{number} = 2.06 \times 10^{-4} \text{Mg} / 4.8721 \times 10^{-10} \text{Mg/bead} = 4.23 \times 10^5 \text{ beads}$$

16.11 Five kg of continuous boron fibres are introduced in a unidirectional orientation into eight kg of an aluminium matrix. Calculate (a) the density of the composite, (b) the modulus of elasticity parallel to the fibres, and (c) the modulus of elasticity perpendicular to the fibres.

$$f_B = \frac{5 \text{kg} / 2.3 \text{Mg.m}^{-3}}{5 \text{kg} / 2.3 + 8 \text{kg} / 2.699} = 0.423 \quad f_{Al} = 0.577$$

$$(a) \rho_c = f_B \rho_B + f_{Al} \rho_{Al} = (0.423) (2.3) + (0.577) (2.699) = 2.530 \text{Mg.m}^{-3}$$

$$(b) E_c = f_B E_B + f_{Al} E_{Al} = (0.423) (379) + (0.577) (70) \\ = 200.7 \text{GN.m}^{-2}$$

$$(c) 1/E_c = f_B/E_B + f_{Al}/E_{Al} = 0.423/379 + 0.577/70$$

$$E_c = 106.8 \text{GN.m}^{-2}$$

16.12 We want to produce 5 kg of a continuous unidirectional fibre-reinforced composite of HS carbon in a polyimide matrix that has a modulus of elasticity of at least 170 GN.m⁻² parallel to the fibres. How many kilograms of fibres are required? See Chapter 15 for properties of polyimide.

The modulus for HS carbon is 276 GN.m⁻² and for polyimide is 2 GN.m⁻². From the rule of mixtures, we can determine the required volume fraction of fibres:

$$170 \text{GN.m}^{-2} = f_{\text{carbon}} (276 \times \text{GN.m}^{-2}) + (1 - f_{\text{carbon}}) (2 \text{GN.m}^{-2})$$

$$f_{\text{carbon}} = 0.613$$

Then we can find the weight of fibres required to produce 5 kgs = 5×10^{-3} Mg composite:

$$0.613 = \frac{W_{\text{carbon}} / 1.75 \text{ Mg.m}^{-3}}{W_{\text{carbon}}/1.75 + (5 \times 10^{-3} - W_{\text{carbon}})/1.39} \quad \text{or} \quad W_{\text{carbon}} = 0.0033 \text{ Mg} = 3.3 \text{ kg}$$

16.13 We produce a continuous unidirectionally reinforced composite containing 60 vol% HM carbon fibres in an epoxy matrix. The epoxy has a tensile strength of 100 MN.m^{-2} . What fraction of the applied force is carried by the fibres?

From Example 16.8, we find that the fraction carried by the fibres is given by:

$$f = \frac{\sigma_f A_f}{\sigma_f A_f + \sigma_m A_m}$$

We can replace the areas by the volume fractions (assuming that the part has a continuous cross-section). Thus $A_f = 0.6$ and $A_m = 0.4$. The tensile strength of the fibres is 1860 MN.m^{-2} and that of the epoxy matrix is 100 MN.m^{-2} . Thus:

$$f = \frac{(1860)(0.6)}{(1860)(0.6) + (100)(0.4)} = 0.965$$

Over 96% of the force is carried by the fibres.

16.14 A polyester matrix with a tensile strength of 90 MN.m^{-2} is reinforced with Al_2O_3 fibres. What vol% fibres must be added to insure that the fibres carry 75% of the applied load?

The tensile strength of the fibres is 2070 MN.m^{-2} and that of the polyester is 90 MN.m^{-2} . From Example 16.8, and assuming a total area of one cm^2 :

$$f = \frac{\sigma_f A_f}{\sigma_f A_f + \sigma_m A_m} = 0.75$$

$$\frac{2070 A_f}{2070 A_f + 90 (1 - A_f)} = 0.75 \quad \text{or} \quad A_f = 0.115$$

Assuming that the area and volume fractions are the same, the volume fraction of fibres is $f_{\text{alumina}} = 0.115$

16.15 An epoxy matrix is reinforced with 40 vol% E-glass fibres to produce a 20 mm diameter composite that is to withstand a load of 25,000 N. Calculate the stress acting on each fibre. $E_m = 3 \text{ GN.m}^{-2}$, $E_f = 72.4 \text{ GN.m}^{-2}$.

We can assume that the strains in the composite, matrix, and fibres are equal. Thus:

$$\epsilon_c = \epsilon_m = \epsilon_f = \sigma_m / E_m = \sigma_f / E_f$$

The modulus for the E-glass is 72.4 GN.m^{-2} and that for the epoxy is 3 GN.m^{-2} . Therefore the ratio of the stresses is:

$$\sigma_f / \sigma_m = E_f / E_m = 72.4 / 3 = 24.13$$

The fraction of the force carried by the fibres (as described in Example 16.8) is (assuming that area and volume fractions are equal):

$$f = \frac{\sigma_f A_f}{\sigma_f A_f + \sigma_m A_m} = \frac{A_f}{A_f + (\sigma_m / \sigma_f) A_m} = \frac{0.4}{0.4 + (1/24.13)(0.6)} = 0.9415$$

Since the total force is 25,000 N, the force carried by the fibres is:

$$F_f = (0.9415)(25,000 \text{ N}) = 23537 \text{ N}$$

The cross-sectional area of the fibres is:

$$A_f = (f_f)(\pi/4)d^2 = (0.4)(\pi/4)(20 \times 10^{-3})^2 = 1.2566 \times 10^{-4} \text{ m}^2$$

Thus the stress is:

$$\sigma_f = 23537 \text{ N} / 1.2566 \times 10^{-4} \text{ m}^2 = 187.3 \text{ MN.m}^{-2}$$

16.16 A titanium alloy with a modulus of elasticity of 110 GN.m⁻² is used to make a 450 kg part for a manned space vehicle. Determine the weight of a part having the same modulus of elasticity parallel to the fibres, if the part is made of (a) aluminium reinforced with boron fibres and (b) polyester (with a modulus of 4.5 GN.m⁻²) reinforced with high modulus carbon fibres. (c) Compare the specific modulus for all three materials.

The titanium alloy has a density of about 4.507 Mg.m⁻³
The volume of the 450 kg part is therefore

$$V_{\text{part}} = 450 \times 10^{-3} / 4.507 = 0.1 \text{ m}^3$$

$$(a) E_c = 110 \text{ GN.m}^{-2} = f_B E_B + (1 - f_B) E_{Al} = f_B (379) + (1 - f_B) (70)$$

$$f_B = 0.129$$

$$\rho_c = (0.129)(2.36 \text{ Mg.m}^{-3}) + (0.871)(2.699 \text{ Mg.m}^{-3}) = 2.655 \text{ Mg.m}^{-3}$$

To produce a 0.1 m³ part of the composite, the part must weigh:

$$\text{Weight} = (0.1 \text{ m}^3)(2.655 \text{ Mg.m}^{-3}) = 0.2655 \text{ Mg} = 265.5 \text{ kg.}$$

$$(b) E_c = 110 = f_C E_C + (1 - f_C) E_{PET} = f_C (531) + (1 - f_C) (4.5)$$

$$f_C = 0.200$$

$$\rho_c = (0.200)(1.9 \text{ Mg.m}^{-3}) + (0.800)(1.28 \text{ Mg.m}^{-3}) = 1.404 \text{ Mg.m}^{-3}$$

To produce a 0.1 m³ part of the composite, the part must weigh:

$$\text{Weight} = (0.1 \text{ m}^3)(1.404 \text{ Mg.m}^{-3}) = 0.1404 \text{ Mg} = 140.4 \text{ kg}$$

(c) The specific moduli of the three materials are:

$$\text{Ti: } E/\rho = 110 \text{ GN.m}^{-2} / 4.507 \text{ Mg.m}^{-3} = 24.41 \times 10^6 \text{ m}^2 \cdot \text{s}^{-2}$$

$$\text{B-Al: } E/\rho = 110 \text{ GN.m}^{-2} / 2.655 \text{ Mg.m}^{-3} = 41.43 \times 10^6 \text{ m}^2 \cdot \text{s}^{-2}$$

$$\text{C-PET: } E/\rho = 110 \text{ GN.m}^{-2} / 1.404 \text{ Mg.m}^{-3} = 78.35 \times 10^6 \text{ m}^2 \cdot \text{s}^{-2}$$

16.17 Short but aligned Al_2O_3 fibres with a diameter of $20 \mu\text{m}$ are introduced into a 6,6-nylon matrix. The strength of the bond between the fibres and the matrix is estimated to be 7 MN.m^{-2} . Calculate the critical fibre length and compare with the case when $1 \mu\text{m}$ alumina whiskers are used instead of the coarser fibres. What is the minimum aspect ratio in each case?

The critical fibre length is given by $l_c = \sigma_f d / 2\tau_i$. For the alumina fibres, $\tau_i = 7 \text{ MN.m}^{-2}$; $d = 20 \times 10^{-6} \text{ m}$; and $\sigma_f = 2070 \text{ MN.m}^{-2}$. Thus, for alumina fibres:

$$l_c = (2070 \text{ MN.m}^{-2}) (20 \times 10^{-6} \text{ m}) / (2) (7 \text{ MN.m}^{-2}) = 2.96 \times 10^{-3} \text{ m} = 2.96 \text{ mm}$$

$$l_c / d = 2.96 \times 10^{-3} \text{ m} / 20 \times 10^{-6} \text{ m} = 148$$

For alumina whiskers, $d = 1 \times 10^{-6} \text{ m}$; The strength of the whiskers can be much higher than that of the fibres; 20700 MN.m^{-2} can be achieved. Thus, for alumina whiskers:

$$l_c = (20700 \text{ MN.m}^{-2}) (1 \times 10^{-6} \text{ m}) / (2) (7 \text{ MN.m}^{-2}) = 1.48 \times 10^{-3} \text{ m} = 1.48 \text{ mm}$$

$$l_c / d = 1.48 \times 10^{-3} / 1 \times 10^{-6} \text{ m} = 1480$$

16.18 We prepare several epoxy matrix composites using different lengths of $3 \mu\text{m}$ diameter ZrO_2 fibres and find that the strength of the composite increases with increasing fibre length up to 5 mm ; for longer fibres, the strength is virtually unchanged. Estimate the strength of the bond between the fibres and the matrix.

We do not expect much change in the strength when $l > 15l_c$. Therefore:

$$15l_c = 5 \text{ mm} \quad \text{or} \quad l_c = 0.333 \text{ mm}$$

$$\text{In addition, } l_c = \sigma_f d / 2\tau_i = 0.333 \text{ mm}$$

For ZrO_2 fibres, the tensile strength = 2070 MN.m^{-2}

Therefore:

$$\tau_i = \sigma_f d / 2l_c = (2070 \text{ MN.m}^{-2}) (3 \times 10^{-6} \text{ m}) / (2) (0.333 \times 10^{-3} \text{ m}) = 9.32 \text{ MN.m}^{-2}$$

16.19 In one polymer matrix composite, as-produced discontinuous glass fibres are introduced directly into the matrix; in a second case, the fibres are first "sized". Discuss the effect this difference might have on the critical fibre length and the strength of the composite.

By sizing the glass fibres, the surface is conditioned so that improved bonding between the fibres and the matrix is obtained. From Equation 16.9, we expect that improved bonding (τ_i) will reduce the length of fibres required for achieving good strength. Improved bonding will also reduce pull-out of the fibres from the matrix. Therefore the sizing improves the strength and allows small fibres to still be effective.

16.20 A Borsic fibre-reinforced aluminium composite is shown in Figure 16.19. Estimate the volume fractions of tungsten, boron, and the matrix for this composite. Calculate the modulus of elasticity parallel to the fibres for this composite. What would the modulus be if the same size boron fibre could be produced without the tungsten precursor?

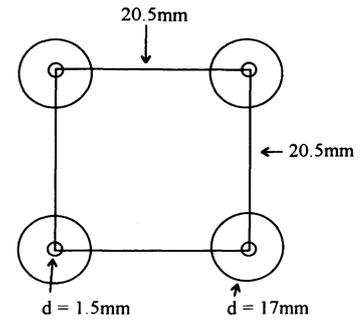
From the photograph, the diameter of the tungsten core is about 1.5 mm , the diameter of the boron fibre is 17 mm , and the distance between the centres of adjacent fibres is 20.5 mm . If we assume that the fibres

produce a square arrangement (see sketch), then

$$\begin{aligned} A_{\text{total}} &= (20.5 \text{ mm})^2 = 420.25 \text{ mm}^2 \\ A_{\text{tungsten}} &= (\pi/4) (1.5 \text{ mm})^2 = 1.77 \text{ mm}^2 \\ A_{\text{boron}} &= (\pi/4) (17 \text{ mm})^2 - 1.77 \text{ mm}^2 = 225.21 \text{ mm}^2 \\ A_{\text{Al}} &= 420.25 - 1.77 - 225.21 = 193.27 \text{ mm}^2 \end{aligned}$$

We can then determine the volume fractions:

$$\begin{aligned} f_{\text{tungsten}} &= 1.77 / 420.25 = 0.0042 \\ f_{\text{boron}} &= 225.21 / 420.25 = 0.5358 \\ f_{\text{Al}} &= 193.27 / 420.25 = 0.4600 \end{aligned}$$



We can now estimate the modulus of elasticity of the composite using the rule of mixtures:

$$\begin{aligned} E_{\text{composite}} &= (0.0042) (407) + (0.5358) (379) + (0.4600) (70) \\ &= 236.98 \text{ GMN.m}^{-2} \end{aligned}$$

If the tungsten filament was absent, then $f_{\text{boron}} = 0.5401$ and the modulus is:

$$E_{\text{composite}} = (0.5401) (379) + (0.4599) (70) = 236.89 \text{ GN.m}^{-2}$$

The tungsten makes virtually no difference in the stiffness of the overall composite. Its function is to serve as the precursor for the boron.

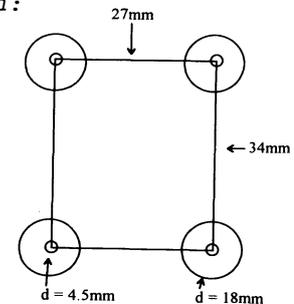
16.21 A silicon nitride matrix reinforced with silicon carbide fibres containing a HS carbon precursor is shown in Figure 16.19. Estimate the volume fractions of the SiC, Si₃N₄, and carbon in this composite. Calculate the modulus of elasticity parallel to the fibres for this composite. What would the modulus be if the same size SiC fibre could be produced without the carbon precursor?

From the photograph, the diameter of the carbon core is about 4.5 mm, the diameter of the SiC fibre is 18 mm, and the fibres produce a "rectangular" area 27 mm x 34 mm (see sketch). Then:

$$\begin{aligned} A_{\text{total}} &= (27 \text{ mm}) (34 \text{ mm}) = 918 \text{ mm}^2 \\ A_{\text{carbon}} &= (\pi/4) (4.5 \text{ mm})^2 = 15.90 \text{ mm}^2 \\ A_{\text{SiC}} &= (\pi/4) (18 \text{ mm})^2 - 15.90 \text{ mm}^2 = 238.57 \text{ mm}^2 \\ A_{\text{nitride}} &= 918 - 15.90 - 238.57 = 663.53 \text{ mm}^2 \end{aligned}$$

We can then determine the volume fractions:

$$\begin{aligned} f_{\text{carbon}} &= 15.90 / 918 = 0.017 \\ f_{\text{SiC}} &= 238.57 / 918 = 0.260 \\ f_{\text{nitride}} &= 663.53 / 918 = 0.723 \end{aligned}$$



We can now estimate the modulus of elasticity of the composite using the rule of mixtures:

$$\begin{aligned} E_{\text{composite}} &= (0.017) (276) + (0.260) (483) + (0.723) (310) \\ &= 354 \text{ GN.m}^{-2} \end{aligned}$$

If the carbon filament was absent, then $f_{\text{SiC}} = 0.277$ and the modulus is:

$$E_{\text{composite}} = (0.277) (483) + (0.723) (380) = 408.53 \text{ GN.m}^{-2}$$

The carbon makes virtually no difference in the stiffness of the overall composite. Its function is to serve as the precursor for the silicon carbide.

16.22 Explain why bonding between carbon fibres and an epoxy matrix should be excellent, whereas bonding between silicon nitride fibres and a silicon carbide matrix should be poor.

In the carbon/epoxy composite, we are interested in developing high strength, with the stresses carried predominantly by the strong carbon fibres. In order to transfer the applied loads from the weak epoxy to the strong carbon fibres, good bonding is required.

In the Si₃N₄/SiC composite, we are interested primarily in developing improved fracture toughness. Now we must design the microstructure to absorb and dissipate energy. By assuring that bonding is poor, the silicon nitride fibres can pull out of the silicon carbide matrix. This pull-out requires energy, thus improving the fracture toughness of the ceramic matrix composite.

16.23 A polyimide matrix is to be reinforced with 70 vol% carbon fibres to give a minimum modulus of elasticity of 275 GN.m⁻². Recommend a process for producing the carbon fibres required. Estimate the tensile strength of the fibres that are produced.

The modulus of polyimide is 2 GN.m⁻². The required modulus of the carbon fibres can be found from the rule of mixtures:

$$E_{\text{composite}} = f_{\text{carbon}}E_{\text{carbon}} + f_{\text{PI}}E_{\text{PI}}$$

$$275 \text{ GN.m}^{-2} = (0.7)E_{\text{carbon}} + (0.3)(2 \text{ GN.m}^{-2})$$

$$E_{\text{carbon}} = 392 \text{ GN.m}^{-2}$$

From Figure 16.20, we find that, to obtain this modulus in the carbon fibres, they must be pyrolysed at 2650°C. This in turn means that the tensile strength will be about 1625 MN.m⁻².

16.24 An electrical capacitor is produced by sandwiching 19 layers of 0.02 mm thick Teflon between 20 layers of 0.08 mm thick silver. Determine the electrical conductivity of the capacitor (a) parallel to the sheets and (b) perpendicular to the sheets. The electrical conductivity of silver is 68 x 10⁶ ohm⁻¹.m⁻¹ and that of Teflon is 10⁻¹⁸ ohm⁻¹.m⁻¹.

First we need to find the volume fractions of the two materials. We can assume that the linear ratio is equal to the volume ratio.

$$V_{\text{Teflon}} = (19 \text{ sheets})(0.02 \text{ mm}) = 0.38 \text{ mm}$$

$$V_{\text{silver}} = (20 \text{ sheets})(0.08 \text{ mm}) = 1.60 \text{ mm}$$

$$\text{total} = 1.98 \text{ mm}$$

The volume fractions are then:

$$f_{\text{Teflon}} = 0.38 / 1.98 = 0.1919 \quad f_{\text{silver}} = 0.8081$$

(a) Parallel to the sheets:

$$\sigma_{\text{composite}} = (0.1919)(10^{-18}) + (0.8081)(68 \times 10^6) = 5.495 \times 10^7 \text{ ohm}^{-1}.\text{m}^{-1}$$

(b) Perpendicular to the sheets:

$$1/\sigma_{\text{composite}} = 0.1919 / 10^{-18} + 0.8081 / 68 \times 10^6 = 1.919 \times 10^{17}$$

$$\sigma = 5.211 \times 10^{-18} \text{ ohm}^{-1}.\text{m}^{-1}$$

16.25 A microlaminate, Arall, is produced using 5 sheets of 0.4 mm thick aluminium and 4 sheets of 0.2 mm thick epoxy reinforced with unidirectionally aligned Kevlar fibres. The volume percent of Kevlar fibres in these intermediate sheets is 55%. Calculate the modulus of elasticity of the microlaminate parallel and perpendicular to the unidirectionally aligned Kevlar fibres. What are the principal advantages of the Arall material compared with those of unreinforced aluminium?

First we can find the volume fractions of each material. The volumes (expressed in a linear direction) are:

$$\begin{aligned} V_{Al} &= (5 \text{ sheets}) (0.4 \text{ mm/sheet}) &&= 2 \text{ mm} \\ V_{Kevlar} &= (0.55) (4 \text{ sheets}) (0.2 \text{ mm/sheet}) &&= 0.44 \text{ mm} \\ V_{epoxy} &= (0.45) (4 \text{ sheets}) (0.2 \text{ mm/sheet}) &&= 0.36 \text{ mm} \\ &&&\text{total} = 2.8 \text{ mm} \end{aligned}$$

$$\begin{aligned} f_{Al} &= 2 / 2.8 = 0.714 \\ f_{Kevlar} &= 0.44 / 2.8 = 0.157 \\ f_{epoxy} &= 0.36 / 2.8 = 0.129 \end{aligned}$$

From the rule of mixtures, the modulus parallel to the laminate is:

$$\begin{aligned} E_{composite} &= (0.714) (70) + (0.157) (124) + (0.129) (3) \\ &= 69.84 \text{ GN.m}^{-2} \end{aligned}$$

Perpendicular to the laminate:

$$\begin{aligned} 1/E_{composite} &= 0.714/70 + 0.157/124 + 0.129/3 = 0.05446 \\ E_{composite} &= 18.36 \text{ GN.m}^{-2} \end{aligned}$$

16.26 A laminate composed of 0.1 mm thick aluminium sandwiched around a 20 mm thick layer of polystyrene foam (Styrofoam®) is produced as an insulation material. Calculate the thermal conductivity of the laminate parallel and perpendicular to the layers. The thermal conductivity of aluminium is 238.6 W.m⁻¹.K⁻¹ and that of the foam is 0.032 W.m⁻¹.K⁻¹.

First we find the volume fractions:

$$\begin{aligned} f_{Al} &= 2(0.1 \text{ mm}) / [(2)(0.1 \text{ mm}) + 20 \text{ mm}] = 0.0099 \\ f_{foam} &= 0.9901 \end{aligned}$$

The thermal conductivity parallel to the laminate is:

$$K_{parallel} = (0.0099) (238.6) + (0.9901) (0.032) = 2.394 \text{ W.m}^{-1}.\text{K}^{-1}$$

Perpendicular to the laminate:

$$\begin{aligned} 1 / K_{perpendicular} &= 0.0099 / 238.6 + 0.9901 / 0.032 = 30.941 \\ K_{perpendicular} &= 0.032 \text{ W.m}^{-1}.\text{K}^{-1} \end{aligned}$$

16.27 A 0.1 mm-thick sheet of a polymer with a modulus of elasticity of 0.5 GN.m⁻² psi is sandwiched between two 4 mm thick sheets of glass with a modulus of elasticity of 83 GN.m⁻². Calculate the modulus of elasticity of the composite parallel and perpendicular to the sheets.

The volume fractions are:

$$\begin{aligned} f_{polymer} &= 0.1 \text{ mm} / (0.1 \text{ mm} + 4 \text{ mm} + 4 \text{ mm}) = 0.01234 \\ f_{glass} &= 0.98766 \end{aligned}$$

The modulus parallel to the laminate is:

$$E_{\text{parallel}} = (0.01234)(5) + (0.98766)(83) = 82.04 \text{ GN.m}^{-2}$$

Perpendicular to the laminate:

$$1 / E_{\text{perpendicular}} = (0.01234 / 5) + (0.98766 / 83) = 0.01437$$

$$E_{\text{perpendicular}} = 69.60 \text{ GN.m}^{-2}$$

This material is "safety" glass and is used in automobile windshields to keep the windshield from shattering.

16.28 A U.S. quarter is 23 mm in diameter and is about 2 mm thick. Copper costs about \$2.40 per kg and nickel costs about \$9.00 per kg. Compare the material cost in a composite quarter versus a quarter made entirely of nickel.

From chapter 16, in a quarter, the thickness (and hence the volume) ratio is 1/6 Ni: 2/3 Cu: 1/6 Ni. The volume fraction of each is:

$$f_{\text{Cu}} = 0.667 \quad f_{\text{Ni}} = 0.333$$

The volume of the quarter, as well as the copper and nickel, are:

$$V_{\text{quarter}} = (\pi/4)(23 \times 10^{-3}\text{m})^2(2 \times 10^{-3}\text{m}) = 8.309 \times 10^{-7}\text{m}^3$$

$$V_{\text{Cu}} = (8.309 \times 10^{-7}\text{m}^3)(0.667) = 5.542 \times 10^{-7}\text{m}^3$$

$$V_{\text{Ni}} = (8.309 \times 10^{-7}\text{m}^3)(0.333) = 2.767 \times 10^{-7}\text{m}^3$$

The weights of copper and nickel in the coin are:

$$W_{\text{Cu}} = (5.542 \times 10^{-7}\text{m}^3)(8.93 \text{ Mg.m}^{-3}) = 4.949 \times 10^{-6}\text{Mg}$$

$$W_{\text{Ni}} = (2.767 \times 10^{-7}\text{m}^3)(8.902 \text{ Mg.m}^{-3}) = 2.463 \times 10^{-6}\text{Mg}$$

The cost of each material in the coin is:

$$\$/\text{Cu} = (4.949 \times 10^{-3}\text{kg})(\$2.40/\text{kg}) = \$0.0119$$

$$\$/\text{Ni} = (2.463 \times 10^{-3}\text{kg})(\$9.00/\text{kg}) = \$0.0222$$

The total cost of the composite (for materials only) = \$0.0341

If the entire coin were made of nickel, then

$$(8.309 \times 10^{-7}\text{m}^3)(8.902 \times 10^3\text{kg.m}^{-3})(\$9.00/\text{kg}) = \$0.0666$$

By using the composite coin, the cost of the materials is about half that of a pure nickel coin, yet the coin appears to have the nickel (or silvery) color.

16.29 Calculate the density of a honeycomb structure composed of the following elements. The two 2-mm thick cover sheets are produced using an epoxy matrix prepreg containing 55 vol% E-glass fibres. The aluminium honeycomb is 20 mm thick; the cells are in the shape of 5 mm squares and the walls of the cells are 0.1 mm thick. Estimate the density of the structure. Compare the weight of a 1 m x 2 m panel of the honeycomb compared to a solid aluminium panel with the same dimensions.

Each cell of aluminium can be considered to be a hollow square shape where the dimensions of the cell are 5 mm x 5 mm x 20 mm, with a wall thickness belonging uniquely to that cell of 0.1 mm / 2 = 0.05 mm.

$$V_{Al} = (4 \text{ sides}) (0.05 \text{ mm}) (5 \text{ mm}) (20 \text{ mm}) = 20 \text{ mm}^3$$

The cover sheet dimensions that just cover the single cell described above are 5 mm x 5 mm x 2 mm. The volume is:

$$V_{cover} = (2 \text{ sheets}) (5 \text{ mm}) (5 \text{ mm}) (2 \text{ mm}) = 100 \text{ mm}^3$$

The total "height" of the cell, including the cover sheets, is 20 mm + 2(2 mm) = 24 mm. The total volume of the cell is:

$$V_{total} = (5 \text{ mm}) (5 \text{ mm}) (24 \text{ mm}) = 600 \text{ mm}^3$$

The volume fractions of these constituents are:

$$\begin{aligned} f_{Al \text{ in Cell}} &= 20 / 600 = 0.0333 \\ f_{cover} &= 100 / 600 = 0.1667 \\ f_{void} &= 0.80 \end{aligned}$$

The densities of the three constituents can be determined. The density of the aluminium in the cells is 2.699 Mg.m⁻³ and the density of the void space within the cells is zero. But the cover sheets are themselves composites:

$$\begin{aligned} \rho_{cover} &= f_{glass}\rho_{glass} + f_{epoxy}\rho_{epoxy} = (0.55)(2.55 \text{ Mg.m}^{-3}) + (0.45)(1.25 \text{ Mg.m}^{-3}) \\ &= 1.965 \text{ Mg.m}^{-3} \end{aligned}$$

Therefore the overall density of the honeycomb structure is:

$$\begin{aligned} \rho_{honeycomb} &= f_{Al \text{ in Cell}}\rho_{Al \text{ in Cell}} + f_{cover}\rho_{cover} + f_{void}\rho_{void} \\ &= (0.0333)(2.699) + (0.1667)(1.965) + (0.80)(0) = 0.417 \text{ Mg.m}^{-3} \end{aligned}$$

The weight of a 1 m x 2 m panel of the honeycomb is:

$$\begin{aligned} W_{honeycomb} &= (24 \times 10^{-3} \text{ m}) (1 \text{ m}) (2 \text{ m}) (0.417 \text{ Mg.m}^{-3}) = 2.0016 \times 10^{-2} \text{ Mg} \\ &= 20.016 \text{ kg} \end{aligned}$$

If the panel were made of solid aluminium, with the same dimensions, the panel would weigh:

$$\begin{aligned} W_{solid} &= (24 \times 10^{-3} \text{ m}) (1 \text{ m}) (2 \text{ m}) (2.669 \text{ Mg.m}^{-3}) = 1.2811 \times 10^{-1} \text{ Mg} \\ &= 128.11 \text{ kg} \end{aligned}$$

The weight savings using the honeycomb is enormous.

Chapter 17 Construction Materials

17.1 A sample of wood with dimensions 75 mm x 100 mm x 300 mm has a dry density of 0.35 Mg.m^{-3} . (a) Calculate the volume of water that must be absorbed by the sample to contain 120% water. (b) Calculate the density after the wood absorbs this amount of water.

$$V = 0.075 \times 0.1 \times 0.3 = 0.00225 \text{ m}^3$$

$$\text{dry weight} = 0.35 \times 0.00225 \times 10^6 = 787.5 \text{ g}$$

$$\text{@120\% water} = \frac{\text{weight of water}}{\text{weight of dry wood}} \times 100$$

$$(a) \text{ water} = (1.2)(787.5) = 945 \text{ g}$$

$$= 945 \text{ g} \times (1 \text{ l}/1000 \text{ g}) = 0.945 \text{ l}$$

(b) If the volume remains the same, then

$$\text{density} = \frac{787.5 \text{ g of dry wood} + 945 \text{ g of water}}{0.00225 \text{ m}^3} = 0.77 \text{ Mg.m}^{-3}$$

17.2 The density of a sample of oak is 0.90 Mg.m^{-3} . Calculate (a) the density of completely dry oak and (b) the percent water in the original sample.

$$\rho_{12\% \text{ water}} = 0.68 \text{ Mg.m}^{-3} \text{ (Table 17.1)}$$

(a) Therefore, in 100 cm^3 of wood at 12% H_2O , there are 68 g.

$$12\% \text{ water} = \frac{\text{green weight} - \text{dry weight}}{\text{dry weight}} = \frac{68 - \text{dry weight}}{\text{dry weight}} \times 100$$

$$\text{dry weight} = 68 / 1.12 = 60.71 \text{ g}$$

(b) When the density is 0.90 Mg.m^{-3} , there are 90 g of green wood per 100 cm^3 . The water is therefore $90 - 60.71 \text{ g}$, or 29.29 g .

$$\% \text{H}_2\text{O} = \frac{90 \text{ g} - 60.71 \text{ g}}{60.71 \text{ g}} \times 100 = 48.2\%$$

17.3 Boards of maple 25 mm thick, 150 mm wide, and 5 m long are used as the flooring for a 20 m x 20 m hall. The boards were cut from logs with a tangential-longitudinal cut. The floor is laid when the boards have a moisture content of 12%. After some particularly humid days, the moisture content in the boards increases to 45%. Determine the dimensional change in the flooring parallel to the boards and perpendicular to the boards. What will happen to the floor? How can this problem be corrected?

Perpendicular:

$$C_{\text{tangential}} = 0.00353 \text{ mm/mm}^{-1}.\% \text{H}_2\text{O} \text{ for maple}$$

$$\Delta x = x_o [c(M_f - M_i)] = 150 [0.00353(45 - 12)] = 17.5 \text{ mm in } 150 \text{ mm}$$

$$\text{Over a 20 m span: } \Delta x = \frac{(20\text{m})(1000\text{mm}\cdot\text{m}^{-1})(17.55\text{ mm})}{150\text{ mm}}$$

$$= 2.34\text{ m}$$

The floor will therefore buckle due to the large amount of expansion of the boards perpendicular to the flooring.

Parallel:

For most woods, only about a 0.2% change in dimensions occurs longitudinally. Thus the total change in the length of the boards will be about

$$\Delta y = (0.002)(20\text{ m}) = 0.04\text{ m} = 40\text{ mm}$$

17.4 A wall 10 m long is built using radial-longitudinal cuts of 125 mm wide pine, with the boards arranged in a vertical fashion. The wood contains a moisture content of 55% when the wall is built; however the humidity level in the room is maintained to give 45% moisture in the wood. Determine the dimensional changes in the wood boards and estimate the size of the gaps that will be produced as a consequence of these changes.

$$C_{\text{tangential}} = 0.00141\text{ mm/mm}^{-1}\cdot\%H_2O\text{ for pine}$$

$$\Delta x = (10\text{ m})(0.00141\text{ mm}\cdot\text{mm}^{-1}\cdot\%H_2O)(45-55) = -0.141\text{ m}$$

The total number of boards in the width of the wall is:

$$\text{number of boards} = (10\text{ m})(1000\text{ mm}\cdot\text{m}^{-1})/(125\text{ mm}/\text{board}) = 80\text{ boards}$$

Therefore there are 79 gaps between the boards. The average width of the gaps is:

$$\text{gap} = 141\text{ mm} / 79\text{ gaps} = 1.78\text{ mm}$$

17.5 We have been asked to prepare 100 m³ of normal concrete using a volume ratio of cement-sand-coarse aggregate of 1:2:4. The water-cement ratio (by weight) is to be 0.5. The sand contains 6 wt% water and the coarse aggregate contains 3 wt% water. No entrained air is expected. (a) Determine the number of sacks of cement that must be ordered, the kg of sand and aggregate required, and the amount of water needed. (b) Calculate the total weight of the concrete per cubic metre. (c) What is the weight ratio of cement-sand-coarse aggregate?

The densities of cement, sand and aggregate are given in Table 17.6.

(a) 1 sack of cement contains 50 kg = 0.05 Mg
 Volume of 1 sack cement = mass/density = 0.05/1.75 = 0.02857 m³
 That requires:

sand: 2 x 0.02857 m³ = 0.05714 m³ = 0.05714 m³ x 2.56 Mg·m⁻³ = 0.1463 Mg
 aggregate: 4 x 0.02857 m³ = 0.11429 m³ = 0.11429 m³ x 2.72 Mg·m⁻³ = 0.3109 Mg

Total amount of water required per sack of cement = 0.05 x 0.05 Mg = 0.025 Mg = 0.025 m³

The volume of concrete/sack of cement = volume (cement + sand + aggregate + water)

$$= 0.02857 + 0.05714 + 0.11429 + 0.025\text{ m}^3 = 0.225\text{ m}^3$$

Since 100 m^3 of concrete is required, we need $100/0.225 = 444$ sacks of cement. Therefore the mass of sand and aggregate required are:

$$\begin{aligned} \text{sand } & 444 \times 0.1463 \text{ Mg} = 64.96 \text{ Mg} \\ \text{aggregate } & 444 \times 0.3109 \text{ Mg} = 138.0 \text{ Mg} \end{aligned}$$

But these contain water, hence amounts to be ordered are
sand $64.96 \times 100/94 = 69.11 \text{ Mg} = 69 \text{ 110 kg}$
aggregate $138.0 \times 100/97 = 142.27 \text{ Mg} = 142 \text{ 270 kg}$

Amount of water required = total amount of water - water in sand - water in aggregate = $444 \times (0.025 - 0.1453 \times 0.06 - 0.3109 \times 0.03) = 1.888 \text{ Mg} = 1.8888 \text{ m}^3$

(b) mass of concrete per 100 m^3 = mass (cement + sand + aggregate + water) = $(444 \times 0.05) + 64.96 + 138.00 + (444 \times 0.05/2) = 236.26 \text{ Mg}$

$$\text{Weight of concrete per } \text{m}^3 = 236.26 \text{ Mg}/100 \text{ m}^3 = 2.36 \text{ Mg} \cdot \text{m}^{-3}$$

(c) Weight ratio of cement-sand-aggregate = $0.05 : 0.1463 : 0.3109 = 1 : 2.92 : 6.2$

17.6 First, we can determine the volume of each material required, using the $1 : 2.5 : 4.5$ ratio to determine the weights per 40 kg sack of cement, and dividing by the density to determine the volume.

$$\begin{aligned} \text{As } 50 \text{ kg} &= 0.050 \text{ Mg} \\ \text{cement: } & 0.050 \text{ Mg} / 1.75 \text{ Mg} \cdot \text{m}^{-3} = 0.02857 \text{ m}^3 \\ \text{sand: } & 2.5 \times 0.050 \text{ Mg} / 2.56 \text{ Mg} \cdot \text{m}^{-3} = 0.04883 \text{ m}^3 \\ \text{aggregate: } & 4.5 \times 0.050 \text{ Mg} / 2.72 \text{ Mg} \cdot \text{m}^{-3} = 0.08272 \text{ m}^3 \\ \text{water: } & 0.45 \times 0.050 \text{ Mg} / 1 \text{ Mg} \cdot \text{m}^{-3} = 0.02250 \text{ m}^3 \end{aligned}$$

$$\text{Volume of per } 50 \text{ kg sack of cement} = 0.18262 \text{ m}^3$$

But 5% of the concrete is expected to be entrained air. The volume x per 50 kg sack of cement is:

$$x/(0.18262 + x) = 0.05 \text{ or } x = 0.00961 \text{ m}^3$$

Therefore the total volume of concrete per 50 kg sack of cement is:

$$\text{Volume of concrete} = 0.18262 + 0.00961 = 0.19223 \text{ m}^3$$

In 10 m^3 of concrete:

$$\begin{aligned} \text{cement} &= 10 \text{ m}^3 / 0.19223 \text{ m}^3 \cdot (50 \text{ kg sack})^{-1} = 52 \text{ sacks} \\ \text{sand} &= 2.5 \times 0.050 \text{ Mg} \times (10 \text{ m}^3 / 0.19223 \text{ m}^3) = 6.5 \text{ Mg} = 6500 \text{ kg} \\ \text{aggregate} &= 4.5 \times 0.050 \text{ Mg} \times (10 \text{ m}^3 / 0.19223 \text{ m}^3) = 10.9 \text{ Mg} = 10900 \text{ kg} \\ \text{water} &= 0.45 \times 0.050 \text{ Mg} \times (10 \text{ m}^3 / 0.19223 \text{ m}^3) = 1.09 \text{ Mg} = 1090 \text{ kg} \end{aligned}$$

But we must also adjust for the water present in the wet sand (3%) and wet aggregate (2%). For example, to find the amount of wet sand, we could either multiply the dry sand by 1.03 or divide by 0.97:

$$\begin{aligned} \text{wet sand} &= 6.5 \text{ Mg} / 0.97 = 6.70 \text{ Mg} = 6700 \text{ kg}; \text{ water} = 0.20 \text{ Mg} \\ \text{wet aggregate} &= 10.9 \text{ Mg} / 0.98 = 11.12 \text{ Mg} = 11120 \text{ kg}; \text{ water} = 0.22 \text{ Mg} \end{aligned}$$

Therefore, the ingredients for the concrete mix include:

$$\begin{aligned} \text{cement} &= 52 \text{ sacks} \\ \text{sand} &= 6500 \text{ kg} \\ \text{aggregate} &= 10900 \text{ kg} \\ \text{water} &= (1.09 - 0.20 - 0.22 \text{ Mg}) / 1 \text{ Mg} \cdot \text{m}^{-3} = 0.67 \text{ m}^3 \end{aligned}$$

Chapter 18 Electrical Behaviour of Materials

18.1 A current of 10 A is passed through a 1 mm diameter wire 1 km long. Calculate the power loss if the wire is made of (a) aluminium, (b) silicon, and (c) silicon carbide. (see Table 18.1)

$$\text{Power} = I^2 R = I^2 \ell / \sigma A = (10 \text{ A})^2 (1000 \text{ m}) / (\pi/4) (1 \times 10^{-3} \text{ m})^2 \sigma$$

$$\text{Power} = 1.273 \times 10^{11} / \sigma$$

The electrical conductivity of each material is given in Table 18.1:

$$(a) P_{Al} = 1.273 \times 10^{11} / 3.77 \times 10^5 = 3380 \text{ watt}$$

$$(b) P_{Si} = 1.273 \times 10^{11} / 5 \times 10^{-4} = 2.546 \times 10^{14} \text{ watt}$$

$$(c) P_{SiC} = 1.273 \times 10^{11} / 1 \text{ to } 10 \\ = 1.273 \times 10^{10} \text{ to } 1.273 \times 10^{11} \text{ watt}$$

18.2 The power lost in a 2 mm diameter copper wire is to be less than 250 W when a 5-A current is flowing in the circuit. What is the maximum length of the wire?

$$P = I^2 R = I^2 \ell / \sigma A = 250 \text{ W}$$

$$\ell = 250 \sigma A / I^2 = (250) (5.98 \times 10^7) (\pi/4) (2 \times 10^{-3})^2 / (5)^2 = 1.88 \times 10^3 \text{ m}$$

18.3 A current density of 10^9 A.m^{-2} is applied to a gold wire 50 m in length. The resistance of the wire is found to be 2 Ω . Calculate the diameter of the wire and the voltage applied to the wire.

$$J = I/A = \sigma V/\ell = 10^9 \text{ A.m}^{-2}$$

$$V = 10^9 \ell / \sigma = (10^9) (50 \text{ m}) / 4.26 \times 10^7 = 1174 \text{ V}$$

$$\text{From Ohm's law, } I = V/R = 1174 / 2 = 587 \text{ A}$$

$$A = I/J = 587 / 10^9 = 5.87 \times 10^{-7} \text{ m}^2$$

$$(\pi/4) d^2 = 5.87 \times 10^{-7} \text{ m}^2 \quad \text{or } d^2 = 7.47 \times 10^{-7} \text{ m}^2 \quad \text{or } d = 0.865 \text{ mm}$$

18.4 We would like to produce a 5000- Ω resistor from boron carbide fibres having a diameter of 0.1 mm. What is the required length of the fibres?

The electrical conductivity is 100 to 200 $\Omega^{-1} \cdot \text{m}^{-1}$.

$$R = \ell / \sigma A = 5000 \Omega$$

If the conductivity is 100 $\Omega^{-1} \cdot \text{m}^{-1}$:

$$\ell = R \sigma A = (5000 \Omega) (100 \Omega^{-1} \cdot \text{m}^{-1}) (1 \times 10^{-4} \text{ m}) = 3.93 \times 10^{-3} \text{ m} = 3.93 \text{ mm}$$

If the conductivity is 200 $\Omega^{-1} \cdot \text{m}^{-1}$:

$$\ell = R \sigma A = (5000) (200) (\pi/4) (1 \times 10^{-4})^2 = 7.85 \text{ mm}$$

The fibres should be 3.93 to 7.85 mm in length.

18.5 Suppose we estimate that the mobility of electrons in silver is $75 \times 10^{-3} \text{ m}^2 \cdot \text{V}^{-1} \cdot \text{s}^{-1}$. Estimate the fraction of the valence electrons that are carrying an electrical charge.

The total number of valence electrons is:

$$n_T = \frac{(4 \text{ atoms/cell})(1 \text{ electron/atom})}{(0.40862 \times 10^{-9} \text{ m})^3} = 5.86 \times 10^{28} \text{ m}^{-3}$$

The number of charge carriers is:

$$n = \sigma / q\mu = (6.80 \times 10^7) / (1.6 \times 10^{-19})(7.5 \times 10^{-3}) = 5.67 \times 10^{28}$$

The fraction of the electrons that carry the electrical charge is:

$$n/n_T = 5.67 \times 10^{28} / 5.86 \times 10^{28} = 0.968$$

18.6 A current density of $5 \times 10^7 \text{ A} \cdot \text{m}^{-2}$ is applied to a magnesium wire. If half of the valence electrons serve as charge carriers, calculate the average drift velocity of the electrons.

The total number of valence electrons is:

$$n_T = \frac{(2 \text{ atoms/cell})(2 \text{ electrons/atom})}{(0.32087 \times 10^{-9})^2 (0.5209 \times 10^{-9}) \cos 30} = 8.61 \times 10^{28} \text{ m}^{-3}$$

The actual number of charge carriers is then 4.305×10^{28} .

$$v = J/nq = (5 \times 10^7 \text{ A} \cdot \text{m}^{-2}) / (4.305 \times 10^{28})(1.6 \times 10^{-19}) = 7.259 \text{ mm} \cdot \text{s}^{-1}$$

18.7 We apply a voltage of 10 V to an aluminium wire 2 mm in diameter and 20 m long. If 10% of the valence electrons carry the electrical charge, calculate the average drift velocity of the electrons in $\text{km} \cdot \text{h}^{-1}$.

The total number of valence electrons is:

$$n_T = \frac{(4 \text{ atoms/cell})(3 \text{ electrons/atom})}{(0.404958 \times 10^{-9} \text{ m})^3} = 1.81 \times 10^{29} / \text{m}^3$$

The number of electrons carrying the electrical charge is one-tenth of the total number, or $1.81 \times 10^{28} \text{ electrons} \cdot \text{m}^{-3}$. The electric field is

$$\xi = V/l = 10 \text{ V} / 20 \text{ m} = 0.5 \text{ V} \cdot \text{m}^{-1}$$

$$\sigma \xi = nqv \quad \text{or} \quad v = \sigma \xi / nq$$

$$v = (3.77 \times 10^7)(0.5) / (1.81 \times 10^{28})(1.6 \times 10^{-19}) = 0.00651 \text{ m} \cdot \text{s}^{-1}$$

$$v = (0.00651 \text{ m} \cdot \text{s}^{-1})(3600 \text{ s} \cdot \text{h}^{-1}) = 0.0234 \text{ km} \cdot \text{h}^{-1}$$

18.8 In a welding process, a current of 400 A flows through the arc when the voltage is 35 V. The length of the arc is about 2.5 mm and the average diameter of the arc is about 4.5 mm. Calculate the current density in the arc, the electric field across the arc, and the electrical conductivity of the hot gases in the arc during welding.

$$R = V/I = 35 \text{ V} / 400 \text{ A} = 0.0875 \Omega$$

The electrical conductivity of the gases in the arc is:

$$\sigma = \ell/RA = \frac{2.5 \text{ mm}}{(0.0875 \Omega) (\pi/4) (4.5)^2}$$

$$= 1.796 \Omega^{-1} \cdot \text{mm}^{-1} = 1796 \Omega^{-1} \text{m}^{-1}$$

The current density J is:

$$J = I/A = 400 \text{ A} / (\pi/4) (4.5)^2 = 25.15 \text{ A} \cdot \text{mm}^{-2} = 25.15 \times 10 \text{ A} \cdot \text{m}^{-2}$$

The electric field is:

$$\xi = V/\ell = 35 \text{ V} / 2.5 \times 10^{-3} = 14000 \text{ V m}^{-1}$$

18.9 Calculate the electrical conductivity of nickel at -50°C and at $+500^\circ\text{C}$.

$$\rho_{\text{room}} = 6.84 \times 10^{-8} \Omega \cdot \text{m}^{-1} \quad a = 0.0069 \text{ }^\circ\text{C}^{-1}$$

$$\rho_{500} = (6.84 \times 10^{-8}) [1 + (0.0069) (500 - 25)] = 2.92 \times 10^{-7} \Omega \cdot \text{m}$$

$$\sigma_{500} = 1/\rho = 1/2.92 \times 10^{-7} = 3.418 \times 10^6 \Omega^{-1} \cdot \text{m}^{-1}$$

$$\rho_{-50} = (6.84 \times 10^{-8}) [1 + (0.0069) (-50 - 25)] = 3.3003 \times 10^{-8} \Omega \cdot \text{m}$$

$$\sigma_{-50} = 1/3.003 \times 10^{-8} = 3.03 \times 10^7 \Omega^{-1} \cdot \text{m}^{-1}$$

18.10 The electrical resistivity of pure chromium is found to be $18 \times 10^{-8} \Omega \cdot \text{m}$. Estimate the temperature at which the resistivity measurement was made.

$$\rho_{\text{room}} = 1.29 \times 10^{-7} \Omega \cdot \text{m} \quad a = 0.0030 \text{ }^\circ\text{C}^{-1}$$

$$18 \times 10^{-8} = (1.29 \times 10^{-7}) [1 + (0.0030) (T - 25)]$$

$$1.395 - 1 = (0.003) (T - 25)$$

$$T = 156.8^\circ\text{C}$$

18.11 After finding the electrical conductivity of cobalt at 0°C , we decide we would like to double that conductivity. To what temperature must we cool the metal?

$$\rho_{\text{room}} = 6.24 \times 10^{-8} \Omega \cdot \text{m} \quad a = 0.006 \text{ }^\circ\text{C}^{-1}$$

$$\rho_{\text{zero}} = (6.24 \times 10^{-8}) [1 + (0.006) (0 - 25)] = 5.304 \times 10^{-8}$$

We wish to double the conductivity, or halve the resistivity to $2.652 \times 10^{-8} \Omega \cdot \text{m}$. The required temperature is:

$$2.652 \times 10^{-8} = (6.24 \times 10^{-8}) [1 + (0.006) (T - 25)]$$

$$- 0.575 = 0.006(T - 25) \quad \text{or} \quad T = -70.8^\circ\text{C}$$

18.12 From Figure 18.11(b), estimate the defect resistivity coefficient for tin in copper.

The conductivity and resistivity of pure copper are, from Table 18.1:

$$\sigma = 5.98 \times 10^7 \Omega^{-1} \cdot \text{m}^{-1} \quad \rho = 1/\sigma = 1.67 \times 10^{-8} \Omega \cdot \text{m}$$

For 0.2 wt% Sn in copper:

$$x_{Sn} = \frac{(0.2 / 118.69)}{(0.2 / 118.69) + (99.8 / 63.54)} = 0.00107$$

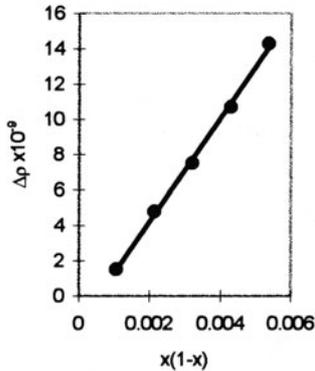
$$x_{Sn}(1-x_{Sn}) = (0.00107)(1 - 0.00107) = 0.00107$$

For 0.2% Sn, Figure 18.11(b) shows that the conductivity is 92% that of pure copper, or

$$\begin{aligned} \sigma &= (5.98 \times 10^7)(0.92) = 5.50 \times 10^7 \Omega^{-1} \cdot m^{-1} \\ \rho &= 1/\sigma = 1.82 \times 10^{-8} \Omega \cdot m \\ \Delta\rho &= 1.82 \times 10^{-8} - 1.67 \times 10^{-8} = 1.5 \times 10^{-9} \Omega \cdot m \end{aligned}$$

The following table includes the calculations for other compositions:

wt%Sn	x_{Sn}	$x_{Sn}(1-x_{Sn})$	% σ	σ	ρ	$\Delta\rho$
0	0	0	100	5.98×10^7	1.67×10^{-8}	0
0.2	0.00107	0.00107	92	5.50×10^7	1.82×10^{-8}	1.5×10^{-9}
0.4	0.00215	0.00215	78	4.66×10^7	2.15×10^{-8}	4.8×10^{-9}
0.6	0.00322	0.00321	69	4.13×10^7	2.42×10^{-8}	7.5×10^{-9}
0.8	0.00430	0.00428	61	3.65×10^7	2.74×10^{-8}	10.7×10^{-9}
1.0	0.00538	0.00535	54	3.23×10^7	3.10×10^{-8}	14.3×10^{-9}



These data are plotted. The slope of the graph is "b":

$$\begin{aligned} b &= \frac{13.5 \times 10^{-9} - 3.0 \times 10^{-9}}{0.0050 - 0.0015} \\ &= 3 \times 10^{-6} \Omega \cdot m \end{aligned}$$

18.13 The electrical resistivity of a beryllium alloy containing 5 at% of an alloying element is found to be $50 \times 10^{-8} \Omega \cdot m$ at $400^\circ C$. Determine the contributions to resistivity due to temperature and due to impurities by finding the expected resistivity of pure beryllium at $400^\circ C$, the resistivity due to impurities, and the defect resistivity coefficient. What would be the electrical resistivity if the beryllium contained 10 at% of the alloying element at $200^\circ C$?

From the data in Table 18.3, the resistivity at $400^\circ C$ should be:

$$\rho_T = (4 \times 10^{-8}) [1 + (0.025)(400 - 25)] = 41.5 \times 10^{-8} \Omega \cdot m$$

Consequently the resistance due to impurities is:

$$\begin{aligned} \rho &= \rho_T + \rho_d \\ 50 \times 10^{-8} &= 41.5 \times 10^{-8} + \rho_d \\ \rho_d &= 8.5 \times 10^{-8} \Omega \cdot m \end{aligned}$$

Since there are 5 at% impurities present, $x = 0.05$, and the defect resistivity coefficient is:

$$\rho_d = bx(1-x) \quad \text{or} \quad b = \rho_d / x(1-x)$$

$$b = 8.5 \times 10^{-8} / (0.05)(1 - 0.05) = 178.9 \times 10^{-8} \Omega.m$$

The resistivity at 200°C in an alloy containing 10 at% impurities is:

$$\begin{aligned} \rho_{200} &= \rho_T + \rho_d \\ &= (4 \times 10^{-8}) [1 + (0.025)(200 - 25)] + 178.9 \times 10^{-8} (0.1)(1 - 0.1) \\ &= 21.5 \times 10^{-8} + 16.1 \times 10^{-8} = 37.6 \times 10^{-8} \Omega.m \end{aligned}$$

18.14 Is Equation 18.7 valid for the copper-zinc system? If so, calculate the defect resistivity coefficient for zinc in copper.

The conductivity and resistivity of pure copper are:

$$\sigma = 5.98 \times 10^7 \Omega^{-1}.m^{-1} \quad \text{or} \quad \rho = 1.67 \times 10^{-8} \Omega.m$$

For 10 wt% Zn in copper:

$$x_{Zn} = \frac{(10 / 65.38)}{(10 / 65.38) + (90 / 63.54)} = 0.0975$$

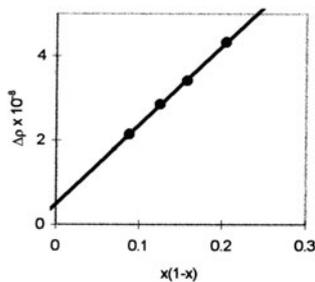
$$x_{Zn}(1-x_{Zn}) = (0.0975)(1 - 0.0975) = 0.088$$

From Figure 18.11(a), the conductivity of the Cu-10% Zn alloy at zero deformation is about 44% that of pure copper, or

$$\begin{aligned} \sigma &= (5.98 \times 10^7)(0.44) = 2.63 \times 10^7 \Omega^{-1}.m^{-1} \\ \rho &= 1/\sigma = 3.8 \times 10^{-8} \Omega.m \\ \Delta\rho &= 3.8 \times 10^{-8} - 1.67 \times 10^{-8} = 2.13 \times 10^{-8} \Omega.m \end{aligned}$$

The following table includes the calculations for other compositions:

wt%Zn	x_{Zn}	$x_{Zn}(1-x_{Zn})$	% σ	σ	ρ	$\Delta\rho$
0	0	0	100	5.98×10^7	1.67×10^{-8}	0
10	0.0975	0.088	44	2.63×10^7	3.80×10^{-8}	2.13×10^{-8}
15	0.146	0.125	37	2.21×10^7	4.52×10^{-8}	2.85×10^{-8}
20	0.196	0.158	33	1.97×10^7	5.08×10^{-8}	3.41×10^{-8}
30	0.294	0.208	28	1.67×10^7	5.99×10^{-8}	4.32×10^{-8}



These data are plotted. The slope of the graph is "b":

$$\begin{aligned} b &= \frac{0.4 \times 10^{-8} - 0.2 \times 10^{-8}}{0.19 - 0.08} \\ &= 1.8 \times 10^{-7} \Omega.m \end{aligned}$$

18.15 GaV₃ is to operate as a superconductor in liquid helium (at 4 K); what is the maximum magnetic field that can be applied to the material?

From Table 18.5:

$$T_c = 16.8 \text{ K} \quad H_o = 2.8 \times 10^7 \text{ A.m}^{-1}$$

$$H_c = H_o [1 - (T/T_c)^2] = 2.8 \times 10^7 [1 - (4/16.8)^2] = 2.64 \times 10^7 \text{ A.m}^{-1}$$

18.16 Nb₃Sn and GaV₃ are candidates for a superconductive application when the magnetic field is $1.2 \times 10^7 \text{ A.m}^{-1}$. Which would require the lower temperature in order to be superconductive?

$$1.2 \times 10^7 = H_0 [1 - (T/T_c)^2]$$

$$\text{For Nb}_3\text{Sn: } 1.2 \times 10^7 = 2 \times 10^7 [1 - (T/18.05)^2]$$

$$T = 11.42 \text{ K}$$

$$\text{For GaV}_3: 1.2 \times 10^7 = 2.8 \times 10^7 [1 - (T/16.8)^2]$$

$$T = 12.7 \text{ K}$$

18.17 A filament of Nb₃Sn 0.05 mm in diameter operates in a magnetic field of 80000 A.m⁻¹ at 4 K. What is the maximum current that can be applied to the filament in order for the material to behave as a superconductor?

From Figure 18.15, the maximum current density for Nb₃Sn in a field of 80000 A.m⁻¹ is about 2 × 10¹⁰ A.m⁻².

$$I = JA = (2 \times 10^{10} \text{ A.m}^{-2}) (\pi/4) (0.05 \times 10^{-3} \text{ m})^2 = 39.3 \text{ A}$$

18.18 Assume that most of the electrical charge transferred in MgO is caused by the diffusion of Mg²⁺ ions. Determine the mobility and electrical conductivity of MgO at 25°C and at 1500°C. (See Table 5.1)

At 25°C = 298 K:

$$D_{\text{Mg}} = 2.49 \times 10^5 \exp[-330,700/(8.314)(298)] = 2.68 \times 10^{-63} \text{ m}^2 \cdot \text{s}^{-1}$$

$$\mu = \frac{ZqD}{kT} = \frac{(2)(1.6 \times 10^{-19})(2.68 \times 10^{-63})}{(1.38 \times 10^{-23})(298)} = 2.09 \times 10^{-61} \text{ m}^2 \cdot \text{V}^{-1} \cdot \text{s}^{-1}$$

We can determine that the lattice parameter for MgO is 0.396 nm (since $a_o = 2r_{\text{Mg}} + 2r_{\text{O}}$). There are four Mg ions per unit cell, so the number of Mg ions per cm³ is:

$$n = (4)/(0.396 \times 10^{-9})^3 = 6.44 \times 10^{28} \text{ m}^{-3}$$

$$\sigma = nZq\mu = (6.44 \times 10^{28})(2)(1.6 \times 10^{-19})(2.09 \times 10^{-61}) = 4.31 \times 10^{-51} \Omega^{-1} \cdot \text{m}^{-1}$$

At 1500°C = 1773 K:

$$D_{\text{Mg}} = 2.49 \times 10^5 \exp[-330,700/(8.314)(1500+273)] = 4.50 \times 10^{-15} \text{ m}^2 \cdot \text{s}^{-1}$$

$$\mu = \frac{(2)(1.6 \times 10^{-19})(4.50 \times 10^{-15})}{(1.38 \times 10^{-23})(1773)} = 5.88 \times 10^{-14} \text{ m}^2 \cdot \text{V}^{-1} \cdot \text{s}^{-1}$$

$$n = (4) / (0.396 \times 10^{-9})^3 = 6.44 \times 10^{28} \text{ m}^{-3}$$

$$\sigma = (6.44 \times 10^{28})(2)(1.6 \times 10^{-19})(5.88 \times 10^{-14}) = 1.21 \times 10^{-3} \Omega^{-1} \cdot \text{m}^{-1}$$

Conductivity increases about fifty orders of magnitude when the temperature increases to 1500°C.

18.19 Assume that most of the electrical charge transferred in Al₂O₃ is caused by the diffusion of Al³⁺ ions. Determine the mobility and electrical conductivity of Al₂O₃ at 500°C and at 1500°C. (See Table 5.1 and Example 14.1).

At 500°C = 773 K:

$$D_{Al} = 2.8 \times 10^{-3} \exp[-477200/(8.314)(773)] = 1.58 \times 10^{-35} \text{ m}^2 \cdot \text{s}^{-1}$$

$$\mu = \frac{ZqD}{kT} = \frac{(3)(1.6 \times 10^{-19})(1.58 \times 10^{-34})}{(1.38 \times 10^{-23})(773)} = 7.11 \times 10^{-34} \text{ m}^2 \cdot \text{V}^{-1} \cdot \text{s}^{-1}$$

Example 14.1 showed that there are 12 Al ions per unit cell. The volume of the unit cell is $253.82 \times 10^{-30} \text{ m}^3$. Thus the number of Al ions per m^3 is:

$$n = 12 / 253.82 \times 10^{-30} = 4.73 \times 10^{28} \text{ m}^{-3}$$

$$\sigma = nAq\mu = (4.73 \times 10^{28})(3)(1.6 \times 10^{-19})(7.1 \times 10^{-34}) = 1.61 \times 10^{-23} \Omega^{-1} \cdot \text{m}^{-1}$$

At 1500°C = 1773 K:

$$D_{Al} = 2.8 \times 10^{-3} \exp[-477200/(8.314)(1773)] = 2.44 \times 10^{-17} \text{ m}^2 \cdot \text{s}^{-1}$$

$$\mu = \frac{(3)(1.6 \times 10^{-19})(2.44 \times 10^{-17})}{(1.38 \times 10^{-23})(1773)} = 4.79 \times 10^{-16} \text{ m}^2 \cdot \text{V}^{-1} \cdot \text{s}^{-1}$$

$$n = 12 / 253.82 \times 10^{-30} = 4.73 \times 10^{28} \text{ m}^{-3}$$

$$\sigma = (4.73 \times 10^{28})(3)(1.6 \times 10^{-19})(4.79 \times 10^{-16}) = 1.09 \times 10^{-5} \Omega^{-1} \cdot \text{m}^{-1}$$

Conductivity increases about 18 orders of magnitude when the temperature increases to 1500°C.

18.20 Calculate the electrical conductivity of a fibre-reinforced polyethylene part that is reinforced with 20 vol% of continuous, aligned nickel fibres.

From Table 18.1, $\sigma_{PE} = 10^{-13} \Omega^{-1} \cdot \text{m}^{-1}$ and $\sigma_{Ni} = 1.46 \times 10^7 \Omega^{-1} \cdot \text{m}^{-1}$

$$\sigma_{\text{composite}} = f_{PE}\sigma_{PE} + f_{Ni}\sigma_{Ni} = (0.8)(10^{-13}) + (0.2)(1.46 \times 10^7) = 2.92 \times 10^6 \Omega^{-1} \cdot \text{m}^{-1}$$

18.21 For germanium, silicon, and tin, compare, at 25°C, (a) the number of charge carriers per cubic millimetre, (b) the fraction of the total electrons in the valence band that are excited into the conduction band, and (c) the constant n_0 . for germanium, silicon, and tin.

For germanium:

$$n_{Ge} = \frac{(8 \text{ atoms.cell}^{-1})(4 \text{ electrons.atom}^{-1})}{(0.56575 \times 10^{-9} \text{ m})^3} = 1.767 \times 10^{29} \text{ m}^{-3} = 1.767 \times 10^{20} \text{ mm}^{-3}$$

From Table 18.6, we can find the conductivity and mobilities for germanium. The number of excited electrons is then:

$$n_{\text{conduction}} = \sigma/q(\mu_e + \mu_h) = 2 / (1.6 \times 10^{-19})(0.38 + 0.18) = 2.23 \times 10^{19} \text{ m}^{-3} = 2.23 \times 10^{10} \text{ mm}^{-3}$$

$$\text{fraction} = 2.23 \times 10^{10} / 1.767 \times 10^{20} = 1.262 \times 10^{-10}$$

$$n_0 = n / \exp(-E_g/2kT) = 2.23 \times 10^{10} / \exp[-0.67 / (2)(8.63 \times 10^{-5})(298)] = 1.013 \times 10^{16} \text{ mm}^{-3}$$

For silicon:

$$n_{si} = \frac{(8 \text{ atoms.cell}^{-1})(4 \text{ electrons.atom}^{-1})}{(0.54307 \times 10^{-9} \text{ m})^3} = 1.998 \times 10^{29} \text{ m}^{-3}$$

$$= 1.998 \times 10^{20} \text{ mm}^{-3}$$

$$n_{\text{conduction}} = \sigma/q(\mu_e + \mu_h) = 5 \times 10^{-4} / (1.6 \times 10^{-19})(0.19 + 0.05)$$

$$= 1.302 \times 10^{16} \text{ m}^{-3} = 1.302 \times 10^7 \text{ mm}^{-3}$$

$$\text{fraction} = 1.302 \times 10^7 / 1.998 \times 10^{20} = 6.517 \times 10^{-14}$$

$$n_o = n / \exp(-E_g/2kT)$$

$$= 1.302 \times 10^7 / \exp[-1.11 / (2)(8.63 \times 10^{-5})(298)]$$

$$= 2.895 \times 10^{16} \text{ mm}^{-3}$$

For tin:

$$n_{sn} = \frac{(8 \text{ atoms.cell}^{-1})(4 \text{ electrons.atom}^{-1})}{(0.64912 \times 10^{-9} \text{ m})^3} = 1.170 \times 10^{29} \text{ m}^{-3}$$

$$= 1.170 \times 10^{20} \text{ mm}^{-3}$$

$$n_{\text{conduction}} = \sigma/q(\mu_e + \mu_h) = 9 \times 10^6 / (1.6 \times 10^{-19})(0.25 + 0.24)$$

$$= 1.148 \times 10^{26} \text{ m}^{-3} = 1.148 \times 10^{17} \text{ mm}^{-3}$$

$$\text{fraction} = 1.148 \times 10^{17} / 1.170 \times 10^{20} = 9.812 \times 10^{-4}$$

$$n_o = n / \exp(-E_g/2kT)$$

$$= 1.148 \times 10^{17} / \exp[-0.08 / (2)(8.63 \times 10^{-5})(298)]$$

$$= 5.44 \times 10^{17}$$

18.22 For germanium, silicon, and tin, compare the temperature required to double the electrical conductivity from the room temperature value.

For germanium, we wish to increase the conductivity from $2 \Omega^{-1} \cdot \text{m}^{-1}$ to $4 \Omega^{-1} \cdot \text{m}^{-1}$. From Problem 18.21, $n_o = 1.013 \times 10^{16} \text{ mm}^{-3} = 1.013 \times 10^{27} \text{ m}^{-3}$

$$\sigma = nq(\mu_e + \mu_h) = n_o q(\mu_e + \mu_h) \exp(-E_g/2kT)$$

$$4 = (1.013 \times 10^{27})(1.6 \times 10^{-19})(0.38 + 0.18) \exp[-0.67/(2)(8.63 \times 10^{-5})T]$$

$$4.407 \times 10^{-6} = \exp(-3882/T) \quad \text{or} \quad T = 315 \text{ K} = 42^\circ\text{C}$$

For silicon, we wish to increase the conductivity from 5×10^{-4} to $10 \times 10^{-4} \Omega^{-1} \cdot \text{m}^{-1}$. From Problem 18.21, $n_o = 2.895 \times 10^{25} \text{ m}^{-3}$

$$\sigma = nq(\mu_e + \mu_h) = n_o q(\mu_e + \mu_h) \exp(-E_g/2kT)$$

$$10 \times 10^{-4} = (2.895 \times 10^{25})(1.6 \times 10^{-19})(0.19 + 0.05)$$

$$\exp[-1.107/(2)(8.63 \times 10^{-5})T]$$

$$8.995 \times 10^{-10} = \exp(-6414/T)$$

$$-20.829 = -6414/T \quad \text{or} \quad T = 308 \text{ K} = 35^\circ\text{C}$$

For tin, we wish to increase the conductivity from 9×10^6 to $18 \times 10^6 \Omega^{-1} \cdot \text{m}^{-1}$. From Problem 18.21, $n_o = 5.44 \times 10^{26} \text{ m}^{-3}$.

$$\sigma = nq(\mu_e + \mu_h) = n_o q(\mu_e + \mu_h) \exp(-E_g/2kT)$$

$$18 \times 10^6 = (5.44 \times 10^{26})(1.6 \times 10^{-19})(0.25 + 0.24)$$

$$\exp[-0.08/(2)(8.63 \times 10^{-5})T]$$

$$0.422 = \exp(-463.499/T)$$

$$-0.863 = -463.499/T \quad \text{or} \quad T = 537 \text{ K} = 264^\circ\text{C}$$

18.23 When an electric field is applied to a semiconductor, 5×10^6 electrons/mm³ serve as charge carriers. When the field is removed, 30 electrons/mm³ remain after 10^{-6} s. Calculate (a) the recombination time and (b) the time required for 99.9% of the electrons and holes to recombine.

$$(a) \quad n = n_0 \exp(-t/\tau)$$

$$30 = 5 \times 10^6 \exp(-10^{-6} / \tau)$$

$$6 \times 10^{-6} = \exp(-10^{-6} / \tau)$$

$$-12.024 = -10^{-6} / \tau \quad \text{or} \quad \tau = 8.32 \times 10^{-8} \text{ s}$$

$$(b) \quad (5 \times 10^6) (0.001) = 5 \times 10^6 \exp(-t / 8.32 \times 10^{-8})$$

$$0.001 = \exp(-t / 8.32 \times 10^{-8})$$

$$-6.9078 = -t/8.32 \times 10^{-8} \quad \text{or} \quad t = 5.75 \times 10^{-7} \text{ s}$$

18.24 Calculate the number of extrinsic charge carriers per cubic metre in an n-type semiconductor when one of every 1,000,000 atoms in silicon is replaced by an antimony atom. Estimate the conductivity of the semiconductor in the exhaustion zone.

$$n = \frac{(8 \text{ atoms/cell})(1 \text{ Sb atom}/10^6 \text{ Si atoms})}{(0.54307 \times 10^{-9} \text{ m})^3} = 5 \times 10^{22} \text{ m}^{-3}$$

$$\sigma = nq\mu_e = (5 \times 10^{22})(1.6 \times 10^{-19})(0.19) = 1520 \Omega^{-1} \cdot \text{m}^{-1}$$

18.25 Determine the electrical conductivity of silicon when 0.0001 at% antimony is added as a dopant and compare it to the electrical conductivity when 0.0001 at% indium is added.

$$0.0001 \text{ at}\% = 1 \text{ impurity atom per } 10^6 \text{ host atoms.}$$

For antimony additions (an n-type semiconductor):

$$n = \frac{(8 \text{ atoms/cell})(1 \text{ Sb atom}/10^6 \text{ Si atoms})}{(0.54307 \times 10^{-9} \text{ m})^3} = 5 \times 10^{22} \text{ m}^{-3}$$

$$\sigma = nq\mu_e = (5 \times 10^{22})(1.6 \times 10^{-19})(0.19) = 1520 \Omega^{-1} \cdot \text{m}^{-1}$$

For indium additions (a p-type semiconductor):

$$n = \frac{(8 \text{ atoms/cell})(1 \text{ In atom}/10^6 \text{ Si atoms})}{(0.54307 \times 10^{-9} \text{ m})^3} = 5 \times 10^{22} \text{ m}^{-3}$$

$$\sigma = nq\mu = (5 \times 10^{22})(1.6 \times 10^{-19})(0.05) = 400 \Omega^{-1} \cdot \text{m}^{-1}$$

18.26 We would like to produce an extrinsic germanium semiconductor having an electrical conductivity of $2 \times 10^5 \Omega^{-1} \cdot \text{m}^{-1}$ in the exhaustion or saturation region. Determine the amount of phosphorus and the amount of gallium required.

For phosphorus (an n-type semiconductor):

$$n = \sigma / q\mu_e = 2 \times 10^5 / (1.6 \times 10^{-19}) (0.38) = 3.29 \times 10^{24}$$

$$3.29 \times 10^{24} = \frac{(8 \text{ atoms/cell})(x \text{ P atoms}/10^6 \text{ Ge atoms})}{(0.56575 \times 10^{-9} \text{ m})^3}$$

$$x = 74.47 \text{ P atoms}/10^6 \text{ Ge atoms} = 0.007447 \text{ at\% P}$$

For gallium (a p-type semiconductor):

$$n = \sigma / q\mu_h = 2 \times 10^5 / (1.6 \times 10^{-19}) (0.18) = 6.94 \times 10^{24}$$

$$6.94 \times 10^{24} = \frac{(8 \text{ atoms/cell})(x \text{ Ga atoms}/10^6 \text{ Ge atoms})}{(0.56575 \times 10^{-9} \text{ m})^3}$$

$$x = 157.1 \text{ Ga atoms}/10^6 \text{ Ge atoms} = 0.01571 \text{ at\% Ga}$$

18.27 Estimate the electrical conductivity of silicon doped with 0.0002 at% arsenic at 600°C, which is above the exhaustion plateau.

$$n_d = \frac{(8 \text{ atoms/cell})(2 \text{ As atoms}/10^6 \text{ Si atoms})}{(0.54307 \times 10^{-9} \text{ m})^3} = 9.99 \times 10^{22}$$

$$\text{From Problem 18.21, } n_o = 2.895 \times 10^{25} \text{ m}^{-3}$$

$$\begin{aligned} \sigma_{600} &= n_d q \mu_e + q(\mu_e + \mu_h) n_o \exp(-E_g/2kT) \\ &= (9.99 \times 10^{22}) (1.6 \times 10^{-19}) (0.19) \\ &\quad + (1.6 \times 10^{-19}) (0.19 + 0.05) (2.895 \times 10^{25}) \exp[-1.107 / (2) (8.63 \times 10^{-5}) (873)] \\ &= 3037 + 717 = 3754 \text{ } \Omega^{-1} \cdot \text{m}^{-1} \end{aligned}$$

18.28 Determine the amount of arsenic that must be combined with 1 kg of gallium to produce a p-type semiconductor with an electrical conductivity of $5 \times 10^4 \text{ } \Omega^{-1} \cdot \text{m}^{-1}$ at 25°C. The lattice parameter of GaAs is about 0.565 nm and GaAs has the zinc blende structure.

$$n = \sigma / q\mu_h = 5 \times 10^4 / (1.6 \times 10^{-19}) (0.040) = 7.81 \times 10^{24}$$

$$7.81 \times 10^{24} = \frac{(4 \text{ Ga atoms/cell})(x \text{ vacancies/Ga atom})}{(0.565 \times 10^{-9} \text{ m})^3}$$

$$x = 0.000352 \text{ vacancies / cell}$$

Therefore there are 0.999648 As atoms per one Ga atom.

$$\text{at\% As} = \frac{0.999648}{1 + 0.999648} \times 100 = 49.991\%$$

$$\text{wt\% As} = \frac{(49.991)(74.9216)}{(49.991)(74.9216) + (50.009)(69.72)} \times 100 = 51.789\%$$

$$\frac{x \text{ g As}}{x + 1000 \text{ g Ga}} \times 100 = 51.789 \quad \text{or} \quad x = 1074 \text{ g As}$$

18.29 A ZnO crystal is produced in which one interstitial Zn atom is introduced for every 500 Zn lattice sites. Estimate (a) the number of charge carriers per cubic millimeter and (b) the electrical conductivity at 25°C.

A n-type semiconductor is produced. From Example 18.8, the lattice parameter for ZnO is 0.4757 nm.

$$(a) \text{ carriers} = \frac{(4 \text{ Zn/cell})(1 \text{ interstitial}/500 \text{ Zn})(2 e^-/\text{interstitial})}{(0.4758 \times 10^{-9} \text{ m})^3}$$

$$= 1.486 \times 10^7 \text{ interstitials/mm}^3$$

$$(b) \sigma = nq\mu_e = (1.486 \times 10^{17} \text{ mm}^{-3})(10^9 \text{ mm}^3 \cdot \text{m}^{-1})(1.6 \times 10^{-19})(0.018)$$

$$= 4.28 \times 10^5 \Omega^{-1} \cdot \text{m}^{-1}$$

18.30 Each Fe^{3+} ion in FeO serves as an acceptor site for an electron. If there is one vacancy per 750 unit cells of the FeO crystal (with the sodium chloride structure), determine the number of possible charge carriers per cubic millimetre. The lattice parameter of FeO is 0.429 nm.

One vacancy requires that 2 Fe^{3+} ions be substituted for 3 Fe^{3+} ions. A hole is present for each Fe^{3+} ion. In 750 unit cells, there are $4 \times 750 = 3000$ Fe sites in the NaCl-type crystal structure:

$$\text{carriers} = \frac{(4 \text{ Fe sites/cell})(2 \text{ Fe}^{3+}/3000 \text{ Fe sites})(1 \text{ hole}/\text{Fe}^{3+})}{(0.429 \times 10^{-9} \text{ m})^3}$$

$$= 3.3775 \times 10^{25} \text{ m}^{-3} = 3.3775 \times 10^{16} \text{ mm}^{-3}$$

18.31 When a voltage of 5 mV is applied to the emitter of a transistor, a current of 2 mA is produced. When the voltage is increased to 8 mV, the current through the collector rises to 6 mA. By what percentage will the collector current increase when the emitter voltage is doubled from 9 mV to 18 mV?

First we can find the constants I_0 and B in Equation 18.22.

$$\frac{2 \text{ mA}}{6 \text{ mA}} = \frac{I_0 \exp(5 \text{ mV}/B)}{I_0 \exp(8 \text{ mV}/B)}$$

$$0.333 = \exp(-3/B)$$

$$-1.0986 = -3/B \quad \text{or} \quad B = 2.73 \text{ mV}$$

$$I_0 = 2 / \exp(5/2.73) = 0.32 \text{ mA}$$

$$\text{At } 9 \text{ mV, } I = 0.32 \exp(9/2.73) = 8.647 \text{ mA}$$

$$\text{At } 18 \text{ mV, } I = 0.32 \exp(18/2.73) = 233.685 \text{ mA}$$

Therefore the percentage increase in the collector current is:

$$\Delta = \frac{233.685 - 8.647}{8.647} \times 100 = 2600\%$$

18.32 A 0.5 mm diameter fibre 10 mm in length made of boron nitride is placed into a 120-V circuit. Calculate (a) the current flowing in the circuit and (b) the number of electrons passing through the boron nitride fibre per second. What would the current and number of electrons be if the fibre were made of magnesium instead of boron nitride? (see Table 18.1)

$$V/I = R = \ell / (\pi/4) d^2 \sigma$$

(a) For boron nitride:

$$120 / I = (1 \times 10^{-2} \text{ m}) / (\pi/4) (0.5 \times 10^{-3} \text{ m})^2 (10^{-11} \Omega^{-1} \cdot \text{m}^{-1})$$

$$I = 2.36 \times 10^{-14} \text{ A} = 2.36 \times 10^{-14} \text{ C/s}$$

(b) As $1A = 1C.s^{-1}$ the electrons moving through the material per second are:

$$\begin{aligned} \text{electrons/s} &= 2.36 \times 10^{-14} \text{ C/s} / 1.6 \times 10^{-19} \text{ C/electron} \\ &= 1.475 \times 10^5 \end{aligned}$$

(c) For magnesium:

$$120 / I = (1 \times 10^{-2} \text{ m}) / (\pi/4) (0.5 \times 10^{-3} \text{ m})^2 (2.25 \times 10^7 \Omega^{-1} \cdot \text{m}^{-1})$$

$$I = 5.3 \times 10^4 \text{ A} = 5.3 \times 10^4 \text{ C/s}$$

$$\begin{aligned} \text{electrons/s} &= 5.3 \times 10^4 \text{ C/s} / 1.6 \times 10^{-19} \text{ C/electron} \\ &= 3.3 \times 10^{23} \end{aligned}$$

18.33 Calculate the displacement of the electrons or ions for the following conditions:

- electronic polarization in nickel of $2 \times 10^{-7} \text{ C.m}^{-2}$
- electronic polarization in aluminium of $2 \times 10^{-8} \text{ C.m}^{-2}$
- ionic polarization in NaCl of $4.3 \times 10^{-8} \text{ C.m}^{-2}$
- ionic polarization in ZnS of $5 \times 10^{-8} \text{ C.m}^{-2}$

Z is the number of charge centers per m^3 :

(a) For FCC nickel, $a_0 = 0.35167 \text{ nm}$ and the atomic number is 28:

$$Z = \frac{(4 \text{ atoms/cell})(28 \text{ electrons/atom})}{(3.5167 \times 10^{-10} \text{ m})^3} = 2.58 \times 10^{30}$$

$$d = P/Zq = (2 \times 10^{-7} \text{ C/m}^2) / (2.58 \times 10^{30} \text{ m}^3) (1.6 \times 10^{-19} \text{ C/electron})$$

$$d = 4.84 \times 10^{-19} \text{ m}$$

(b) For FCC aluminium, $a_0 = 0.404988 \text{ nm}$ and the atomic number is 13:

$$Z = \frac{(4 \text{ atoms/cell})(13 \text{ electrons/atom})}{(4.04988 \times 10^{-10} \text{ m})^3} = 0.78 \times 10^{30}$$

$$d = P/Zq = (2 \times 10^{-8}) / (0.78 \times 10^{30}) (1.6 \times 10^{-19})$$

$$d = 1.603 \times 10^{-19} \text{ m}$$

(c) For NaCl, $a_0 = 0.55 \text{ nm}$ and there is one charge per ion. There are 4 of each type of ion per cell. The lattice parameter is:

$$a_0 = 2r_{\text{Na}} + 2r_{\text{Cl}} = 2(0.097) + 2(1.81) = 0.556 \text{ nm}$$

$$Z = \frac{(4 \text{ Na ions/cell})(1 \text{ charge/ion})}{(5.56 \times 10^{-10} \text{ m})^3} = 0.024 \times 10^{30}$$

$$d = P/Zq = (4.3 \times 10^{-8}) / (0.024 \times 10^{30}) (1.6 \times 10^{-19})$$

$$d = 1.12 \times 10^{-17} \text{ m}$$

(d) For ZnS, $a_0 = 0.596 \text{ nm}$ and there are two charges per ion. There are 4 of each type of ion per cell. The lattice parameter is:

$$a_0 = (4r_{\text{Zn}} + 4r_{\text{S}}) / \sqrt{3} = [(4)(0.074) + (4)(0.184)] / \sqrt{3} = 0.596 \text{ nm}$$

$$Z = \frac{(4 \text{ ZnS ions/cell})(2 \text{ charge/ion})}{(5.96 \times 10^{-10} \text{ m})^3} = 0.038 \times 10^{30}$$

$$d = P/Zq = (5 \times 10^{-8}) / (0.038 \times 10^{30}) (1.6 \times 10^{-19})$$

$$d = 8.22 \times 10^{-18} \text{ m}$$

18.34 A 2 mm thick alumina dielectric is used in a 60-Hz circuit. Calculate the voltage required to produce a polarization of $5 \times 10^{-7} \text{ C.m}^{-2}$.

$$P = (\kappa - 1)\epsilon_0\xi = (\kappa - 1)\epsilon_0V/\ell \quad \text{where } \ell = 2 \text{ mm} = 0.002 \text{ m}$$

$$5 \times 10^{-7} = (9 - 1)(8.85 \times 10^{-12})V / 2 \times 10^{-3}$$

$$V = 14.1 \text{ volts}$$

18.35 Suppose we are able to produce a polarization of $5 \times 10^{-5} \text{ C.m}^{-2}$ in a 5 mm cube of barium titanate. What voltage is produced?

$$P = (\kappa - 1)\epsilon_0 \xi = (\kappa - 1)\epsilon_0V/\ell \quad \text{where } \ell = 5 \text{ mm} = 0.005 \text{ m}$$

$$5 \times 10^{-5} = (3000 - 1)(8.85 \times 10^{-12})V / 0.005$$

$$V = 9.4 \text{ volts}$$

18.36 Calculate the thickness of polyethylene required to store the maximum charge in a 24,000 V circuit without breakdown.

$$\xi_{\max} = 2 \times 10^7 \text{ Vm}^{-2} = 24,000 / \ell$$

$$\ell = 0.0012 \text{ m} = 1.2 \text{ mm}$$

18.37 Calculate the maximum voltage that can be applied to a 1 mm thick barium titanate dielectric without causing breakdown.

$$\xi_{\max} = 1.2 \times 10^7 \text{ V.m}^{-1} = V / 0.001 \text{ m}$$

$$V = 12,000 \text{ volts}$$

18.38 A 120 V circuit is to operate at 10^6 Hz . A Teflon film 10 mm in diameter is a part of the circuit. (a) What thickness of Teflon is required to assure a power loss of no more than 1 W? (b) What thickness would be required if the dielectric was made of polyvinyl chloride?

$$P_L = 5.556 \times 10^{-11} \kappa \tan\delta (V/\ell)^2 f v = 1 \text{ watt}$$

or

$$\ell = 5.556 \times 10^{-11} \kappa \tan\delta V^2 f v / P_L$$

For the problem, $f = 10^6 \text{ Hz}$

$$v = (\pi/4) (0.01 \text{ m})^2 \ell = (7.854 \times 10^{-5}) (\ell) \text{ m}^3$$

$$V = 120 \text{ volts}$$

(a) For Teflon:

$$\ell = (5.556 \times 10^{-11}) (2.1) (0.00007) (120)^2 (10^6) (7.854 \times 10^{-5}) / 1$$

$$\ell = 9.24 \times 10^{-9} \text{ m}$$

(b) For polyvinyl chloride:

$$\ell = (5.556 \times 10^{-11}) (3.2) (0.05) (120)^2 (10^6) (7.854 \times 10^{-5}) / 1$$

$$\ell = 1.005 \times 10^{-5} \text{ m}$$

18.39 Calculate the capacitance of a parallel plate capacitor containing 5 layers of mica, where each mica sheet is 10 mm x 20 mm x 0.05 mm.

$$C = \epsilon_o(\kappa) (n - 1)A/d$$

$$= (8.85 \times 10^{-12} \text{ F.m}^{-1}) (7) (6 - 1) (1 \times 10^{-2} \text{ m} \times 2 \times 10^{-2} \text{ m}) / (5 \times 10^{-5} \text{ m})$$

$$= 1.239 \times 10^{-9} \text{ F} = 0.001239 \mu\text{F}$$

18.40 Determine the number of Al_2O_3 sheets, each 15 mm x 15 mm x 0.01 mm, required to obtain a capacitance of 0.0142 μF in a 10^6 Hz parallel plate capacitor.

$$n - 1 = Cd / \epsilon_o \kappa A$$

$$n - 1 = (0.0142 \times 10^{-6} \text{ F}) (0.01 \times 10^{-3} \text{ m}) / (8.85 \times 10^{-12} \text{ F.m}^{-1}) (6.5) (0.015 \text{ m})^2$$

$$n - 1 = 11 \text{ Al}_2\text{O}_3 \text{ sheets} \quad \text{and} \quad n = 12 \text{ conductor plates}$$

18.41 We would like to construct a barium titanate device with a 2.5 mm diameter that will produce a voltage of 250 V when a 20 N is applied. How thick should the device be?

$$E = 69 \text{ GN.m}^{-2}$$

$$d = 100 \times 10^{-12} \text{ m.V}^{-1}$$

If t = thickness of the device, F is the applied force, and A is the area of the device, then:

$$\xi = V/t = g\sigma = \sigma/Ed = F/AEd$$

$$t = VAEd/F$$

$$= (250 \text{ V}) (\pi/4) (2.5 \times 10^{-3} \text{ m})^2 (69 \times 10^9) (100 \times 10^{-12} \text{ m.V}^{-1}) / 20 \text{ N}$$

$$= 0.423 \text{ mm}$$

18.42 A force of 90 N is applied to the face of a 5 mm x 5 mm x 1 mm thick quartz crystal. Determine the voltage produced by the force. The modulus of elasticity of quartz is 71.7 GN.m⁻².

$$\xi = V/t = \sigma/Ed = F/AEd$$

$$V = Ft/AEd = \frac{(90 \text{ N}) (1 \times 10^{-3} \text{ m})}{(5 \times 10^{-3} \text{ m}) (5 \times 10^{-3} \text{ m}) (71.7 \times 10^9 \text{ N.m}^{-2}) (2.3 \times 10^{-12} \text{ m.V}^{-1})}$$

$$= 21,830 \text{ volts}$$

18.43 Determine the strain produced when a 300 V signal is applied to a barium titanate wafer 2 mm x 2 mm x 0.1 mm thick.

$$d = 100 \times 10^{-12} \text{ m.V}^{-1}$$

$$\epsilon = d\xi = (100 \times 10^{-12} \text{ m.V}^{-1}) (300 \text{ V}/0.1 \times 10^{-3} \text{ m}) = 0.0003$$

18.44 Figure 18.38 shows the hysteresis loop for two ferroelectric materials.

- Determine the voltage required to eliminate polarisation in a 1 mm thick dielectric made from Material A.
- Determine the thickness of a dielectric made from Material B if 10 V is required to eliminate polarisation.
- What electric field is required to produce a polarisation of $8 \times 10^{-8} \text{ C.m}^{-2}$ in material A; and what is the dielectric constant at this polarisation?
- An electric field of 2500 V.m^{-1} is applied to material B. Determine the polarisation and the dielectric constant at this electric field.

(a) coercive field = 4000 V.m^{-1}

$$V = (4000 \text{ V.m}^{-1}) (0.001 \text{ m}) = 4 \text{ volts}$$

(b) coercive field = 3500 V.m^{-1}

$$\text{thickness} = 10 \text{ V} / 3500 \text{ V.m}^{-1} = 0.002857 \text{ m} = 2.857 \text{ mm}$$

(c) field = 5000 V.m^{-1}

$$P = (\kappa - 1)\epsilon_0\xi \quad \text{or} \quad \kappa = 1 + P/\epsilon_0\xi$$

$$\kappa = 1 + (8 \times 10^{-8} \text{ C.m}^{-2}) / (8.85 \times 10^{-12} \text{ F.m}^{-1}) (5000 \text{ V.m}^{-1})$$

$$\kappa = 2.81$$

(d) polarization = $12 \times 10^{-8} \text{ C.m}^{-2}$

$$\kappa = 1 + P/\epsilon_0\xi$$

$$\kappa = 1 + (12 \times 10^{-8} \text{ C.m}^{-2}) / (8.85 \times 10^{-12} \text{ F.m}^{-1}) (2500 \text{ V.m}^{-1})$$

$$\kappa = 6.42$$

Chapter 19 Magnetic Behaviour of Materials

19.1 Calculate and compare the maximum magnetisation we would expect in iron, nickel, cobalt, and gadolinium. There are seven electrons in the 4f level of gadolinium.

Iron: The number of atoms/m³ is:

$$\frac{2 \text{ atoms/cell}}{(2.866 \times 10^{-10} \text{ m})^3} = 0.085 \times 10^{30} \text{ atoms/m}^3$$

$$M = (0.085 \times 10^{30}) (4 \text{ magnetons/atom}) (9.27 \times 10^{-24} \text{ A m}^2) \\ = 3.15 \times 10^6 \text{ A.m}^{-1}$$

Nickel: The number of atoms/m³ is:

$$\frac{4 \text{ atoms/cell}}{(3.5167 \times 10^{-10} \text{ m})^3} = 0.09197 \times 10^{30} \text{ atoms/m}^3$$

$$M = (0.09197 \times 10^{30}) (2 \text{ magnetons/atom}) (9.27 \times 10^{-24} \text{ A m}^2) \\ = 1.705 \times 10^6 \text{ A.m}^{-1}$$

Cobalt: The number of atoms/m³ is:

$$\frac{2 \text{ atoms/cell}}{(2.5071 \times 10^{-10} \text{ m})^2 (4.0686 \times 10^{-10} \text{ m}) \cos 30} = 0.0903 \times 10^{30} \text{ atoms/m}^3$$

$$M = (0.0903 \times 10^{30}) (3 \text{ magnetons/atom}) (9.27 \times 10^{-24} \text{ A m}^2) \\ = 2.51 \times 10^6 \text{ A.m}^{-1}$$

Gadolinium: The number of atoms/m³ is:

$$\frac{2 \text{ atoms/cell}}{(3.6336 \times 10^{-10} \text{ m})^2 (5.781 \times 10^{-10} \text{ m}) \cos 30} = 0.0303 \times 10^{30} \text{ atoms/m}^3$$

$$M = (0.0303 \times 10^{30}) (7 \text{ magnetons/atom}) (9.27 \times 10^{-24} \text{ A m}^2) \\ = 1.96 \times 10^6 \text{ A.m}^{-1}$$

19.2 An alloy of nickel and cobalt is to be produced to give a magnetisation of $2 \times 10^6 \text{ A.m}^{-1}$. The crystal structure of the alloy is FCC with a lattice parameter of 0.3544 nm. Determine the atomic percent cobalt required, assuming no interaction between the nickel and cobalt.

Let f_{Ni} be the atomic fraction of nickel; $1 - f_{Ni}$ is then the atomic fraction of cobalt. The numbers of Bohr magnetons per cubic metre due to nickel and to cobalt atoms are:

$$\text{Ni: } (4 \text{ atoms/cell}) (2 \text{ magnetons/atom}) f_{Ni} / (3.544 \times 10^{-10} \text{ m})^3 \\ = 0.1797 \times 10^{30} f_{Ni}$$

$$\text{Co: } (4 \text{ atoms/cell}) (3 \text{ magnetons/atom}) (1 - f_{Ni}) / (3.544 \times 10^{-10} \text{ m})^3 \\ = 0.2696 \times 10^{30} (1 - f_{Ni})$$

The magnetisation, M , is given by:

$$M = [(0.1797 \times 10^{30})f_{Ni} + (0.2696 \times 10^{30})(1-f_{Ni})](9.27 \times 10^{-24})$$

$$M = -833373 f_{Ni} + 2499192 = 2 \times 10^6$$

$$\therefore f_{Ni} = 0.60 \quad f_{Co} = 0.40$$

19.3 Estimate the magnetisation that might be produced in an alloy containing nickel and 70 at% copper, assuming that no interaction occurs.

We can estimate the lattice parameter of the alloy from those of the pure nickel and copper and their atomic fractions:

$$a_0 = (0.3)(0.3294) + (0.7)(0.36151) = 0.3519 \text{ nm}$$

If the copper does not provide magnetic moments that influence magnetisation, then

$$M = \frac{(4 \text{ atoms/cell})(0.3 \text{ fraction Ni})(2 \text{ magnetons/Ni atom})(9.27 \times 10^{-24})}{(3.519 \times 10^{-10} \text{ m})^3}$$

$$M = 5.10 \times 10^5 \text{ A.m}^{-1}$$

19.4 An Fe-80% Ni alloy has a maximum relative permeability of 300,000 when an inductance of 0.35 tesla is obtained. The alloy is placed in a 20 turn coil that is 20 mm in length. What current must flow through the conductor coil to obtain this field?

$$\text{Magnetic permeability } \mu = \mu_r \mu_0 = 300000 \times 4\pi \times 10^{-7} = 0.377 \text{ T.m.A}^{-1}.$$

$$\text{but, } B = \mu H \quad \therefore H = (0.35 \text{ T}) / (0.377 \text{ T.m.A}^{-1}) = 0.928 \text{ A.m}^{-1}$$

$$\text{Then, } I = Hl/n = (0.928 \text{ A.m}^{-1})(20 \times 10^{-3} \text{ m}) / (20 \text{ turns}) = 0.00093 \text{ A}$$

19.5 An Fe-49% Ni alloy has a maximum permeability of 64,000 when a magnetic field of 10 A.m⁻¹ is applied. What inductance is obtained and what current is needed to obtain this inductance in a 200 turn, 30 mm long coil?

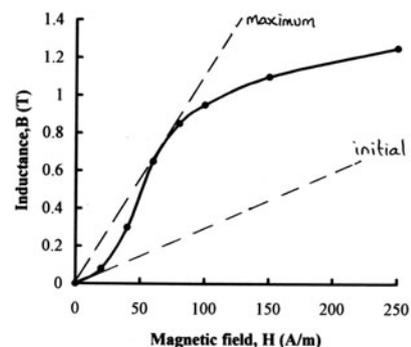
$$\mu = \mu_r \mu_0 = 64,000 \times 4\pi \times 10^{-7} = 0.0804 \text{ T.m.A}^{-1}$$

$$B = \mu H = (0.0804 \text{ T.m.A}^{-1})(10 \text{ A.m}^{-1}) = 0.804 \text{ T}$$

$$I = Hl/n = (10 \text{ A.m}^{-1})(30 \times 10^{-3} \text{ m}) / (200 \text{ turns}) = 1.5 \text{ mA}$$

19.6 The following data describe the effect of the magnetic field on the inductance in a silicon steel. Calculate (a) the initial permeability and (b) the maximum permeability for the material.

$H (\text{A.m}^{-1})$	$B (\text{T})$
0	0
20	0.08
40	0.3
60	0.65
80	0.85
100	0.95
150	1.10
250	1.25



The data is plotted; from the graph, the initial and maximum permeability are calculated from the slopes of the plots:

(a) initial permeability = 0.003 T.m.A^{-1}

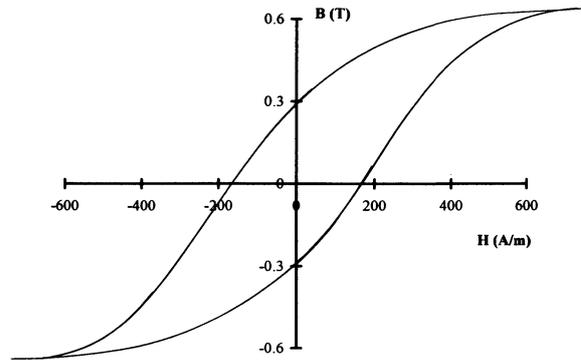
(b) maximum permeability = 0.011 T.m.A^{-1}

19.7 A magnetic material has a coercive field of 167 A.m^{-1} , a saturation magnetization of 0.616 tesla , and a residual inductance of 0.3 tesla . Sketch the hysteresis loop for the material.

$$M_{sat} = B_{sat} = 0.616 \text{ T}$$

$$B_r = 0.3 \text{ T}$$

$$H_c = 167 \text{ A.m}^{-1}$$

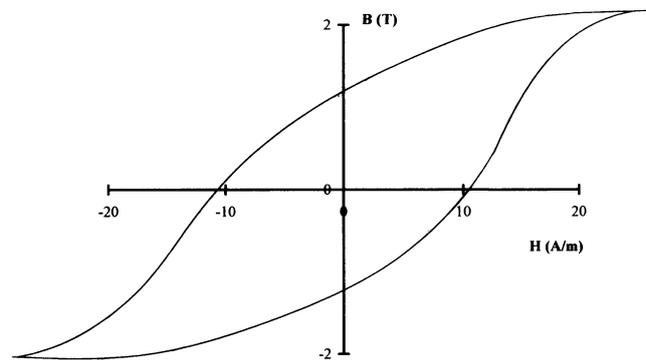


19.8 A magnetic material has a coercive field of 10.74 A.m^{-1} , a saturation magnetization of 2.158 tesla , and a remanence induction of 1.183 tesla . Sketch the hysteresis loop for the material.

$$B_{sat} = M_{sat} = 2.158 \text{ T}$$

$$B_r = 1.183 \text{ T}$$

$$H_c = 10.74 \text{ A.m}^{-1}$$



19.9 Using Figure 19.16, determine the following properties of the magnetic material.

- | | |
|-----------------------------|-------------------------------|
| a. remanence | d. initial permeability |
| b. saturation magnetisation | e. maximum permeability |
| c. coercive field | f. power (maximum BH product) |

(a) remanence = 1.3 T

(b) saturation magnetisation = 1.4 T

(c) coercive field = 63500 A.m^{-1}

(d) initial permeability = $0.7 \text{ T} / 100000 \text{ A.m}^{-1} = 7 \times 10^{-6} \text{ T.m.A}^{-1}$

(e) maximum permeability = $1.4 \text{ T} / 75000 \text{ A.m}^{-1} = 1.9 \times 10^{-5} \text{ T.m.A}^{-1}$

(f) we can try several BH products in the 4th quadrant:

$$\begin{aligned} 1.2 \text{ T} \times 35800 &= 42960 \text{ T.A.m}^{-1} \\ 1.0 \text{ T} \times 54600 &= 54600 \text{ T.A.m}^{-1} \\ 0.8 \text{ T} \times 57600 &= 46080 \text{ T.A.m}^{-1} \end{aligned}$$

The maximum BH product, or power, is about 54600 T.A.m⁻¹.

19.10 Using Figure 19.17, determine the following properties of the magnetic material.

- | | |
|-----------------------------|-------------------------------|
| a. remanence | d. initial permeability |
| b. saturation magnetisation | e. maximum permeability |
| c. coercive field | f. power (maximum BH product) |

(a) remanence = 0.55 T

(b) saturation magnetisation = 0.58 T

(c) coercive field = 44,000 A.m⁻¹

(d) initial permeability = 0.2 T / (50000 A.m⁻¹)
= 4 x 10⁻⁶ T.m.A⁻¹

(e) maximum permeability = 0.5 T / (35000 A.m⁻¹)
= 1.4 x 10⁻⁵ T.m.A⁻¹

(f) we can try several BH products in the 4th quadrant:

$$\begin{aligned} 0.45 \text{ T} \times 23000 \text{ A.m}^{-1} &= 10350 \text{ T.A.m}^{-1} \\ 0.40 \text{ T} \times 32800 \text{ A.m}^{-1} &= 13120 \text{ T.A.m}^{-1} \\ 0.35 \text{ T} \times 36100 \text{ A.m}^{-1} &= 12635 \text{ T.A.m}^{-1} \\ 0.30 \text{ T} \times 38975 \text{ A.m}^{-1} &= 11692 \text{ T.A.m}^{-1} \end{aligned}$$

The maximum BH product, or power, is about 13120 T.A.m⁻¹.

19.11 Estimate the power of the Co₅Ce material shown in Figure 19.14.

H	B	BH
0 A.m ⁻¹	0.75 T	0 T.A.m ⁻¹
200000 A.m ⁻¹	0.65 T	130000 T.A.m ⁻¹
225000 A.m ⁻¹	0.45 T	101250 T.A.m ⁻¹
250000 A.m ⁻¹	0 T	0 T.A.m ⁻¹

19.12 Why are eddy current losses important design factors in ferromagnetic materials but less important in ferrimagnetic materials?

The ferrimagnetic materials are ceramics, which have a very low electrical conductivity; consequently eddy currents are not introduced into the magnetic material. The ferromagnetic materials are metals or alloys; they are conductors, permitting eddy currents to be introduced and thus causing eddy current losses.

19.13 What advantage does the Fe-3% Si material have compared to Supermalloy for use in electric motors?

The Fe-3% Si has a larger saturation inductance than Supermalloy, allowing more work to be done. However Fe-3% Si does require larger fields, since the coercive field for Fe-3% Si is large, and the per-

meability of Fe-3% Si is small compared with that of Supermalloy.

19.14 The coercive field for pure iron is related to the grain size of the iron by the relationship $H_c = 1.83 + 4.14/\sqrt{A}$, where A is the area of the grain in two dimensions (mm^2) and H_c is in A.m^{-1} . If only the grain size influences the 99.95% iron given in Table 19.3, estimate the size of the grains in the material. What happens when the iron is annealed to increase the grain size?

$$H_c = 71.60 \text{ A.m}^{-1}$$

Thus, from the equation,

$$71.60 = 1.83 + 4.14 / \sqrt{A}$$

$$\sqrt{A} = 4.14 / 69.77 = 0.0593 \quad \text{or} \quad A = 0.0035 \text{ mm}^2$$

When the iron is annealed, the grain size increases, A increases, and the coercive field H_c decreases.

19.15 Suppose we replace 10% of the Fe^{2+} ions in magnetite with Cu^{2+} ions. Determine the total magnetic moment per cubic metre.

From Example 19.6, the lattice parameter is $8.37 \times 10^{-10} \text{ m}$.

$$V_{\text{unit cell}} = (8.37 \times 10^{-10} \text{ m})^3 = 5.86 \times 10^{-28} \text{ m}^3$$

In the octahedral sites, the fraction of copper atoms is 0.1, while the fraction of Fe^{2+} ions is 0.9. The magnetic moment is then:

$$\text{moment} = \frac{(8 \text{ subcells}) [0.1 \text{ Cu}(1 \text{ magneton}) + 0.9 \text{ Fe}(4 \text{ magneton})] (9.27 \times 10^{-24} \text{ A.m}^{-1})}{5.86 \times 10^{-28} \text{ m}^3}$$

$$\text{moment} = 4.68 \times 10^5 \text{ A.m}^2 \text{ per m}^3$$

19.16 Suppose that the total magnetic moment per cubic metre in a spinel structure in which Ni^{2+} ions have replaced a portion of the Fe^{2+} ions is $4.6 \times 10^5 \text{ A.m}^{-1}$. Calculate the fraction of the Fe^{2+} ions that have been replaced and the wt% Ni present in the spinel.

From Example 19.6, the volume of the unit cell is $5.86 \times 10^{-28} \text{ m}^3$. If we let f_{Ni} be the fraction of the octahedral sites occupied by nickel, then $(1-f_{\text{Ni}})$ is the fraction of the sites occupied by iron. Then:

$$\text{moment} = 4.6 \times 10^5 =$$

$$\frac{(8 \text{ subcells}) [(1-f_{\text{Ni}})(4 \text{ magnetons}) + (f_{\text{Ni}})(2 \text{ magnetons})] (9.27 \times 10^{-24})}{5.86 \times 10^{-28} \text{ m}^3}$$

$$3.6348 = 4 - 4f_{\text{Ni}} + 2f_{\text{Ni}}$$

$$\therefore f_{\text{Ni}} = 0.183$$

Thus the number of each type of atom or ion in the unit cell is:

$$\begin{aligned} \text{oxygen:} & (4 \text{ atoms/subcell}) (8 \text{ subcells}) = 32 \\ \text{Fe}^{3+} & (2 \text{ ions/subcell}) (8 \text{ subcells}) = 16 \\ \text{Fe}^{2+} & (0.817) (1 \text{ ion/subcell}) (8 \text{ subcells}) = 6.54 \\ \text{Ni}^{2+} & (0.183) (1 \text{ ion/subcell}) (8 \text{ subcells}) = 1.46 \end{aligned}$$

The total number of ions in the unit cell is 56; the atomic fraction of each ion is:

$$f_{\text{oxygen}} = 32/56 = 0.5714 \quad f_{\text{Fe}^{3+}} = 16/56 = 0.2857$$

$$f_{\text{Fe}^{2+}} = 6.54/56 = 0.1168 \quad f_{\text{Ni}^{2+}} = 1.48/56 = 0.0261$$

The weight percent nickel is (using the molecular weights of oxygen, iron and nickel):

$$\%Ni = \frac{(0.0261)(58.71)}{(0.5714)(16) + (0.2857)(55.847) + (0.1168)(55.847) + (0.0261)(58.71)}$$

$$= 4.62 \text{ wt}\%$$

Chapter 20 Optical Behaviour

20.1 What voltage must be applied to a tungsten filament to produce a continuous spectrum of X-rays having a minimum wavelength of 0.09 nm?

$$E = \frac{hc}{\lambda} = \frac{(6.62 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m}\cdot\text{s}^{-1})}{(0.09 \times 10^{-9} \text{ m})} = 2.206 \times 10^{-15} \text{ J}$$

$$E = (2.206 \times 10^{-15} \text{ J}) / (1.6 \times 10^{-19} \text{ J/eV}) = 13790 \text{ V}$$

20.2 A tungsten filament is heated with a 12,400 V power supply; what is (a) the wavelength and (b) frequency of the highest energy X-rays that are produced?

$$E = (12,400 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV}) = 1.984 \times 10^{-15} \text{ J}$$

$$1.984 \times 10^{-15} \text{ J} = hc/\lambda = \frac{(6.62 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m}\cdot\text{s}^{-1})}{\lambda}$$

$$(a) \quad \lambda = 1.00 \times 10^{-10} \text{ m} = 0.100 \text{ nm}$$

$$(b) \quad \nu = c/\lambda = 3 \times 10^8 \text{ m}\cdot\text{s}^{-1} / 1.00 \times 10^{-10} \text{ m} = 3.0 \times 10^{18} \text{ s}^{-1}$$

20.3 What is the minimum voltage required to produce K_α X-rays in nickel?

The wavelength of K_α X-rays in nickel is 0.166 nm = $1.66 \times 10^{-10} \text{ m}$

$$E = \frac{(6.62 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m}\cdot\text{s}^{-1})}{(1.66 \times 10^{-10} \text{ m})(1.6 \times 10^{-19} \text{ J}\cdot\text{eV}^{-1})} = 7477 \text{ V}$$

20.4 Based on the characteristic X rays that are emitted, determine the difference in energy between electrons in (a) the K and L shells, (b) the K and M shells, and (c) the L and M shells of tungsten.

The energy difference between the K and L shells produces K_α X-rays. The wavelength of these X-rays is 0.0211 nm:

$$E (K-L) = \frac{(6.62 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m}\cdot\text{s}^{-1})}{(2.11 \times 10^{-11} \text{ m})} = 9.41 \times 10^{-15} \text{ J}$$

The energy difference between the K and M shells produces K_β X-rays. The wavelength of these X-rays is 0.0184 nm:

$$E (K-M) = \frac{(6.62 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m}\cdot\text{s}^{-1})}{(18.4 \times 10^{-11} \text{ m})} = 1.08 \times 10^{-14} \text{ J}$$

The energy difference between the L and M shells produces L_α X-rays. The wavelength of these X-rays is 0.1476 nm:

$$E (L-M) = \frac{(6.62 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m}\cdot\text{s}^{-1})}{(1.476 \times 10^{-10} \text{ m})} = 1.35 \times 10^{-15} \text{ J}$$

20.5 Figure 20.21 shows the results of an X-ray fluorescent analysis, in which the energy of X-rays emitted from a material are plotted relative to the wavelength of the X-rays. Determine (a) the accelerating voltage used to produce the exciting X-rays and (b) the identity of the elements in the sample.

(a) The highest energy X-rays produced have a wavelength (λ_{sw1}) of about 0.05 nm. The accelerating voltage is therefore:

$$E = \frac{(6.62 \times 10^{-34} \text{ J}\cdot\text{s}) (3 \times 10^8 \text{ m}\cdot\text{s}^{-1})}{(0.05 \times 10^{-9} \text{ m}) (1.6 \times 10^{-19} \text{ J}\cdot\text{eV}^{-1})} = 24,825 \text{ V}$$

(b) The wavelengths of the characteristic X-rays are listed below. By comparison with the wavelengths of characteristic X-rays from different elements, Table 20.1, we can match the observed X-rays with the X-rays of the elements to obtain the composition of the sample.

<u>observed</u>	<u>expected</u>	<u>element</u>
0.14 nm	0.1392 nm	Cu K_{β}
1.555	0.1542	Cu K_{α}
0.19	0.1910	Mn K_{β}
0.21	0.2104	Mn K_{α}
0.67	0.6768	Si K_{β}
0.71	0.7125	Si K_{α}

The alloy must contain copper, manganese, and silicon.

20.6 Figure 20.22 shows the energies of X-rays produced from an energy-dispersive analysis of radiation emitted from a specimen in a scanning electron microscope. Determine the identity of the elements in the sample.

The energy of the first observed peak is about 2200 eV; the wavelength corresponding to this energy is:

$$\lambda = hc/E = \frac{(6.62 \times 10^{-34} \text{ J}\cdot\text{s}) (3 \times 10^8 \text{ ms}^{-1})}{3.52 \times 10^{-16} \text{ J}}$$

$$= 5.642 \times 10^{-10} \text{ m} = 0.564 \text{ nm}$$

Similarly we can find the wavelength corresponding to the energies of the other characteristic peaks. The table below lists the energies and calculated wavelengths for each peak and compares the wavelength to the characteristic radiation for different elements, from Table 20.1.

<u>energy</u>	<u>calculated λ</u>	<u>expected λ</u>	<u>element</u>
$3.52 \times 10^{-16} \text{ J}$	0.564 nm	0.5724 nm	Mo L_{α}
$8.40 \times 10^{-16} \text{ J}$	0.2364	0.2291	Cr K_{α}
$9.60 \times 10^{-16} \text{ J}$	0.2069	0.2084	Cr K_{β}
$11.20 \times 10^{-16} \text{ J}$	0.1773	0.1790	Co K_{α}
$12.50 \times 10^{-16} \text{ J}$	0.1591	0.1621	Co K_{β}
$27.70 \times 10^{-16} \text{ J}$	0.0717	0.0711	Mo K_{α}
$31.50 \times 10^{-16} \text{ J}$	0.0630	0.0632	Mo K_{β}

The sample must contain molybdenum, chromium, and cobalt.

20.7 CaWO_4 has a relaxation time of $4 \times 10^{-6} \text{ s}$. Determine the time required for the intensity of this phosphorescent material to decrease to 1% of the original intensity after the stimulus is removed.

$$\ln (I/I_0) = - t/\tau$$

$$\begin{aligned} \ln(0.01) &= -t / 4 \times 10^{-6} \text{ s} \\ -4.605 &= -t / 4 \times 10^{-6} \\ t &= 18.4 \times 10^{-6} \text{ s} \end{aligned}$$

20.8 The intensity of a phosphorescent material is reduced to 90% of its original intensity after 1.95×10^{-7} s. Determine the time required for the intensity to decrease to 1% of its original intensity.

We can use the information in the problem to find the relaxation time for the material.

$$\begin{aligned} \ln(I/I_0) &= \ln(0.9) = - (1.95 \times 10^{-7}) / \tau \\ -0.1054 &= - (1.95 \times 10^{-7}) / \tau \\ \tau &= 1.85 \times 10^{-6} \text{ s} \end{aligned}$$

Then we can find the time required to reduce the intensity to $I/I_0 = 0.01$:

$$\begin{aligned} \ln(0.01) &= -t / 1.85 \times 10^{-6} \\ -4.605 &= -t / 1.85 \times 10^{-6} \\ t &= 8.52 \times 10^{-6} \text{ s} \end{aligned}$$

20.9 By appropriately doping yttrium aluminium garnet with neodymium, electrons are excited within the 4f energy shell of the Nd atoms. Determine the approximate energy transition if the Nd:YAG serves as a laser, producing a wavelength of 532 nm. What color would the laser beam possess?

The energy transition is:

$$E = \frac{(6.62 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m}\cdot\text{s}^{-1})}{(532 \times 10^{-9} \text{ m})} = 1.35 \times 10^{-15} \text{ J}$$

The wavelength of 532 nm corresponds to a colour of green.

20.10 Determine whether an incident beam of photons with a wavelength of 750 nm will cause luminescence in the following materials (see Table 18.8).

- a. ZnO b. GaP c. GaAs d. GaSb e. PbS

The incident beam must have an energy greater than the energy gap of the material in order for luminescence to occur. The energy of the incident photons is:

$$E = \frac{(6.62 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m}\cdot\text{s}^{-1})}{(750 \times 10^{-9} \text{ m})(1.6 \times 10^{-19} \text{ J}\cdot\text{eV}^{-1})} = 1.655 \text{ eV}$$

From Table 18.8, the energy gaps of the five materials are:

ZnO: 3.2 eV
GaP: 2.24 eV
GaAs: 1.35 eV
GaSb: 0.67 eV
PbS: 0.37 eV

Consequently the photons, having energy 1.655 eV, will be able to excite

electrons in GaAs, GaSb, and PbS; however electrons will not be excited in ZnO and GaP.

20.11 Determine the wavelength of photons produced when electrons excited into the conduction band of indium-doped silicon (a) drop from the conduction band to the acceptor band and (b) then drop from the acceptor band to the valence band. (see Table 18.7)

The acceptor energy in Si-In is 0.16 eV; the energy gap in pure Si is 1.107 eV. The difference between the energy gap and the acceptor energy level is $1.107 - 0.16 = 0.947$ eV.

(a) The wavelength of photons produced when an electron drops from the conduction band to the acceptor band, an energy difference of 0.947 eV, is:

$$\lambda = hc/E = \frac{(6.62 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m}\cdot\text{s}^{-1})}{(0.947 \text{ eV})(1.6 \times 10^{-19} \text{ J}\cdot\text{eV}^{-1})} = 1.311 \times 10^{-6} \text{ m}$$

(b) The wavelength of photons produced when the electron subsequently drops from the acceptor band to the valence band, an energy difference of 0.16 eV, is:

$$\lambda = hc/E = \frac{(6.62 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m}\cdot\text{s}^{-1})}{(0.16 \text{ eV})(1.6 \times 10^{-19} \text{ J}\cdot\text{eV}^{-1})} = 7.758 \times 10^{-6} \text{ m}$$

20.12 Which, if any, of the semiconducting compounds listed in Table 18-8 are capable of producing an infrared laser beam?

Infrared radiation has a wavelength of between 10^{-4} and 10^{-6} m. Thus the semiconducting compound must have an energy gap that lies between the energies corresponding to these wavelength limits:

$$E = \frac{(6.62 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m}\cdot\text{s}^{-1})}{(10^{-4} \text{ m})(1.6 \times 10^{-19} \text{ J}\cdot\text{eV}^{-1})} = 0.0124 \text{ eV}$$

$$E = \frac{(6.62 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m}\cdot\text{s}^{-1})}{(10^{-6} \text{ m})(1.6 \times 10^{-19} \text{ J}\cdot\text{eV}^{-1})} = 1.24 \text{ eV}$$

Of the semiconducting compounds in Table 18.8, the following have energy gaps between 0.0124 and 1.24 eV and can therefore act as infrared lasers:

InSb InAs PbS PbTe CdSnAs₂

20.13 What type of electromagnetic radiation (ultraviolet, infrared, visible) is produced from (a) pure germanium and (b) germanium doped with phosphorus? (See Tables 18.6 and 18.7)

(a) For pure germanium, the energy gap is 0.67 eV; the wavelength is:

$$\lambda = \frac{(6.62 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m}\cdot\text{s}^{-1})}{(0.67 \text{ eV})(1.6 \times 10^{-19} \text{ J}\cdot\text{eV}^{-1})} = 1.853 \times 10^{-6} \text{ m}$$

This corresponds to the infrared region of the spectrum.

(b) For Ge doped with phosphorus, the energy gap is 0.012 eV.

$$\lambda = \frac{(6.62 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m}\cdot\text{s}^{-1})}{(0.012 \text{ eV})(1.6 \times 10^{-19} \text{ J}\cdot\text{eV}^{-1})} = 1.034 \times 10^{-4} \text{ m}$$

This wavelength is also in the infrared region.

20.14 Which, if any, of the dielectric materials listed in Table 18.9 would reduce the speed of light in air from $3 \times 10^8 \text{ m.s}^{-1}$ to less than $0.5 \times 10^8 \text{ m.s}^{-1}$?

To reduce the speed of light the required amount, the index of refraction must be greater than:

$$n = c/v = 3 \times 10^8 \text{ m.s}^{-1} / 0.5 \times 10^8 \text{ m.s}^{-1} = 6$$

Consequently the dielectric constant κ of the material must be greater than:

$$\kappa = n^2 = 6^2 = 36$$

From Table 18.9, only H_2O , BaTiO_3 , and TiO_2 have dielectric constants greater than 36.

20.15 A beam of photons strikes a material at an angle of 25° to the normal of the surface. Which, if any, of the materials listed in Table 20.2 could cause the beam of photons to continue at an angle of 18° to 20° from the normal of the material's surface?

Assuming that the beam originally is passing through air or a vacuum,

$$n = \sin \alpha / \sin \beta = \sin 25^\circ / \sin \beta$$

To exit at an angle of 18° :

$$n = \sin 25^\circ / \sin 18^\circ = 0.4226 / 0.3090 = 1.367$$

To exit at an angle of 20° :

$$n = \sin 25^\circ / \sin 20^\circ = 0.4226 / 0.3420 = 1.236$$

In Table 20.2, only ice, water, and Teflon have an index of refraction between 1.236 and 1.367.

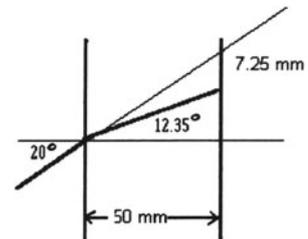
20.16 A laser beam passing through air strikes a 50 mm thick polystyrene block at a 20° angle to the normal of the block. By what distance is the beam displaced from its original path when the beam reaches the opposite side of the block?

The index of refraction for polystyrene is 1.60. Since the incident angle α is 20° , the angle of the beam as it passes through the polystyrene block will be:

$$n = \sin \alpha / \sin \beta = \sin 20^\circ / \sin \beta = 1.6$$

$$\sin \beta = 0.3420 / 1.6 = 0.2138$$

$$\beta = 12.35^\circ$$



From the sketch, we can find the displacement of the beam expected if no refraction occurs:

$$\tan 20^\circ = x / 5 \quad \text{or} \quad x = 5 \tan 20^\circ = (50) (0.3640) = 18.20 \text{ mm}$$

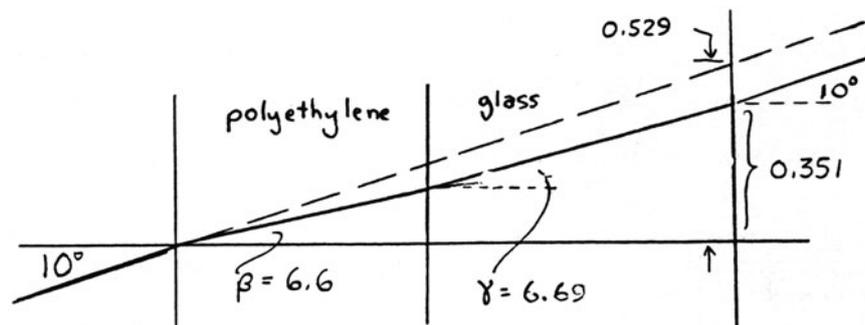
We can also find the displacement of the beam when refraction occurs:

$$\tan 12.35^\circ = y / 5 \quad \text{or} \quad y = 5 \tan 12.35^\circ = (5)(0.2189) = 10.95 \text{ mm}$$

Because of refraction, the beam is displaced $18.20 - 10.95 = 7.25 \text{ mm}$ from its path had no refraction occurred.

20.17 A beam of photons in air strikes a composite material consisting of a 10 mm thick sheet of polyethylene and a 20 mm thick sheet of soda-lime glass. The incident beam is 10° from the normal of the composite. Determine the angle of the beam with respect to the normal as the beam (a) passes through the polyethylene, (b) passes through glass, and (c) passes through air on the opposite side of the composite. (d) By what distance is the beam displaced from its original path when it emerges from the composite?

The figure shows how the beam changes directions, and the amount that the beam is displaced from the normal to the point of entry, as it passes through each interface.



(a) As the beam passes from air into polyethylene (which has an index of refraction of 1.52),

$$\sin \beta = \sin \alpha / n = \sin 10^\circ / 1.52 = 0.1736 / 1.52 = 0.1142$$

$$\beta = 6.6^\circ$$

(b) When the beam enters the glass (which has an index of refraction of 1.50), the new angle is:

$$n_g / n_{PE} = \sin \beta / \sin \gamma$$

$$1.50 / 1.52 = \sin 6.6^\circ / \sin \gamma$$

$$\sin \gamma = 0.11647 \quad \text{or} \quad \gamma = 6.69^\circ$$

(c) When the beam emerges from the glass back into air, the final angle is:

$$n_{air} / n_g = \sin \gamma / \sin \kappa$$

$$1.00 / 1.50 = \sin 6.69^\circ / \sin \kappa$$

$$\sin \kappa = 0.1747 \quad \text{or} \quad \kappa = 10^\circ$$

(d) When the beam reaches the polyethylene-glass interface, it has been displaced:

$$\tan 6.6^\circ = x / 10 \text{ mm} \quad \text{or} \quad x = 1.16 \text{ mm}$$

When the beam then reaches the glass-air interface, it has been displaced an additional:

$$\tan 6.69^\circ = y / 20 \text{ mm} \quad \text{or} \quad y = 2.35 \text{ mm}$$

The total displacement is therefore $x + y = 3.51 \text{ mm}$. If the beam had not been refracted, the displacement would have been:

$$\tan 10^\circ = z / 30 \text{ mm} \quad \text{or} \quad z = 5.29 \text{ mm}$$

The beam has therefore been displaced $5.29 - 3.51 = 1.78 \text{ mm}$ from its original path.

20.18 A glass fibre ($n = 1.5$) is coated with PTFE. Calculate the maximum angle that a beam of light can deviate from the axis of the fibre without escaping from the inner portion of the fibre.

To keep the beam from escaping from the fibre, the angle β must be 90° . Therefore the maximum angle that the incoming beam can deviate from the fibre axis is:

$$n_{\text{teflon}} / n_{\text{glass}} = \sin \alpha / \sin \beta$$

$$1.35 / 1.50 = \sin \alpha / \sin 90^\circ$$

$$\sin \alpha = 0.90 \quad \text{or} \quad \alpha = 64.16^\circ$$

The maximum angle is therefore $90 - 64.16 = 25.84^\circ$.

20.19 A material has a linear absorption coefficient of $5.91 \times 10^4 \text{ m}^{-1}$ for photons of a particular wavelength. Determine the thickness of the material required to absorb 99.9% of the photons.

$$I / I_0 = 0.001 = \exp(-\mu x) = \exp(-5.91 \times 10^4 x)$$

$$\ln(0.001) = -6.9078 = -5.91 \times 10^4 x$$

$$x = 0.117 \text{ mm}$$

20.20 What filter material would you use to isolate the K_α peak of the following X rays -- iron, manganese, nickel? Explain your answer.

Iron: use a manganese filter. The absorption edge for Mn is 0.1896 nm, which lies between the iron K_α peak of 0.1937 nm and the K_β peak of 0.1757 nm.

Manganese: use a chromium filter. The absorption edge for Cr is 0.2070 nm, which lies between the manganese K_α peak of 0.2104 nm and the K_β peak of 0.1910 nm.

Nickel: use a cobalt filter. The absorption edge for Co is 0.1608 nm, which lies between the nickel K_α peak of 0.1660 nm and the K_β peak of 0.1500 nm.

20.21 A beam of photons passes through air and strikes a soda-lime glass that is part of an aquarium containing water. What fraction of the beam is reflected by the front face of the glass? What fraction of the remaining beam is reflected by the back face of the glass?

The fraction of the beam reflected by the front face is:

$$R = \frac{n_{\text{glass}} - n_{\text{air}}}{n_{\text{glass}} + n_{\text{air}}}^2 = \frac{1.50 - 1.00}{1.50 + 1.00}^2 = 0.04$$

The fraction of the remaining beam reflected from the back face of the glass is:

$$R = \frac{n_{\text{water}} - n_{\text{glass}}}{n_{\text{water}} + n_{\text{glass}}}^2 = \frac{1.33 - 1.50}{1.33 + 1.50}^2 = 0.0036$$

20.22 We find that 20% of the original intensity of a beam of photons is transmitted from air through a 10 mm thick material having a dielectric constant of 2.3 and back into air. Determine (a) the fraction of the beam that is (a) reflected at the front surface, (b) absorbed in the material, and (c) reflected at the back surface. (d) Determine the linear absorption coefficient of the photons in the material.

The dielectric material has an index of refraction of:

$$n = \sqrt{\kappa} = \sqrt{2.3} = 1.5166$$

(a) The fraction of the beam reflected at the front surface is:

$$R = \frac{n_{\text{material}} - n_{\text{air}}}{n_{\text{material}} + n_{\text{air}}}^2 = \frac{1.5166 - 1.00}{1.5166 + 1.00}^2 = 0.04214$$

(b) The fraction transmitted through the material is 0.2; therefore the linear absorption coefficient of the materials is:

$$I_t/I_o = (1-R)^2 \exp(-\mu x) \\ = (1 - 0.04214)^2 \exp[-\mu(10 \times 10^{-3} \text{ m})] = 0.20$$

$$\exp(-10 \times 10^{-3} \mu) = 0.21798$$

$$-10 \times 10^{-3} \mu = \ln(0.21798) = -1.523$$

$$\mu = 152.3 \text{ m}^{-1}$$

After reflection, the intensity of the remaining beam is

$$I_{\text{after reflection}} = 1 - 0.04215 = 0.95785I_o$$

Before reflection at the back surface, the intensity of the beam is:

$$I_{\text{after absorption}} = 0.95785 \exp[(-1.523)(1)] = 0.2089I_o$$

The fraction of the beam that is absorbed is therefore

$$I_{\text{absorbed}} = 0.95785 - 0.2089 = 0.74895I_o$$

(c) The fraction of the beam reflected off the back surface is:

$$I_o = I_{\text{reflected, front}} + I_{\text{absorbed}} + I_{\text{reflected, back}} + I_{\text{transmitted}}$$

$$I_o = 0.04214I_o + 0.74895I_o + I_{\text{reflected, back}} + 0.20I_o$$

$$I_{\text{reflected, back}} = 0.0089I_o$$

(d) See part b; $\mu = 152.3 \text{ m}^{-1}$

20.23 Figure 20.23 shows the intensity of the radiation obtained from a copper X-ray generating tube as a function of wavelength. The accompanying table shows the linear absorption coefficient for a nickel filter for several wavelengths. If the Ni filter is 0.05 mm thick, calculate and plot the

intensity of the transmitted X-ray beam versus wavelength.

The intensity after absorption is

$$I/I_0 = \exp(-\mu x) = \exp(-0.05 \times 10^{-3} \mu)$$

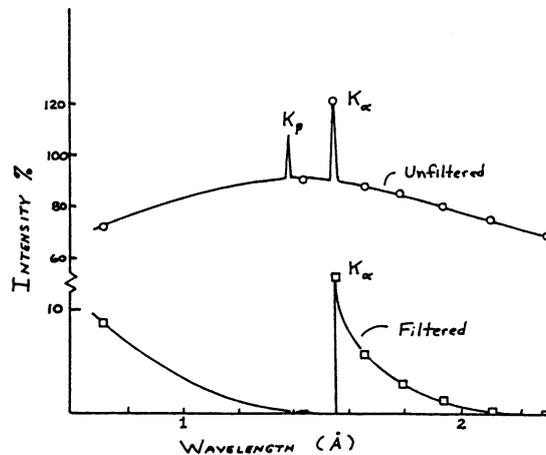
We can then select various wavelengths of X-rays and, from the table, determine the μ for each wavelength. From our equation, we can then calculate the I/I_0 expected for each wavelength. Finally we can multiply I/I_0 by the initial intensity, obtained from Figure 20.23. For $\lambda = 0.0711$ nm, these calculations are:

$$\mu = 42200 \text{ m}^{-1} \quad I_f/I_0 = \exp[(-42200)(0.05 \times 10^{-3})] = 0.121$$

$$I_0 = 72 \quad I_f = (0.121)(72) = 8.7$$

λ nm	μ/m^{-1}	I_0	I/I_0	I_f
0.0711	42200	72	0.121	8.7
0.1436	290000	90	5.04×10^{-7}	0.000045
0.1542	44000	120	0.11	13.3
0.1659	54300	88	0.066	5.8
0.179	67000	86	0.035	3
0.1937	83000	80	0.016	1.3
0.2103	103000	75	0.006	0.4
0.2291	130000	68	0.00125	0.1

The graph compares the original intensity to the final, filtered intensity of the X-ray beam. Note that the characteristic K_β peak from the copper is eliminated, while much of the K_α peak is transmitted.



Chapter 21 Thermal Properties of Materials

21.1 Calculate the heat required to raise the temperature of 1 kg of the following materials by 50°C.

- a. lead b. nickel c. Si₃N₄ d. 6,6-nylon

The heat is the specific heat times the weight times the temperature change. Calories can be converted to joules by multiplying by 4.184.

$$(a) c_{Pb} = (159 \text{ J.kg}^{-1}.\text{K}^{-1}) (1 \text{ kg}) (50 \text{ K}) = 7,950 \text{ J}$$

$$(b) c_{Ni} = (444 \text{ J.kg}^{-1}.\text{K}^{-1}) (1 \text{ kg}) (50 \text{ K}) = 22200 \text{ J}$$

$$(c) c_{\text{silicon nitride}} = (712 \text{ J.kg}^{-1}.\text{K}^{-1}) (1 \text{ kg}) (50 \text{ K}) = 35600 \text{ J}$$

$$(d) c_{6,6 \text{ nylon}} = (1674 \text{ J.kg}^{-1}.\text{K}^{-1}) (1 \text{ kg}) (50 \text{ K}) = 83700 \text{ J}$$

21.2 Calculate the temperature of a 100 g sample of the following materials, originally at 25°C, when 12500 J are introduced.

- a. tungsten b. titanium c. Al₂O₃ d. low density polyethylene

$$\Delta T = \frac{\text{Heat}}{(c) (\text{mass})} \text{ and } \Delta T = T_f - T_0$$

$$(a) \text{ W: } \Delta T = 12500 / (134 \times 0.1) = 932.8 \text{ K} = 932.8 \text{ }^\circ\text{C}$$

$$T_f = \Delta T + T_0 = 932.8 + 25 = 957.8 \text{ }^\circ\text{C}$$

$$(b) \text{ Ti: } \Delta T = 12500 / (523 \times 0.1) = 239.0 \text{ K} = 239.0 \text{ }^\circ\text{C}$$

$$\therefore T_f = 264 \text{ }^\circ\text{C}$$

$$(c) \text{ Al}_2\text{O}_3: \Delta T = 12500 / (837 \times 0.1) = 149.3 \text{ K} = 149.3 \text{ }^\circ\text{C}$$

$$\therefore T_f = 174.3 \text{ }^\circ\text{C}$$

$$(d) \text{ LDPE: } \Delta T = 12500 / (2302 \times 0.1) = 54.3 \text{ K} = 54.3 \text{ }^\circ\text{C}$$

$$T_f = 79.3 \text{ }^\circ\text{C}$$

21.3 An alumina insulator for an electrical device is also to serve as a heat sink. A 10°C temperature rise in an alumina insulator 10 mm x 10 mm x 0.2 mm is observed during use. Determine the thickness of a high density polyethylene insulator that would be needed to provide the same performance as a heat sink. The density of alumina is 3.96 Mg.m⁻³.

The heat absorbed by the alumina is:

$$\text{heat} = (837 \text{ J.kg}^{-1}.\text{K}^{-1}) (10 \text{ K}) (3.96 \times 10^3 \text{ kg.m}^{-3}) (10^{-2} \text{ m} \times 2 \times 10^{-2} \text{ m} \times 10^{-4} \text{ m})$$

$$= 6.63 \times 10^{-1} \text{ J}$$

The same amount of heat must be absorbed by the polyethylene, which has a density of about 0.96 Mg.m⁻³:

$$\text{heat} = (1842 \text{ J.kg}^{-1}.\text{K}^{-1}) (10 \text{ K}) (0.96 \times 10^3 \text{ kg.m}^{-3}) (10^{-2} \text{ m} \times 10^{-2} \text{ m} \times t)$$

$$6.63 \times 10^{-1} \text{ J} = 1768 t$$

$$t = 3.75 \times 10^{-4} \text{ m} = 0.375 \text{ mm}$$

21.4 A 200 g sample of aluminium is heated to 400°C and is then quenched into 2 litres of water at 20°C. Calculate the temperature of the water after the aluminium and water reach equilibrium. Assume no temperature losses from the system.

The amount of heat gained by the water equals the amount lost by the aluminium. If the equilibrium temperature is T_e :

$$(900 \text{ J.kg}^{-1}.\text{K}^{-1}) (400 - T_e) (0.2 \text{ kg}) = (4186 \text{ J.kg}^{-1}.\text{K}^{-1}) (T_e - 20) (2 \text{ kg})$$

$$72000 - 180 T_e = 8372 T_e - 167,440$$

$$T_e = 29^\circ\text{C}$$

21.5 A 2 m long soda lime glass sheet is produced at 1400°C. Determine its length after it cools to 25°C.

$$\Delta l = l_o \alpha \Delta T = (2 \text{ m}) (9 \times 10^{-6} \text{ }^\circ\text{C}^{-1}) (1400 - 25)$$

$$\Delta l = 0.02475 \text{ m}$$

$$l_o = l_f - \Delta l = 2 - 0.02475 = 1.975 \text{ m}$$

21.6 A copper casting is to be produced having the final dimensions of 25 mm x 300 mm x 600 mm. Determine the size of the pattern that must be used to make the mould into which the liquid copper is poured during the manufacturing process.

$$\Delta l = l_o \alpha (T_m - T_o) = l_o (1.66 \times 10^{-5}) (1084.9 - 25)$$

$$\Delta l = 0.01759 l_o$$

$$l_f = l_o + 0.01759 l_o$$

$$l_f = 600 + (0.01759) (600) = 610.6 \text{ mm}$$

$$l_f = 300 + (0.01759) (300) = 305.3 \text{ mm}$$

$$l_f = 25 + (0.01759) (25) = 25.4 \text{ mm}$$

21.7 An aluminium casting is made by the permanent mould process. In this process, the liquid aluminium is poured into a grey cast iron mould that is heated to 350°C. We wish to produce an aluminium casting that is 375 mm long at 25°C. Calculate the length of the cavity that must be machined into the grey cast iron mould.

The aluminium casting shrinks between the solidification temperature (660.4°C) and room temperature (25°C). However, the grey cast iron mould expands when it is heated from 25°C to 350°C during the casting process. The original length of the cavity in the mould is therefore given by the amount of contraction of the aluminium minus the amount of expansion of the mould:

$$l = l_o + \Delta l_{\text{grey iron}} - \Delta l_{\text{aluminium}}$$

$$375 = l_o + l_o [(1.2 \times 10^{-5}) (350 - 25) - (2.5 \times 10^{-5}) (660.4 - 25)]$$

$$375 = l_o + 0.0039 l_o - 0.015885 l_o = 0.988 l_o$$

$$l_o = 379.6 \text{ mm}$$

21.8 We coat a 1 m long, 2 mm diameter copper wire with a 0.5 mm thick epoxy insulation coating. Determine the length of the copper and the coating when their temperature increases from 25°C to 250°C. What is likely to happen to the epoxy coating as a result of this heating?

Both the copper and the epoxy expand when heated. The final length of each material, assuming that they are not bonded to one another, would be:

$$l_f = \Delta l \alpha \Delta T + l_o$$

$$l_{Cu} = (1 \text{ m}) (1.66 \times 10^{-5}) (250 - 25) + 1 = 1.003735 \text{ m}$$

$$l_{epoxy} = (1 \text{ m}) (5.5 \times 10^{-5}) (250 - 25) + 1 = 1.012375 \text{ m}$$

The epoxy expands nearly 1 cm more than does the underlying copper. If the copper and epoxy are well bonded, the epoxy coating will buckle, debond, and perhaps even flake off.

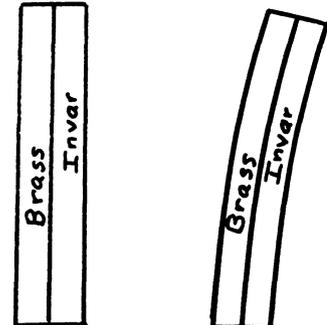
21.9 We produce a 250 mm long bimetallic composite material composed of a strip of yellow brass bonded to a strip of Invar. Determine the length to which each material would like to expand when the temperature increases from 20°C to 150°C. Draw a sketch showing what will happen to the shape of the bimetallic strip.

If the two metals are not bonded to one another, the amount each would like to expand is:

$$l_{brass} = 250 + (250) (1.89 \times 10^{-5}) (150 - 20) = 250.614 \text{ mm}$$

$$l_{Invar} = 250 + (250) (1.54 \times 10^{-6}) (150 - 20) = 250.050 \text{ mm}$$

The brass expands more than the Invar; if the two are bonded together, the bimetallic strip will bend, with the Invar on the inside radius of curvature of the strip.



21.10 A nickel engine part is coated with SiC to provide corrosion resistance at high temperatures. If no residual stresses are present in the part at 20°C, determine the thermal stresses that develop when the part is heated to 1000°C during use. (see Table 14.4)

The net difference in the amount of expansion is given by:

$$\Delta \alpha = \alpha_{nickel} - \alpha_{SiC} = (13 - 4.3) \times 10^{-6} = 8.7 \times 10^{-6}$$

The thermal stresses $\sigma_{thermal}$ in the SiC coating are:

$$\sigma_{thermal} = E \Delta \alpha \Delta T$$

$$= (414 \times 10^3 \text{ MN.m}^{-2}) (8.7 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}) (1000 - 20) = 3530 \text{ MN.m}^{-2}$$

The nickel expands more than the SiC; therefore the stresses acting on the SiC are tensile stresses. The tensile strength of SiC is only on the order of 175 MN.m⁻² (Table 14.4), so the coating will likely crack.

21.11 Alumina fibres 20 mm long are incorporated into an aluminium matrix. Assuming good bonding between the ceramic fibres and the aluminium, estimate the thermal stresses acting on the fibre when the temperature of the composite increases 250°C. Are the stresses on the fibre tensile or compressive? (see Table 14.4)

The net difference in the expansion coefficients of the two materials is:

$$\Delta\alpha = \alpha_{Al} - \alpha_{alumina} = (25 - 6.7) \times 10^{-6} = 18.3 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$$

The thermal stresses on the alumina are:

$$\begin{aligned} \sigma_{\text{thermal}} &= E\Delta\alpha\Delta T \\ &= (386 \times 10^3 \text{ MN.m}^{-2})(18.3 \times 10^{-6} \text{ } ^\circ\text{C}^{-1})(250^\circ\text{C}) = 1766 \text{ MN.m}^{-2} \end{aligned}$$

The aluminium expands more than the alumina; thus the alumina fibres are subjected to tensile stresses. The alumina has a tensile strength of only about 210 MN.m⁻² (Table 14.4); consequently the fibres are expected to crack.

21.12 A 600 mm long copper bar with a yield strength of 210 MN.m⁻² is heated to 120°C and immediately fastened securely to a rigid framework. Will the copper deform plastically during cooling to 25°C? How much will the bar deform if it is released from the framework after cooling?

If room temperature is 25°C, then the thermal stresses that develop in the restrained copper as it cools is:

$$\begin{aligned} \sigma_{\text{thermal}} &= E\alpha\Delta T = (124.83 \times 10^3 \text{ MN.m}^{-2})(1.66 \times 10^{-5})(120 - 25) \\ \sigma_{\text{thermal}} &= 197 \text{ MN.m}^{-2} \end{aligned}$$

The thermal stresses are less than the yield strength; consequently, no plastic deformation occurs in the copper as it cools. When the copper is released from its restraint, the residual stresses will be relieved by elastic deformation. The strain stored in the material by contraction will be:

$$\epsilon = \sigma/E = 197 / 124.83 \times 10^3 = 0.001577 \text{ m.m}^{-1}$$

The change in length of the copper bar is

$$\Delta l = (600 \text{ mm})(0.001577 \text{ mm.mm}^{-1}) = 0.95 \text{ mm}$$

21.13 Repeat problem 21.12, but using a silicon carbide rod rather than a copper rod. (See Table 14.4)

SiC has a modulus of 414 x 10³ MN.m⁻² (Table 14.4). The thermal stresses are:

$$\begin{aligned} \sigma_{\text{thermal}} &= E\alpha\Delta T = (414 \times 10^3 \text{ MN.m}^{-2})(4.3 \times 10^{-6})(120 - 25) \\ \sigma_{\text{thermal}} &= 169 \text{ MN.m}^{-2} \end{aligned}$$

The thermal stresses are less than the tensile strength of SiC (about 175 MN.m⁻², Table 14.4). Thus the elastic strain stored in the SiC is:

$$\epsilon = \sigma/E = 169 / 414 \times 10^3 = 0.0004082 \text{ m.m}^{-1}$$

The change in length of the SiC rod is

$$\Delta l = (600) (0.0004082 \text{ m.m}^{-1}) = 0.24 \text{ mm}$$

21.14 A 30 mm thick plate of silicon carbide separates liquid aluminium (held at 700°C) from a water cooled steel shell maintained at 20°C. Calculate the heat Q transferred to the steel per mm² of silicon carbide each second.

The temperature change through the $\Delta x = 30$ mm thick SiC is $\Delta T = 700 - 20 = 680^\circ\text{C}$. The temperature gradient is thus:

$$\Delta T/\Delta x = 680 / 30 = 22.67 \text{ }^\circ\text{C.mm}^{-1}$$

The thermal conductivity is $88 \text{ W.m}^{-1}.\text{K}^{-1} = 88 \times 10^3 \text{ W.mm}^{-1}.\text{K}^{-1}$ thus:

$$Q/A = (88 \times 10^3) (22.67) = 1.995 \text{ J.s}^{-1}.\text{mm}^{-2}$$

21.15 A sheet of 0.25 mm thick polyethylene is sandwiched between two 900 mm x 900 mm x 3 mm sheets of soda-lime glass to produce a window. Calculate (a) the heat lost through the window each day when room temperature is 25°C and the outside air is 0°C and (b) the heat entering through the window each day when room temperature is 25°C and the outside air is 40°C.

The rule of mixtures will allow us to calculate the thermal conductivity of this laminar composite. The volume fractions of each constituent are determined from the thicknesses:

$$f_{PE} = 0.25 \text{ mm} / (0.25 + 3 + 3) = 0.25 / 6.25 = 0.04$$

$$f_g = (2)(3) / 6.25 = 0.96$$

$$\frac{1}{K_c} = f_{PE} \frac{1}{K_{PE}} + f_g \frac{1}{K_g}$$

$$1 / K = 0.04 / 0.33 + 0.96 / 0.96 = 1.212$$

$$K = 1/1.212 = 0.8919 \text{ W.m}^{-1}.\text{K}^{-1}$$

The surface area of the glass is 0.9 m x 0.9 m, or $A = 0.81 \text{ m}^2$

The thickness of the composite is:

$$\Delta x = (0.25 \times 10^{-3}\text{m} + 3 \times 10^{-3}\text{m} + 3 \times 10^{-3}\text{m}) = 6.25 \times 10^{-3} \text{ m}$$

(a) The heat loss to the outside is:

$$\begin{aligned} Q &= K_{\text{composite}} A \Delta T / \Delta x \\ &= (0.8919 \text{ W.m}^{-1}.\text{K}^{-1}) (0.81 \text{ m}^2) (25 \text{ K} / 6.25 \times 10^{-3}\text{m}) \\ &= 2890 \text{ W} = 2890 \text{ J.s}^{-1} \end{aligned}$$

or:

$$Q = (2890 \text{ J.s}^{-1}) (3600 \text{ s/h}) (24 \text{ h/day}) = 2.5 \times 10^8 \text{ J per day}$$

(b) The heat entering the room from outside is:

$$\begin{aligned} Q &= (0.8919 \text{ W.m}^{-1}.\text{K}^{-1}) (0.81 \text{ m}^2) (40 - 25) / 6.25 \times 10^{-3} \\ &= 1734 \text{ W} = 1734 \text{ J.s}^{-1} \\ &= 1.5 \times 10^8 \text{ J per day} \end{aligned}$$

21.16 We would like to build a heat deflection plate that permits heat to be transferred rapidly parallel to the sheet but very slowly perpendicular to the sheet. Consequently we incorporate 1 kg of copper wires, each 1 mm in diameter, into 5 kg of a polyimide polymer matrix. Estimate the thermal conductivity parallel and perpendicular to the sheet.

We can first calculate the volume fractions of the two constituents in the composite. The volume of each material is:

$$V_{Cu} = 1000 \text{ g} / 8.93 \text{ Mg.m}^{-3} = 111.98 \times 10^{-6} \text{ m}^3$$

$$V_{PI} = 5000 \text{ g} / 1.14 \text{ Mg.m}^{-3} = 4385.96 \times 10^{-6} \text{ m}^3$$

$$f_{Cu} = 111.98 \times 10^{-6} / (111.98 \times 10^{-6} + 4385.96 \times 10^{-6}) = 0.025$$

$$f_{PI} = 0.975$$

Parallel to the wires:

$$K = (0.025)(402 \text{ W.m}^{-1}.\text{K}^{-1}) + (0.975)(0.21 \text{ W.m}^{-1}.\text{K}^{-1})$$

$$= 10.25 \text{ W.m}^{-1}.\text{K}^{-1}$$

Perpendicular to the wires:

$$1 / K = 0.025 / 402 + 0.975 / 0.21 = 6.22 \times 10^{-5} + 4.64286 = 4.64292$$

$$K = 0.2154 \text{ W.m}^{-1}.\text{K}^{-1}$$

The thermal conductivity is much higher parallel to the conductive copper wires than perpendicular to the wires.

21.17 Suppose we just dip a 10 mm diameter, 100 mm long rod of aluminium into one litre of water at 20°C. The other end of the rod is in contact with a heat source operating at 400°C. Determine the length of time required to heat the water to 25°C if 75% of the heat is lost by radiation from the bar.

The heat required to raise the temperature of the water by 5°C is:

$$\text{Heat} = (4186 \text{ J.kg}^{-1}.\text{K}^{-1})(1 \text{ kg})(25 - 20) = 20930 \text{ J}$$

However, since 75% of the heat is lost by radiation, we must supply a total of

$$\text{Heat} = 4 \times 20930 = 83720 \text{ J}$$

The heat flux Q is Joules per area per time; thus

$$\text{Heat} / t = KA\Delta T / \Delta x$$

$$\frac{83720}{t} = \frac{(238 \text{ W.m}^{-1}.\text{K}^{-1})(\pi/4)(10 \times 10^{-3} \text{ m})^2(400 - 20)}{(100 \times 10^{-3} \text{ m})}$$

$$t = 1179 \text{ s} = 19.6 \text{ min}$$

21.18 Determine the thermal shock parameter for silicon nitride, hot pressed silicon carbide, and alumina and compare it with the thermal shock resistance as defined by the maximum quenching temperature difference. (See Table 14.4)

For Si_3N_4 :

$$\text{TSP} = \sigma_f K / E\alpha = \frac{(550 \text{ MN.m}^{-2})(15 \text{ W.m}^{-1}.\text{K}^{-1})}{(310 \times 10^3 \text{ MN.m}^{-2})(3.3 \times 10^{-6} \text{ K}^{-1})} = 8064 \text{ J.s}^{-1}.\text{m}^{-1}$$

For SiC:

$$TSP = \sigma_f K / E\alpha = \frac{(175 \text{ MN.m}^{-2})(88 \text{ W.m}^{-1}.\text{K}^{-1})}{(414 \times 10^3 \text{ MN.m}^{-2})(4.3 \times 10^{-6} \text{ K}^{-1})} = 8651 \text{ J.s}^{-1}.\text{m}^{-1}$$

For alumina:

$$TSP = \sigma_f K / E\alpha = \frac{(210 \text{ MN.m}^{-2})(16 \text{ W.m}^{-1}.\text{K}^{-1})}{(386 \times 10^3 \text{ MN.m}^{-2})(6.7 \times 10^{-6} \text{ K}^{-1})} = 1299 \text{ J.s}^{-1}.\text{m}^{-1}$$

The maximum quenching difference for silicon nitride is 500°C, for silicon carbide is 350°C, and for alumina is 200°C. The maximum quenching difference correlates reasonably well with the thermal shock parameter.

21.19 Grey cast iron has a higher thermal conductivity than ductile or malleable cast iron. Review Chapter 12 and explain why this difference in conductivity might be expected.

The thermal conductivities of the constituents in the cast irons are:

$$\begin{aligned} K_{\text{graphite}} &= 335 \text{ W.m}^{-1}.\text{K}^{-1} \\ K_{\text{ferrite}} &= 75 \text{ W.m}^{-1}.\text{K}^{-1} \\ K_{\text{cementite}} &= 50 \text{ W.m}^{-1}.\text{K}^{-1} \end{aligned}$$

The grey cast iron contains interconnected graphite flakes, while the graphite nodules in ductile and malleable iron are not interconnected. Graphite, or carbon, has a higher thermal conductivity than does the "steel" matrix of the cast iron. Consequently heat can be transferred more rapidly through the iron-graphite "composite" structure of the grey iron than through the ductile and malleable irons.

Chapter 22 Corrosion and Wear

22.1 A grey cast iron pipe is used in the natural gas distribution system for a city. The pipe fails and leaks, even though no corrosion noticeable to the naked eye has occurred. Offer an explanation for why the pipe failed.

Because no corrosion is noticeable, the corrosion byproduct apparently is still in place on the pipe, hiding the corroded area. The circumstances suggest graphitic corrosion, an example of a selective chemical attack. The graphite flakes in the grey iron are not attacked by the corrosive soil, while the iron matrix is removed or converted to an iron oxide or hydroxide trapped between the graphite flakes. Although the pipe appears to be sound, the attacked area is weak, porous, and spongy. The natural gas can leak through the area of graphitic corrosion and eventually cause gas accumulations leading to an explosion.

22.2 A brass plumbing fitting produced from a Cu-30% Zn alloy operates in the hot water system of a large office building. After some period of use, cracking and leaking occur. On visual examination no metal appears to have been corroded. Offer an explanation for why the fitting failed.

The high zinc brasses are susceptible to dezincification, particularly when the temperature is increased, as in the hot water supply of the building. One of the characteristics of dezincification is that copper is redeposited in the regions that are attacked, obscuring the damage. However the redeposited copper is spongy, brittle, and weak, permitting the fitting to fail and leak. Therefore dezincification appears to be a logical explanation.

22.3 Suppose 10 g of Sn^{2+} are dissolved in 1000 ml of water to produce an electrolyte. Calculate the electrode potential of the tin half-cell.

The concentration of the electrolyte is:

$$C = 10 \text{ g} / 118.69 \text{ g.mol}^{-1} = 0.0842 \text{ M}$$

The electrode potential from the Nernst equation is:

$$E = -0.14 + (0.0592/2)\log(0.0842) = -0.172 \text{ V}$$

Note that the logarithm is to the base 10 in this equation.

22.4 A half-cell produced by dissolving copper in water produces an electrode potential of +0.32 V. Calculate the amount of copper that must have been added to 1000 ml of water to produce this potential.

From the Nernst equation, with $E_0 = 0.34 \text{ V}$ and the atomic mass of copper of 63.54 g.mol^{-1} , we can find the number of grams "x" added to 1000 ml of the solution. For Cu^{2+} , $n = 2$.

$$0.32 = 0.34 + (0.0592/2)\log(x / 63.54)$$
$$\log(x / 63.54) = (-0.02)(2) / 0.0592 = -0.67568$$

$$x / 63.54 = 0.211 \quad \text{or} \quad x = 13.4 \text{ g Cu per 1000 ml H}_2\text{O}$$

22.5 An electrode potential in a platinum half-cell is 1.10 V. Determine the concentration of Pt^{4+} ions in the electrolyte.

From the Nernst equation, with $E_0 = 1.20 \text{ V}$ and the atomic mass of platinum of $195.09 \text{ g.mol}^{-1}$, we can find the amount "x" of Pt added per 1000 ml of solution. For Pt, $n = 4$.

$$1.10 = 1.20 + (0.0592/4) \log(x / 195.09)$$

$$\log(x / 195.09) = -6.7568$$

$$x / 195.09 = 0.000000175$$

$$x = 0.00003415 \text{ g Pt per } 1000 \text{ ml } \text{H}_2\text{O}$$

22.6 A current density of 500 A.m^{-2} is applied to a 0.015 m^2 cathode. What period of time is required to plate out a 1-mm thick coating of silver onto the cathode?

The current in the cell is $I = iA = (500 \text{ A.m}^{-2})(0.015 \text{ m}^2) = 7.5 \text{ A}$

The weight of silver that must be deposited to produce a 1 mm thick layer is:

$$w = (0.015 \text{ m}^2)(0.001 \text{ m})(10.49 \text{ Mg.m}^{-3}) = 157.35 \text{ g}$$

From the Faraday equation, with $n = 1$ for silver:

$$157.35 \text{ g} = (7.5 \text{ A})(t)(107.868 \text{ g.mol}^{-1}) / (1)(96,500 \text{ C})$$

$$t = 18,769 \text{ s} = 5.21 \text{ h}$$

22.7 We wish to produce 100 g of platinum per hour on a 0.01 m^2 cathode by electroplating. What plating current density is required? Determine the current required.

In the Faraday equation, $n = 4$ for platinum, which has an atomic mass of $195.09 \text{ g.mol}^{-1}$. The mass of platinum that must be deposited per second is $100 \text{ g} / 3600 \text{ s.h}^{-1} = 0.02778 \text{ g.s}^{-1}$.

$$0.02778 \text{ g.s}^{-1} = (i)(0.1 \text{ m}^2)(195.09 \text{ g.mol}^{-1}) / (4)(96,500 \text{ C})$$

$$i = 549 \text{ A.m}^{-2}$$

The current must then be:

$$I = iA = (549 \text{ A.m}^{-2})(0.1 \text{ m}^2) = 55 \text{ A}$$

22.8 A 1 m square steel plate is coated on both sides with a 0.05 mm-thick layer of zinc. A current density of 200 A.m^{-2} is applied to the plate in an aqueous solution. Assuming that the zinc corrodes uniformly, determine the length of time required before the steel is exposed.

The surface area includes both sides of the plate:

$$A = (2 \text{ sides})(1 \text{ m})(1 \text{ m}) = 2 \text{ m}^2$$

The mass of zinc that must be removed by corrosion is:

$$w = (2 \text{ m}^2)(0.05 \times 10^{-3} \text{ m})(7.133 \text{ Mg.m}^{-3}) = 713.3 \text{ g}$$

From Faraday's equation, where $n = 2$ for zinc:

$$713.3 \text{ g} = \frac{(200 \text{ A.m}^{-2})(2 \text{ m}^2)(t)(65.38 \text{ g.mol}^{-1})}{(2)(96,500 \text{ C})}$$

$$t = 5264 \text{ s} = 1.462 \text{ h}$$

22.9 A 50 mm inside diameter, 4 m long copper distribution pipe in a plumbing system is accidentally connected to the power system of a manufacturing plant, causing a current of 0.05 A to flow through the pipe. If the wall thickness of the pipe is 3 mm, estimate the time required before the pipe begins to leak, assuming a uniform rate of corrosion.

If the pipe corroded uniformly, essentially all of the pipe would have to be consumed before leaking. The volume of material in the pipe, which has an inside diameter of 50 mm and an outside diameter of 53 mm is:

$$V = (\pi/4)[(0.056 \text{ m})^2 - (0.05 \text{ m})^2](4\text{m}) = 0.002 \text{ m}^3$$

The density of copper is 8.96 Mg.m^{-3} . The mass of material to be corroded is:

$$w = V\rho = (0.000971 \text{ m}^3)(8.96 \text{ Mg.m}^{-3}) = 17900 \text{ g}$$

From Faraday's law, with $n = 2$ for copper:

$$17700 \text{ g} = (0.05 \text{ A})(t)(63.54 \text{ g.mol}^{-1}) / (2)(96,500 \text{ C})$$

$$t = 1.09 \times 10^9 \text{ s} = 34.5 \text{ years}$$

22.10 A steel surface $0.1 \text{ m} \times 0.1 \text{ m}$ is coated with a 0.02 mm thick layer of chromium. After one year of exposure to an electrolytic cell, the chromium layer is completely removed. Calculate the current density required to accomplish this removal.

The volume and weight of chromium that must be removed are:

$$V = (0.1 \text{ m})(1 \text{ m})(0.02 \times 10^{-3} \text{ m}) = 2 \times 10^{-6} \text{ m}^3$$

$$w = (2 \times 10^{-6} \text{ m}^3)(7.19 \text{ Mg.m}^{-3}) = 14.38 \text{ g}$$

There are $31.536 \times 10^6 \text{ s}$ per year. The surface area of the chromium is $(0.1 \text{ m})(0.1 \text{ m}) = 0.1 \text{ m}^2$. From Faraday's law, with $n = 3$ for chromium:

$$14.38 \text{ g} = \frac{(i)(0.1 \text{ m}^2)(31.536 \times 10^6 \text{ s})(51.996 \text{ g.mol}^{-1})}{(3)(96,500 \text{ C})}$$

$$i = 0.0254 \text{ A.m}^{-2}$$

The current is $(0.0254 \text{ A.m}^{-2})(0.1 \text{ m}^2) = 2.54 \times 10^{-3} \text{ A} = 2.54 \text{ mA}$

22.11 A corrosion cell is composed of a 0.03 m^2 copper sheet and a 0.02 m^2 iron sheet, with a current density of 6000 A.m^{-2} applied to the copper. Which material is the anode? What is the rate of loss of metal from the anode per hour?

In the Cu-Fe cell, the iron is the anode.

$$I = i_{\text{Cu}}A_{\text{Cu}} = i_{\text{Fe}}A_{\text{Fe}}$$

$$(6000 \text{ A.m}^{-2})(0.03 \text{ m}^2) = i_{\text{Fe}}(0.02 \text{ m}^2)$$

$$i_{Fe} = 9000 \text{ A.m}^{-2}$$

The weight loss of iron per hour (3600 s) is:

$$w = (9000 \text{ A.m}^{-2}) (0.02 \text{ m}^2) (3600 \text{ s}) (55.847 \text{ g.mol}^{-1}) / (2) (96,500 \text{ C}) \\ = 187.5 \text{ g of iron lost per hour}$$

22.12 A corrosion cell is composed of a 0.002 m² copper sheet and a 0.04 m² iron sheet, with a current density of 7000 A.m⁻² applied to the copper. Which material is the anode? What is the rate of loss of metal from the anode per hour?

In the Cu-Fe cell, the iron is the anode.

$$I = i_{Cu}A_{Cu} = i_{Fe}A_{Fe} \\ (7000 \text{ A.m}^{-2}) (0.002 \text{ m}^2) = i_{Fe} (0.04 \text{ m}^2) \\ i_{Fe} = 350 \text{ A.m}^{-2}$$

The weight loss of iron per hour (3600 s) is:

$$w = (350 \text{ A.m}^{-2}) (0.04 \text{ m}^2) (3600 \text{ s}) (55.847 \text{ g.mol}^{-1}) / (2) (96,500 \text{ C}) \\ = 14.58 \text{ g of iron lost per hour}$$

Note that the rate of iron lost per hour when the anode area is large is much smaller than the rate of iron loss when the anode area is small (Problem 22.11).

22.13 Alclad is a laminar composite composed of two sheets of commercially pure aluminium (alloy 1100) sandwiched around a core of 2024 aluminium alloy. Discuss the corrosion resistance of the composite. Suppose that a portion of one of the 1100 layers was machined off, exposing a small patch of the 2024 alloy. How would this affect the corrosion resistance? Explain. Would there be a difference in behavior if the core material were 3003 aluminium? Explain.

The Alclad composed of 2024 and 1100 alloys should have good corrosion resistance under most circumstances. The 1100 alloy has good corrosion resistance, since it is nearly pure aluminium, when it completely covers the underlying 2024 alloy. Furthermore, if the 1100 layer is disturbed by machining, scratching, or other means, the 1100 alloy serves as the anode and protects the 2024 alloy. The surface area of the 1100 alloy is large, and even corrosion of the 1100 alloy will be slow.

When the 3003 alloy is coated with 1100 alloy, a disturbance of the surface is more critical. The 3003 alloy will serve as the anode; since the surface area of the 3003 anode is likely to be small compared to the surface area of the 1100 alloy, corrosion may occur rapidly.

22.14 The leaf springs for an automobile are formed from a high carbon steel. For best corrosion resistance, should the springs be formed by hot working or cold working? Explain. Would corrosion still occur even if you use the most desirable forming process? Explain.

If we form the springs cold, residual stresses are likely to be introduced into the product, leading to a stress cell and corrosion of the spring. Forming the springs hot will reduce the level of any residual stresses introduced into the spring and minimize the stress cell.

However, the steel will contain ferrite and pearlite (forming a composition cell), not to mention grain boundaries, inclusions, and other potential sites for corrosion cells. Corrosion is still likely to occur even if the springs were produced by hot working.

22.15 Several types of metallic coatings are used to protect steel, including zinc, lead, tin, cadmium, aluminium, and nickel. In which of these cases will the coating provide protection even when the coating is locally disrupted? Explain.

Aluminium, zinc, and cadmium are anodic compared to iron; consequently these three elements should provide protection (serving as sacrificial anodes) to the iron even if the coating is disrupted.

Nickel, tin, and lead are cathodic compared to iron; when these coatings are disrupted, small anodic regions of iron are exposed and corrosion may occur rapidly.

22.16 An austenitic stainless steel corrodes in all of the heat affected zone surrounding the fusion zone of a weld. Explain why corrosion occurs and discuss the type of welding process or procedure that might have been used. What might you do to prevent corrosion in this region?

Since the entire heat affected zone has corroded, the entire heat affected region must have been sensitized during the welding process. Sensitization means that chromium carbides have precipitated at the austenite grain boundaries during joining, reducing the chromium content in the austenite near the carbides. The low chromium content austenite serves as the anode and corrosion occurs.

Based on our observation of the corrosion, we can speculate that the austenitic stainless steel is not a low carbon steel (that is, the steel contains more than 0.03%C). The welding process was such that the heat affected zone experienced long exposure times and slow rates of cooling. Nearest the fusion zone, the steel was all austenite at the peak temperatures developed during welding; however the slow rate of cooling provided ample time for carbide precipitation as the region cooled between 870 and 425°C. A bit further from the fusion zone, the steel was exposed to the 870 to 425°C temperature range for a long time, permitting carbides to precipitate and sensitize the steel.

The long times and slow cooling rates may have been a result of the welding process -- a low rate of heat input process, or inefficient process, will introduce the heat slowly, which in turn heats up the surrounding base metal which then acts as a poor heat sink. Preheating the steel prior to welding would also result in the same problems.

The stainless steel should be welded as rapidly as possible, using a high rate of heat input process, with no preheat of the steel prior to welding. The steel should be a low carbon steel to assure that carbides do not precipitate even when the cooling rates are slow. If problems persist, a quench anneal heat treatment might be used to redissolve the carbides.

22.17 A steel nut is securely tightened onto a bolt in an industrial environment. After several months, the nut is found to contain numerous cracks, even though no externally applied load acts on the nut. Explain why cracking might have occurred.

When the nut is tightly secured onto the bolt, residual stresses are likely to be introduced into the assembly. The presence of numerous

cracks suggests that stress corrosion cracking may have occurred as a result of these stresses.

22.18 The shaft for a propeller on a ship is carefully designed so that the applied stresses are well below the endurance limit for the material. Yet after several months, the shaft cracks and fails. Offer an explanation for why failure might have occurred under these conditions.

The propeller is under a cyclical load during operation, but it is also in a marine environment which may provide a relatively aggressive electrolyte. Failure, it is noted, requires some time to occur. Corrosion-fatigue sounds like a strong possibility in this case. Even though the stress is nominally below the endurance limit for the shaft, corrosion encouraged by the stress will lead to loss of material or development of pits in the shaft. This will increase the stress acting on the shaft and further encourage corrosion. The result is the eventual formation of fatigue cracks, encouraged by corrosion, which cause the shaft to fail.

22.19 An aircraft wing composed of carbon fibre-reinforced epoxy is connected to a titanium forging on the fuselage. Will the anode for a corrosion cell be the carbon fibre, the titanium, or the epoxy? Which will most likely be the cathode? Explain.

Titanium is expected to serve as the anode and corrode, while carbon is expected to be the cathode. Titanium is more anodic than carbon, or graphite. Both are electrical conductors, they are in physical contact with one another at the junction between the wing and the fuselage, and both can be exposed to the environment.

The epoxy should not participate in the electrochemical cell; epoxy is an electrical insulator.

22.20 The inside surface of a cast iron pipe is covered with tar, which provides a protective coating. Acetone in a chemical laboratory is drained through the pipe on a regular basis. Explain why, after several weeks, the pipe begins to corrode.

During use, the acetone serves as a solvent for the tar; the protective tar coating is eventually dissolved and the cast iron pipe is then exposed to any corrosive material that is drained through the pipe.

22.21 A cold worked copper tube is soldered, using a lead-tin alloy, into a steel connector. What types of electrochemical cells might develop due to this connection? Which of the materials would you expect to serve as the anode and suffer the most extensive damage due to corrosion? Explain.

Several cells may develop. Composition cells include those between the solder and the steel, with the steel serving as the anode and the solder as the cathode. The steel then corrodes.

A composition cell may also develop between the copper and the solder. In this case, the solder will act as the anode.

Microcomposition cells may also develop. The steel contains ferrite and cementite; the ferrite will act as the anode. In addition, the lead-tin solder is a two-phase alloy containing nearly pure tin (β) and a solid solution of tin in lead (α). Lead is most likely to serve as the anode with respect to tin.

A concentration cell is also possible, particularly in the crevice between the copper tube and the steel. The material adjacent to the crevice will act as the anode.

Finally, the copper tube is cold worked; the cold working may cause a stress cell to develop. This may be accentuated by the soldering process; during soldering, the copper tube at the soldered joint will heat, perhaps to a high enough temperature to allow stress relieving to occur. This again helps to provide the stress cell between the cold worked and stress relieved portions of the tube.

22.22 Pure tin is used to provide a solder connection for copper in many electrical uses. Which metal will most likely act as the anode?

From the galvanic series, we find that tin is anodic to copper; therefore the tin anode is expected to corrode while the copper cathode is protected.

22.23 Sheets of annealed nickel, cold-worked nickel, and recrystallized nickel are placed into an electrolyte. Which would be most likely to corrode? Which would be least likely to corrode? Explain.

The cold worked nickel sheet is expected to have the poorest corrosion resistance due to the residual stresses introduced during the cold working process.

The annealed nickel sheet should be most resistant to corrosion; the grain size is expected to be particularly large and no residual stresses are expected; consequently a stress cell is unlikely. In addition, the annealed sheet is expected to have the most uniform composition, that is, the least segregation, so a composition cell is also unlikely.

The recrystallized nickel sheet should have intermediate corrosion resistance; the residual stresses should have been eliminated as a result of the heat treatment but the grain size may be smaller than in the annealed sheet.

22.24 A pipeline carrying liquid fertilizer crosses a small creek. A large tree washes down the creek and is wedged against the steel pipe. After some time, a hole is produced in the pipe at the point where the tree touches the pipe, with the diameter of the hole larger on the outside of the pipe than on the inside of the pipe. The pipe then leaks fertilizer into the creek. Offer an explanation for why the pipe corroded.

One possibility for the corrosion is a concentration cell caused by microbial corrosion. The point of contact between the tree and the pipe produces a low oxygen environment and also a location at which various bacteria may grow. As the bacteria grow on the pipe, a low oxygen environment is further encouraged. A galvanic cell is produced, with the pipe beneath the fallen tree (and thus the bacteria) serving as the anode and the remainder of the pipe acting as the cathode. Localized corrosion will then continue until a hole is corroded through the wall of the pipe.

22.25 Two sheets of a 0.4%C steel (BS 080A40) steel are joined together with an aluminium rivet (Figure 22.20). Discuss the possible corrosion cells that might be created as a result of this joining process. Recommend a joining process that might minimize some of these cells.

Composition cells: Aluminium may act as the anode in comparison to the steel, thus causing corrosion of the aluminium. In addition, ferrite may serve as the anode to cementite within the steel.

Stress cells: The aluminium rivet is deformed when the joint is produced, causing the most highly cold worked portion of the rivet to act as the anode. In addition, grain boundaries in both the steel and the aluminium may act as anodes for a stress cell.

Concentration cells: Crevice corrosion may occur between the two steel sheets and also between the aluminium rivet and the steel sheets.

A fusion welding process, in which a filler material having a composition similar to that of the steel, might be the best way to minimize most of these cells.

22.26 Figure 22.21 shows a cross section through an epoxy-encapsulated integrated circuit, including a small microgap between the copper lead frame and the epoxy polymer. Suppose chloride ions from the manufacturing process penetrate the package. What types of corrosion cells might develop? What portions of the integrated circuit are most likely to corrode?

Composition cells can develop between gold and aluminium (with the aluminium serving as the anode and corroding); gold and copper (with the copper serving as the anode and corroding); and aluminium and silicon (with aluminium serving as the anode and corroding).

22.27 A current density of 1000 A.m^{-2} is applied to the iron in an iron-zinc corrosion cell. Calculate the weight loss of zinc per hour (a) if the zinc has a surface area of 0.01 m^2 and the iron has a surface area of 0.1 m^2 and (b) if the zinc has a surface area of 0.1 m^2 and the iron has a surface area of 0.01 m^2 .

$$(a) I = i_{\text{Fe}} A_{\text{Fe}} = (1000 \text{ A.m}^{-2}) (0.1 \text{ m}^2) = 100 \text{ A}$$

$$w_{\text{Zn}} = (100 \text{ A}) (3600 \text{ s}) (65.38 \text{ g.mol}^{-1}) / (2) (96,500 \text{ C}) \\ = 122 \text{ g of Zn lost per hour}$$

$$(b) I = i_{\text{Fe}} A_{\text{Fe}} = (1000 \text{ A.m}^{-2}) (0.01 \text{ m}^2) = 10 \text{ A}$$

$$w_{\text{Zn}} = (10 \text{ A}) (3600 \text{ s}) (65.38 \text{ g.mol}^{-1}) / (2) (96,500 \text{ C}) \\ = 12.2 \text{ g of Zn lost per hour}$$

The loss of zinc is accelerated when the zinc anode area is small.

22.28 Determine the Pilling-Bedworth ratio for the following metals and predict the behavior of the oxide that forms on the surface. Is the oxide protective, does it flake off the metal, or is it permeable? (see Appendix A for the metal density)

The Pilling-Bedworth relation is

$$P-B = MW_{\text{oxide}} \rho_{\text{metal}} / n MW_{\text{metal}} \rho_{\text{oxide}}$$

From metal density data listed in Appendix A, the calculated P-B ratios are shown in the table below.

	Oxide density (Mg.m ⁻³)	Metal density (Mg.m ⁻³)	n	P-B ratio
Mg-MgO	3.60	1.738	1	0.80
Na-Na ₂ O	2.27	0.967	2	0.57
Ti-TiO ₂	5.10	4.507	1	1.47
Fe-Fe ₂ O ₃	5.30	7.87	2	2.12
Ce-Ce ₂ O ₃	6.86	6.6893	2	1.14
Nb-Nb ₂ O ₃	4.47	8.57	2	2.74
W-WO ₃	7.30	19.254	1	3.33

The oxides that form in magnesium and sodium are expected to be nonadherent, or tend to flake off, since the oxide volume is substantially larger than the metal volume. A P-B ratio of less than 1 indicates this condition.

The oxides that form on iron, niobium, and tungsten are expected to be adherent but permeable. A P-B ratio of more than 2 indicates that the oxide volume is much less than that of the metal volume.

The oxides that form on titanium and cesium are expected to be protective; a P-B ratio of 1 to 2 indicates this condition.

22.29 Oxidation of most ceramics is not considered to be a problem. Explain.

Most ceramics are already oxides -- thus materials such as MgO and Al₂O₃ are expected to be inert in an oxidizing atmosphere. Non-oxide ceramics, however, may sometimes be subject to oxidation problems.

22.30 A sheet of copper is exposed to oxygen at 1000°C. After 100 h, 0.00246 g of copper are lost per mm² of surface area; after 250 h, 0.00388 g.mm² are lost, and after 500 h, 0.00550 g.mm² are lost. Determine whether oxidation is parabolic, linear, or logarithmic, then determine the time required for a 7.5 mm sheet of copper to be completely oxidized. The sheet of copper is oxidized from both sides.

Let's assume that the rate is parabolic: We can determine the constant "k" in the oxidation equation $y = \sqrt{kt}$ for each time, first converting the rate in g.mm² to thickness y in mm:

$$y_1 = V/A = (0.00246 \text{ g} / 8.93 \text{ Mg.m}^{-3}) / 1 \text{ mm}^2 = 0.275 \text{ mm}$$

$$y_2 = V/A = (0.00388 \text{ g} / 8.93 \text{ Mg.m}^{-3}) / 1 \text{ mm}^2 = 0.434 \text{ mm}$$

$$y_3 = V/A = (0.00549 \text{ g} / 8.93 \text{ Mg.m}^{-3}) / 1 \text{ mm}^2 = 0.615 \text{ mm}$$

If oxidation is parabolic, the value for k should be the same for each time:

$$0.0275 \text{ mm} = \sqrt{k(100 \text{ h})} \quad \text{or} \quad k = 7.56 \times 10^{-4} \text{ mm}^2 \cdot \text{h}^{-1}$$

$$0.0434 \text{ mm} = \sqrt{k(250 \text{ h})} \quad \text{or} \quad k = 7.53 \times 10^{-4} \text{ mm}^2 \cdot \text{h}^{-1}$$

$$0.0615 \text{ mm} = \sqrt{k(500 \text{ h})} \quad \text{or} \quad k = 7.56 \times 10^{-4} \text{ mm}^2 \cdot \text{h}^{-1}$$

Therefore the rate of oxidation must be parabolic.

To completely oxidize the copper, assuming that the rate of oxidation is the same from both sides of the sheet, we need to determine the time required to oxidize 7.5 mm / 2 sides = 3.75 mm:

$$y = 3.75 \text{ mm} = \sqrt{(7.56 \times 10^{-4} \text{ mm}^2 \cdot \text{h}^{-1})(t)} \quad \text{or} \quad t = 18,601 \text{ h}$$

22.31 At 800°C, iron oxidizes at a rate of 0.0014 g.mm⁻² per hour; at 1000°C, iron oxidizes at a rate of 0.00656 g.mm⁻² per hour. Assuming a parabolic oxidation rate, determine the maximum temperature at which iron can be held if the oxidation rate is to be less than 0.00005 g.mm⁻² per hour.

The rate is given by an Arrhenius equation, $\text{rate} = A \exp(-Q/RT)$. We can find the constants A and Q from the data provided.

$$0.00014 \text{ g.mm}^2\cdot\text{h}^{-1} = A \exp(-Q/(8.314 \text{ J.mol}^{-1}\cdot\text{K}^{-1})(800 + 273\text{K}))$$

$$0.000656 \text{ g.mm}^2\cdot\text{h}^{-1} = A \exp(-Q/(8.314 \text{ J.mol}^{-1}\cdot\text{K}^{-1})(1000 + 273\text{K}))$$

Taking logarithms of both sides:

$$-8.8739 = \ln A - 0.0001121Q$$

$$-7.3293 = \ln A - 0.00009448Q$$

$$1.5445 = 0.00001762 Q \quad \text{or} \quad Q = 87,700 \text{ J.mol}^{-1}$$

$$-8.8739 = \ln A - (0.0001121)(87,700)$$

$$\ln A = 18.703$$

$$A = 2.60$$

To keep the oxidation rate below 0.00005 g.mm⁻², the maximum temperature is:

$$\text{rate} = 2.60 \exp(-87700/RT) = 0.005$$

$$\ln(0.00005/2.60) = -87,700 / (8.314)(T)$$

$$\ln(0.00001923) = -10.859 = -10,548/T$$

$$T = 971 \text{ K} = 698^\circ\text{C}$$

Chapter 23 Failure - Origin, Detection and Prevention

23.1 Investigation of an automobile accident revealed that an axle for a rear wheel was broken. Expert witnesses disagree on whether the axle failed by fatigue, thereby causing the accident, or whether the axle failed in impact as a consequence of the accident. What features on the axle and the fracture surface would you look for in an attempt to settle this dispute?

If the accident was caused by a fatigue fracture of the axle, examination of the axle should reveal that at least part of the fracture surface is very smooth, with beach marks radiating from the origin of the fracture. Examination of the fracture surface at a high magnification, using a scanning electron microscope, would show the presence of very closely spaced striations indicating the distance that the crack front moved during each cycle of the applied load. The axle should not be bent -- the axle should roll smoothly on a flat surface.

If the axle failed by impact, very different phenomena will be observed. The fracture surface will be relatively rough, often with a "finger" of metal that includes the last material to fail. The axle will almost certainly be bent -- it will not roll smoothly on a flat surface. Axles are typically induction hardened steels, with a case depth of about 6 mm; chevron marks in the case will often be observed and will point to the origin of the fracture. Almost always, additional damage will be observed; for example, there may be small indentations on the axle caused by the bearings -- this is sometimes called "brinelling". Typically, severe damage to the wheel or even the tyre may be observed. Often, the peripheral damage can be matched in orientation to the origin of the fracture.

23.2 A turbine blade in a jet engine is found to be the cause of an airplane crash. Expert witnesses disagree on whether the turbine blade failed because a large bird was ingested into the engine, because the engine overheated and creep failure occurred, or because of stress-corrosion cracking due to the presence of sulphides in the engine. What features of the turbine blade, fracture surface, and microstructure would you look for in an attempt to settle this dispute?

Ingestion of a foreign object such as a bird is likely to cause failure by impact -- severe deformation of the turbine blade, with a possible chevron pattern and cleavage planes on the fracture surface, might be observed.

If failure is due to creep caused by over-heating, deformation of the blade again might be expected. However, in this case, examination of the microstructure near the fracture surface may show the presence of voids produced by the creep mechanism.

Failure by stress-corrosion cracking may lead to relatively little overall deformation of the blade. Examination of the microstructure near the fracture surface may reveal additional cracks that did not propagate far enough to cause failure. Examination of these cracks with a scanning electron microscope, using the X-ray features, would show the presence of byproducts of the corrosion process, in this case perhaps complex sulfides, suggesting that stress-corrosion was involved.

23.3 A wire rope made up of many strands of small diameter wire passes over a pulley before being used to lift heavy loads. After several months of use, the wire rope fails while lifting a load. Expert witnesses disagree on whether the strands in the rope failed due to fatigue (when a rope passes over a pulley whose diameter is too small, the stresses are unusually high) or because the load being lifted exceeded the limit of the rope. What features of the rope would you look for in order to determine the cause of failure?

A close examination of the ends of the broken wire should help determine the cause for failure. In fatigue due to a small-diameter pulley, the fractured ends of many of the wire should be smooth, with the fracture surface perpendicular to the length of the wire, and no deformation, or necking, of the wire is expected. Fatigue striations and perhaps even beach marks might be observed on the fracture surface at high magnifications. (Some of the wires might be necked -- these would be the last wires to fail, after the strength of the rope was greatly decreased.) Furthermore, examination of portions of the rope further away from the point of fracture should reveal broken wires that also appear to be caused by fatigue; not enough wires broke at these locations to cause the rope to fail.

If the rope was overloaded, almost all of the broken wires should be at least slightly necked. Examination of the fracture surface at a high magnification might reveal a dimpled surface, again indicating a tensile ductile overload.

23.4 A cast iron lever is used to tighten ropes that hold a heavy load on a truck. After several years of use, the lever breaks during a routine tightening operation. What type of cast iron would you recommend for such an application? What types of fracture mechanisms would be possible for such a case? What features of the fractured lever would you look for to determine the cause of the fracture?

To assure safety and good strength, either a malleable or ductile (spheroidal graphite) cast iron would likely be used to produce the lever. Castings produced from either of these two materials can possess good strength and good ductility, along with reasonable fracture toughness. If overloaded, the lever may deform prior to fracture, providing a warning to the operator and preventing an accident.

Several fracture mechanisms might cause the lever to fail. The lever will be exposed to rather rough handling during its use; the rough handling might cause cracks to develop during its use or may nucleate cracks from casting imperfections, such as inclusions, gas pores, or undesirable graphite shapes (such as flake or vermicular graphite in what is intended to be a ductile iron). Over time and use, the crack may grow until the effective cross-section of the lever is reduced to the point that the lever breaks even with the normal application of a force. In this case, the fracture will be brittle, with little evidence of deformation of the lever prior to fracture.

Fatigue might also cause the lever to fail; the lever is highly stressed each time a load is bound onto the truck. However, once, the truck is loaded, the force acting on the lever is relatively small. Such a lever is probably not used more than a few hundred times each year; consequently a very large number of years might be required before fatigue becomes a problem.

The lever is also exposed to the environment. Corrosion and/or stress-corrosion might cause the cross-section to be reduced or a crack to initiate.

The lever might also simply be overloaded. For example, to make the load tighter, a "cheater bar", or pipe, might be used to make the lever arm longer; this would produce a higher than designed stress on the lever and cause it to fail. Or the cast iron might have a ferritic matrix, when it was designed to have a pearlitic matrix; the lever now would have a lower than expected strength and might fail. In this case, the lever would be expected to deform a substantial amount prior to fracture.

23.5 A complex-shaped clamp made of a copper alloy is made by a cold-forming process. The clamp is used to fasten a heavy electrical transformer to a pole outdoors. Shortly after installation, cracks are found at a location where the clamp had been bent into a U-shape during forming. What are possible fracture mechanisms for this device? What features in the material or on the fracture surface would you look for to determine the cause of the crack?

The most likely cause for this failure is stress corrosion; the clamp was locally cold worked during forming, with the curve of the U-shape receiving the largest cold work and thus containing the highest residual stresses. It would act as the anode in a corrosion cell. The clamp is also used to hold a heavy transformer, which adds additional stress to the fixture. Finally, the clamp is used outside, possibly in a corrosive environment. Due to the stress cell, stress corrosion cracking may have occurred, eventually causing the clamp to fail. Examination of the metal near the fracture may reveal secondary cracks, likely filled with a corrosion byproduct, supporting our theory of stress corrosion cracking.

23.6 Two thick steel plates are joined by an arc welding process as part of an assembly for a missile-carrying transport vehicle. After the vehicle is placed in service, the assembly fails. Inspection of the failure indicates that the crack propagated through the heat affected area of the weld, right next to the actual fusion zone. Suggest possible causes of the failure, including the role of the welding process and the microstructure that the welding process may have produced in the heat affected zone. What recommendations might you have to avoid such failures in the future?

When a steel is welded, the metal next to the fusion zone heats above the A_1 temperature and changes to austenite. Austenite grain growth may occur. When the weld cools, the coarse-grained austenite may then transform to martensite, making the heat affected zone very brittle. Furthermore, if hydrogen is able to enter the heat affected zone, hydrogen embrittlement and cracking can occur in the heat affected zone. When the weld is overstressed, the weldment will fail in a brittle manner.

Factors that might contribute to such a failure include (1) using a high hardenability steel, so that martensite easily forms, (2) achieving high cooling rates in the weld, encouraging the formation of martensite, and (3) using welding processes that produce high cooling rates or introduce hydrogen into the weld that might contribute to embrittlement.

Recommendations for minimizing the problem include using low alloy, low carbon steels (such as HSLA steels), preheating the base metal prior to welding to reduce the cooling rates, postheating the weld to temper any martensite that might form and also to reduce residual stresses that might encourage cracking, using ductile filler metals to minimize residual stresses (the stresses can be reduced by plastic deformation of the fusion zone), and using welding processes that produce low hydrogen contents (such as electron beam welding or inert-gas welding processes).

23.7 The titanium tubes in a heat exchanger operating at 500°C are found to crack and leak after several months of use. Suggest possible causes for the failure and describe the features of the heat exchanger and its microstructure that you would look for in order to confirm the actual cause.

Because the titanium tubes are operating at a high temperature and possibly in a corrosive environment, the first two failure mechanisms to look for in this case might be creep or stress-rupture fracture or stress corrosion fracture. Furthermore, the operating temperature of 500°C is close to the temperature at which the protective titanium oxide film breaks down, leading to oxidation or embrittlement of the alloy.

A metallographic examination will help to reveal the cause for the failure. If creep or stress-rupture is the cause, the microstructure may include voids caused by grain boundary sliding or void coalescence; there may also be extensive plastic deformation of the tubes due to creep. If stress corrosion is the cause, we might expect to find multiple cracks filled with a corrosion byproduct.

23.8 Determine the mass absorption coefficient and the linear absorption coefficient for tungsten X-rays in a copper alloy containing 5 wt% Sn and 15 wt% Pb.

First we can determine the atomic fraction of the three elements in the copper alloy, using the atomic mass of copper, tin, and lead:

$$f_{Cu} = \frac{80/63.54}{(80/63.54) + (5/118.69) + (15/207.19)} = 0.91662$$

$$f_{Sn} = \frac{5/118.69}{(80/63.54) + (5/118.69) + (15/207.19)} = 0.03067$$

$$f_{Pb} = \frac{15/207.19}{(80/63.54) + (5/118.69) + (15/207.19)} = 0.05271$$

From the rule of mixtures and the densities and mass absorption coefficients of each of the three elements,

$$\rho = (0.91662)(8.96) + (0.03067)(7.3) + (0.05271)(11.34) = 9.0345 \text{ Mg}\cdot\text{m}^{-3}$$

$$\mu_m = (0.91662)(32.5) + (0.03067)(117) + (0.05271)(350) = 51.83 \text{ m}^2\cdot\text{Mg}^{-1}$$

$$\mu = \rho\mu_m = (9.0345 \text{ Mg}\cdot\text{m}^{-3})(51.83 \text{ m}^2\cdot\text{Mg}^{-1}) = 468.3 \text{ m}^{-1}$$

23.9 We would like to determine whether there is a large amount of segregation in an Al-15 wt% Si alloy. To assure the best chance for determining this radiographically, determine whether we should use X-ray radiography or neutron radiography by calculating the mass absorption coefficients of the alloy and comparing them with the coefficients of pure aluminium and silicon.

The atomic fractions of Si and Al in the alloy are:

$$f_{Si} = \frac{15/28.08}{(15/28.08) + (85/26.981)} = 0.14498 \quad f_{Al} = 0.85502$$

From the data for pure Si and Al, using the rule of mixtures:

$$\mu_m \text{ (X-rays)} = (0.14498)(15.9) + (0.85502)(15.6) = 15.64 \text{ m}^2\cdot\text{Mg}^{-1}$$

$$\mu_m \text{ (neutrons)} = (0.14498)(0.1) + (0.85502)(0.3) = 0.27 \text{ m}^2\cdot\text{Mg}^{-1}$$

For X-ray radiography, very little difference in the mass absorption coefficient is obtained even if aluminium and silicon completely segregate during solidification; segregation will therefore be undetected by the X-ray radiography. For neutron radiography, there is a large percentage difference between pure aluminium, pure silicon, and the alloy. Particularly if the pure aluminium segregates during solidification, a detectable difference in the intensity of the transmitted beam is expected.

23.10 A hydrogen gas bubble 1 mm in diameter is present in a 20 mm-thick magnesium casting. Compare the intensity of an X-ray beam transmitted through the section containing the bubble with that of a beam transmitted through the section containing no bubble. If the difference in intensity must be larger than 5% in order for the bubble to be detected, will X-ray radiography be successful? Would the bubble be detected if the casting is made of zinc rather than magnesium?

When a 1 mm diameter pore is present in the 20 mm thick casting, the effective thickness of the casting is 19 mm. Neglecting the absorption of X-rays by the hydrogen in the pore, the intensity of the beam passing through the sound material versus that passing through the material containing the pore is:

$$I/I_0 \text{ (sound)} = \exp[(-15.2 \text{ m}^2 \cdot \text{Mg}^{-1}) (20 \times 10^{-3} \text{ m}) (1.74 \text{ Mg} \cdot \text{m}^{-3})] = 0.589$$

$$I/I_0 \text{ (pore)} = \exp[(-15.2 \text{ m}^2 \cdot \text{Mg}^{-1}) (19 \times 10^{-3} \text{ m}) (1.74 \text{ Mg} \cdot \text{m}^{-3})] = 0.605$$

The percentage difference in the transmitted X-ray beam intensity is

$$\% \text{ difference} = \frac{(0.605 - 0.589)}{0.605} \times 100 = 2.6\%$$

This percentage difference may be too small to permit the pore to be detected in the magnesium casting.

However in a zinc casting:

$$I/I_0 \text{ (sound)} = \exp(-35 \text{ m}^2 \cdot \text{Mg}^{-1}) (20 \times 10^{-3} \text{ m}) (7.133 \text{ Mg} \cdot \text{m}^{-3}) = 0.00678$$

$$I/I_0 \text{ (pore)} = \exp(-35 \text{ m}^2 \cdot \text{Mg}^{-1}) (19 \times 10^{-3} \text{ m}) (7.133 \text{ Mg} \cdot \text{m}^{-3}) = 0.00871$$

The percentage difference in the transmitted X-ray beam intensity is

$$\% \text{ difference} = \frac{(0.00871 - 0.00678)}{0.00871} \times 100 = 22\%$$

Although longer exposure times are required compared to magnesium in order to produce a radiograph, the percentage difference between sound and unsound material is large; now the pore might be detected.

23.11 The gas tungsten arc welding process is used to join 6 mm-thick titanium alloy sheet (Ti - 7 wt%Al - 4 wt%Mo). A 0.76 mm diameter tungsten inclusion is melted from the electrode and is lodged in the fusion zone. Compare the intensity of an X-ray beam transmitted through the fusion zone containing the inclusion with that of a beam transmitted through the unaffected titanium alloy.

The atom fractions of each element in the alloy are:

$$f_{\text{Al}} = \frac{7/26.981}{(7/26.981) + (4/95.94) + (89/47.9)} = 0.120$$

$$f_{Mo} = \frac{4/95.94}{(7/26.981) + (4/95.94) + (89/47.9)} = 0.019$$

$$f_{Ti} = \frac{89/47.9}{(7/26.981) + (4/95.94) + (89/47.9)} = 0.861$$

Thus the density and mass absorption coefficients of the titanium alloy are:

$$\rho = (0.120)(2.7) + (0.019)(10.2) + (0.861)(4.54) = 4.4267 \text{ Mg.m}^{-3}$$

$$\mu_m = (0.120)(15.6) + (0.019)(79) + (0.861)(21.7) = 22.06 \text{ m}^2.\text{Mg}^{-1}$$

In the sound metal, $x = 6 \text{ mm}$ and the intensity of the transmitted X-ray beam is:

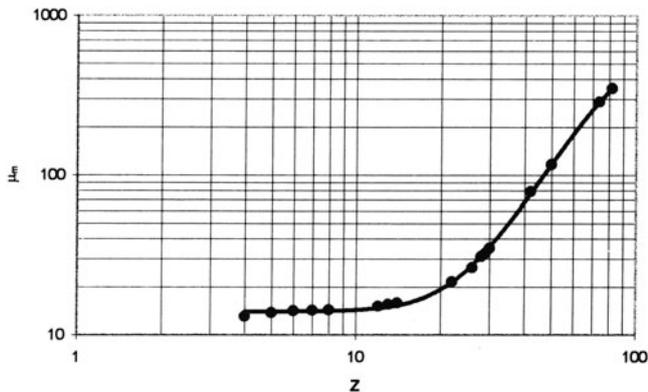
$$I/I_0 = \exp(-4.4267)(22.06 \text{ m}^2.\text{Mg}^{-1})(6 \times 10^{-3} \text{ m}) = 0.556$$

In the portion containing the inclusion, the thickness of the inclusion is 0.76 mm and the thickness of the titanium alloy is $6 - 0.76 = 5.24 \text{ mm}$. The intensity of the transmitted beam is (using the data for the titanium alloy and for pure tungsten):

$$I/I_0 = \exp[(-4.4267)(22.06)(5.24 \times 10^{-3}) + (-288)(19.3)(0.76 \times 10^{-3})] = 0.0088$$

There is a very large difference in the intensity of the transmitted beam; the X-ray film beneath the inclusion will receive hardly any radiation, the film will not be exposed, and a white circle will appear on the film, indicating the presence of the tungsten inclusion.

23.12 On the basis of data in Table 23.1, determine the relationship between mass absorption coefficient and atomic number Z for X rays with $\lambda = 0.0098 \text{ nm}$. Is a similar relationship obtained for the absorption of neutrons?



The sketch shows how the mass absorption coefficient varies with the atomic number for the elements listed in the table for X-rays with a wavelength of 0.0098 nm . For elements with a low atomic number, the mass absorption coefficient is nearly a constant (approximately $15 \text{ m}^2.\text{Mg}^{-1}$). For higher atomic number elements, the mass absorption coefficient increases with about the square of the atomic number.

There is not a similar relationship for neutrons.

23.13 We wish to determine whether there is a crack perpendicular to the surface of a 25-mm thick aluminium plate. We are able to see the crack if there is at least a 2% difference in the intensities of a transmitted X-ray beam. What is the length of the smallest crack that we can detect?

The intensity of the transmitted beam through sound aluminium, using the data for pure aluminium, is:

$$I/I_0 = \exp(-15.6)(2.7)(25 \times 10^{-3}) = 0.349$$

For a crack to be detectable, the intensity of the transmitted beam must be 2% greater than in the sound material, or the intensity should be

$$I/I_0 \text{ (crack)} = (1.02)(0.349) = 0.356$$

The thickness of the aluminium required to give this intensity is:

$$\begin{aligned} I/I_0 = 0.356 &= \exp[(-15.6)(2.7)(x \cdot 10^{-3})] \\ \ln(0.356) &= -1.033 = 0.0421 x \\ x &= 24.5 \text{ mm} \end{aligned}$$

The minimum crack length that can be detected therefore is:

$$\text{minimum crack length} = 25 - 24.53 = 0.47 \text{ mm}$$

23.14 A 4 mm thick composite material is obtained when magnesium is reinforced with 60 vol% carbon fibres, each fibre having a diameter of 0.08 mm. Determine the ratio of intensities of an X-ray beam transmitted through a portion of the composite containing a cracked fibre to that transmitted through a portion of the composite containing no cracked fibres. Would you expect a broken fibre to be easily detected by X-ray radiography?

$$\begin{array}{ll} \mu_m \text{ (Mg)} = 15.2 \text{ m}^2 \cdot \text{Mg}^{-1} & \mu_m \text{ (C)} = 14.2 \text{ m}^2 \cdot \text{Mg}^{-1} \\ \rho \text{ (Mg)} = 1.74 \text{ Mg} \cdot \text{m}^{-3} & \rho \text{ (C)} = 2.2 \text{ Mg} \cdot \text{m}^{-3} \\ f_{\text{Mg}} = 0.4 & f_{\text{C}} = 0.6 \end{array}$$

The density and mass absorption coefficients of the composite are:

$$\begin{aligned} \rho &= (0.4)(1.74) + (0.6)(2.2) = 2.016 \text{ Mg} \cdot \text{m}^{-3} \\ \mu_m &= (0.4)(15.2) + (0.6)(14.2) = 14.60 \text{ m}^2 \cdot \text{Mg}^{-1} \end{aligned}$$

The average intensity of the beam transmitted through the composite, assuming no broken fibres, is:

$$I/I_0 = \exp[(-14.60)(2.016)(4 \times 10^{-3})] = 0.8889$$

If one of the fibres is broken, the effective thickness of the composite is $x = 4 - 0.08 = 3.92$ mm. The intensity of the transmitted beam is:

$$I/I_0 = \exp[(-14.60)(2.016)(3.92 \times 10^{-3})] = 0.8910$$

The percentage increase in the intensity is only

$$\% \text{ increase} = \frac{0.8910 - 0.8889}{0.8889} \times 100 = 0.24\%$$

The broken fibre is not expected to be easily detected.

23.15 The intensity of a radiation source and the absorption capability of a material are often related by the half value layer (HVL) - the thickness of the material that will reduce the intensity of a radiation beam by half. Calculate the HVL for magnesium and copper for (a) tungsten X-rays, (b) cobalt 60 gamma rays, and (c) neutrons.

For the HVL:

$$\begin{aligned} I/I_0 = 0.5 &= \exp(-\mu_m \rho \text{HVL}) \quad \text{or} \quad \ln(0.5) = -\mu_m \rho \text{HVL} \\ \text{HVL} &= 0.69315/\mu_m \rho \end{aligned}$$

The mass absorption coefficient of gamma rays for most materials is $5.5 \text{ m}^2 \cdot \text{Mg}^{-1}$ coefficients for X-rays and neutrons are available in the table.

For magnesium, the HVL values are:

$$\begin{aligned} \text{tungsten X-rays:} & \quad \text{HVL} = 0.69315 / (15.2) (1.74) = 26.2 \text{ mm} \\ \text{cobalt 60:} & \quad \text{HVL} = 0.69315 / (5.5) (1.74) = 72.4 \text{ mm} \\ \text{neutrons:} & \quad \text{HVL} = 0.69315 / (0.1) (1.74) = 3.98 \text{ m} \end{aligned}$$

For copper, the HVL values are:

$$\begin{aligned} \text{tungsten X-rays:} & \quad \text{HVL} = 0.69315 / (32.5) (8.96) = 2.4 \text{ mm} \\ \text{cobalt 60:} & \quad \text{HVL} = 0.69315 / (5.5) (8.96) = 14.1 \text{ mm} \\ \text{neutrons:} & \quad \text{HVL} = 0.69315 / (2.1) (8.96) = 36.8 \text{ mm} \end{aligned}$$

23.16 Strontium has a half-life of 28 years. Calculate (a) the decay constant for Sr^{90} and (b) the percentage of the intensity of the strontium source after 500 years.

(a) Since we know that $I/I_0 = 0.5$ when the time is 28 years.

$$\begin{aligned} I/I_0 &= \exp(-\lambda t) \quad \text{or} \quad 0.5 = \exp(-28\lambda) \\ \ln(0.5) &= -0.69315 = -28\lambda \\ \lambda &= 0.02476 \text{ yr}^{-1} \end{aligned}$$

(b) After 500 years,

$$I/I_0 = \exp(-0.02476) (500) = \exp(-12.38) = 0.0000042$$

23.17 Polonium has a half-life of 138 days. Determine the number of years before the intensity of polonium is reduced to 10% of its original value.

$$\begin{aligned} I/I_0 &= 0.5 = \exp(-138\lambda) \\ \ln(0.5) &= -138\lambda \\ \lambda &= 0.005023 \text{ days}^{-1} \end{aligned}$$

For I/I_0 to reduced to 10%,

$$\begin{aligned} I/I_0 &= 0.10 = \exp(-0.005023t) \\ \ln(0.10) &= -2.3026 = -0.005023t \\ t &= 458.4 \text{ days} = 1.26 \text{ years} \end{aligned}$$

23.18 Calculate the thin-rod ultrasonic velocity in magnesium, silver, and tin and compare with the values given in Table 23.3.

$$\begin{aligned} \rho_{\text{Mg}} &= 1.74 \text{ Mg.m}^{-3}, \quad E_{\text{Mg}} = 45.5 \text{ GN.m}^{-2} \\ v_{\text{Mg}} &= \sqrt{(E/\rho)} = \sqrt{(45.5 \times 10^9 \text{ N.m}^{-2} / 1.74 \times 10^3 \text{ kg.m}^{-3})} \\ &= 5113 \text{ m.s}^{-2} = 5.11 \times 10^3 \text{ m.s}^{-1} \end{aligned}$$

This thin-rod calculation is slightly smaller than the $5.77 \times 10^3 \text{ m.s}^{-1}$ bulk velocity

For Ag

$$\begin{aligned} \sqrt{(E/\rho)} &= \sqrt{(72 \times 10^9 \text{ Nm}^{-2} / 10.49 \times 10^3 \text{ kg.m}^{-3})} \\ &= 2.62 \times 10^3 \text{ m.s}^{-1} \end{aligned}$$

compared with $3.63 \times 10^3 \text{ m.s}^{-1}$ bulk velocity

For Sn

$$V_{sn} = \sqrt{E/\rho} = \sqrt{53.8 \times 10^9 \text{ Nm}^{-2} / 7.3 \times 10^3 \text{ kg.m}^{-3}}$$

$$= 2.71 \times 10^3 \text{ m.s}^{-1}$$

compared with $3.38 \times 10^3 \text{ m.s}^{-1}$ bulk velocity

23.19 An ultrasonic pulse introduced into the wall of a 1000 mm-thick nickel pressure vessel returns to the transducer in 2.4×10^{-5} s. Is this time lapse caused by the opposite wall of the vessel or by a discontinuity within the wall? If caused by a discontinuity, how far beneath the surface of the wall is the discontinuity located?

The ultrasonic velocity is $v_{Ni} = 6.02 \times 10^3 \text{ m.s}^{-1}$. If the opposite wall of the vessel causes the return pulse, the time lapse should be:

$$t = (100 \times 10^{-3}\text{m}) (2) / 6.02 \times 10^3\text{m.s}^{-1} = 3.32 \times 10^{-5} \text{ s}$$

The pulse returned, however, in a shorter period of time. The indication must be caused by a discontinuity. The distance "x" beneath the surface at which the discontinuity is located is

$$v = 6.02 \times 10^3 \text{ m.s}^{-1} = (x) (2) / 2.4 \times 10^{-5} \text{ s}$$

$$x = 72.2 \text{ mm beneath the front wall}$$

23.20 An ultrasonic pulse is introduced into a 80 mm thick material, and a pulse is received by the transducer after 8×10^{-5} s. Calculate the ultrasonic velocity in the material. If this velocity were the result of a test using a thin rod, determine the specific modulus of the material.

$$v = (80 \times 10^{-3}\text{m}) (2) / (8 \times 10^{-5}\text{s}) = 200 \text{ m.s}^{-1}$$

$$v = 2000 \text{ m.s}^{-1} = \sqrt{(E/\rho)}$$

$$\text{specific modulus } E/\rho = v^2 = 4 \times 10^6 \text{ m}^2.\text{s}^{-2}$$

23.21 Figure 23.28 shows the results on an oscilloscope screen of an ultrasonic inspection of a composite material produced by centrifugally casting aluminium on the inside of an stainless steel cylinder. The transducer in this inspection is located on the outside of the cylinder, in contact with the steel. Determine the thickness of both the stainless steel cylinder and the cast aluminium.

From the figure, pulses are received after $t_1 = 6.6 \times 10^{-6}$ s and $t_2 = 10.7 \times 10^{-6}$ s. The first pulse is obtained by the beam passing through the stainless steel, which has an ultrasonic velocity of $5.74 \times 10^3 \text{ m.s}^{-1}$. The thickness "d" of the stainless steel is:

$$d_{\text{stainless}} = (5.740 \text{ m.s}^{-1}) (6.6 \times 10^{-6} \text{ s}) / 2 = 0.0189 \text{ m} = 18.9 \text{ mm}$$

The second pulse passes through both the aluminium and the stainless steel. The time required to pass through the aluminium is $10.7 \times 10^{-6} - 6.6 \times 10^{-6} = 4.1 \times 10^{-6}$ s. Since the ultrasonic velocity in aluminium is $6.25 \times 10^3 \text{ m.s}^{-1}$, then the thickness "d" of the aluminium is:

$$d_{\text{aluminium}} = (6250 \text{ m.s}^{-1}) (4.1 \times 10^{-6} \text{ s}) / 2 = 0.0128 \text{ m} = 12.8 \text{ mm}$$

23.22 A laminar composite is produced by gluing a 6 mm-thick sheet of glass onto a 50 mm thick plate of polyethylene. Determine the length of time required for an ultrasonic pulse to be received from (a) the interface between

the glass and the polymer and (b) from the back surface of the polymer. The transducer is located on the glass side of the composite.

The time required for a pulse to return from the back side of the glass is

$$t_g = (2) (6 \times 10^{-3} \text{m}) / 5.64 \times 10^3 \text{ m.s}^{-1} = 0.213 \times 10^{-5} \text{ s}$$

The time required for a pulse to return from the back side of a 50 mm thick plate of polyethylene is

$$t_{PE} = (2) (50 \times 10^{-3} \text{m}) / 1.96 \times 10^3 \text{ m.s}^{-1} = 5.102 \times 10^{-5} \text{ s}$$

The total time for a pulse to return from the back side of the composite is

$$t_{\text{composite}} = 0.213 \times 10^{-5} + 5.102 \times 10^{-5} = 5.32 \times 10^{-5} \text{ s}$$

23.23 Sketch the signal you expect to receive when an ultrasonic pulse passes through a sandwich structure composed of 4 mm-thick aluminium surrounded by two layers of 2.55 mm-thick nickel.

We would expect, at the minimum, reflections due to the first Ni-Al interface, the second Al-Ni interface, and finally the back surface of the second Ni layer. (There might also be additional reflections; for example the pulse may reflect off the second Al-Ni interface, then the first Ni-Al interface, and finally the second Al-Ni interface again before returning to the transducer -- we will ignore these secondary indications.) We can find the expected times for reception of the return pulses at the transducer from the ultrasonic velocities of each material and the thickness of each layer.

For the first Ni-Al interface, the distance the pulse must travel is 2 times the 2.5 mm thickness of the Ni:

$$t_1 = (2) (2.5 \times 10^{-3} \text{ m}) / (6.02 \times 10^3 \text{ m.s}^{-1}) = 0.831 \times 10^{-6} \text{ s}$$

For the reflection from the second Al-Ni interface, the pulse must travel an additional 2 x 4 mm distance through aluminium. The total time required for the pulse to return to the transducer is:

$$\begin{aligned} t_1 &= (2) (4 \times 10^{-3} \text{ m}) / (6.25 \times 10^3 \text{ m.s}^{-1}) + 0.831 \times 10^{-6} \\ &= 1.280 \times 10^{-6} + 0.831 \times 10^{-6} \\ &= 2.110 \times 10^{-6} \text{ s} \end{aligned}$$

For the reflection from the back of the second Ni sheet, the pulse travels another 2 x 2.5 mm through nickel. The total time required is:

$$\begin{aligned} t_1 &= 0.831 \times 10^{-6} + 1.280 \times 10^{-6} + 0.831 \times 10^{-6} \\ &= 2.9411 \times 10^{-6} \text{ s} \end{aligned}$$

23.24 We would like to monitor the thickness of nominally 0.01 mm thick aluminium foil using a resonance ultrasonic test. What frequency must we select if we want to produce 5 half-wavelengths in the foil?

Five half-wavelengths are:

$$5(\lambda/2) = 0.01 \text{ mm} \quad \text{or} \quad \lambda = 4 \times 10^{-3} \text{ mm} = 4 \times 10^{-6} \text{ m}$$

$$\begin{aligned} v &= v/\lambda = (6.25 \times 10^3 \text{ m.s}^{-1}) / 4 \times 10^{-6} \text{ m} \\ &= 1.56 \times 10^9 \text{ Hz} \end{aligned}$$

23.25 A resonance ultrasonic test is to be used to determine the thickness of a polyethylene coating on copper. A frequency of 31×10^6 Hz is required to produce 3 half-wavelengths in the coating. Calculate the thickness of the coating.

The wavelength is:

$$\lambda = v/v = (1.96 \times 10^3 \text{ m.s}^{-1}) / (31 \times 10^6 \text{ Hz}) = 6.32 \times 10^{-5} \text{ m}$$
$$\lambda/2 = 3.16 \times 10^{-5} \text{ m} = 3.16 \times 10^{-2} \text{ mm}$$

Since there are three half-wavelengths at resonance, the thickness is:

$$\text{thickness} = 3(3.16 \times 10^{-2}) = 0.0948 \text{ mm}$$

23.26 What nondestructive testing method(s) might be helpful in detecting the amount of ferrite produced in an austenitic stainless steel as a result of a welding process?

Ferrite is ferromagnetic, whereas austenite is not ferromagnetic. Some type of magnetic test might be used. One method might be to use magnetic particle testing; an indication would only be observed in the heat affected area of the weld if ferrite formed during the welding process.

23.27 What nondestructive testing method(s) might be helpful in detecting delamination between the plies of a fibre-reinforced composite material?

Both thermography and ultrasonic inspection might be suitable for this application. The delaminated area will be a hot spot in a thermography test; a heat sensitive coating on the composite will indicate higher temperatures at the delamination site as the composite material is heated and cooled. In ultrasonic inspection, a reflection is expected from the delaminated area, whereas no reflection is expected if the bond between the plies is perfect. There is a danger, however, that the fibres may provide so many interfaces that too much of the ultrasonic pulse will be scattered, making it difficult to detect a return pulse from the delaminated area.

23.28 What nondestructive testing method(s) might be helpful in detecting a crack that is parallel to the surface of a steel plate?

Ultrasonic inspection would be most suited; the orientation of the crack is such that a good indication should be received by the transducer; furthermore the size and depth of the crack beneath the surface can be determined ultrasonically. Other methods are either less likely to be successful, are more expensive, or are sensitive to the location of the crack. For example, magnetic particle inspection using the yoke method described in Figure 23.25(c) would not be effective because the lines of magnetic flux would not be disrupted. Eddy current inspection would only be effective if the crack were near the surface. The crack is likely too thin to be detected by radiography in this orientation; unless the crack is connected to the surface, liquid penetrant testing would not work.

23.29 What nondestructive testing method(s) might be helpful in detecting a crack that is perpendicular to and intersects the surface of an aluminium casting?

In this case, radiography, eddy current inspection, and liquid penetrant inspection would all work. The geometry is not correct for a simple pulse-echo ultrasonic test, and since aluminium is not ferromagnetic, magnetic particle inspection would not work.

Suggested Solutions to Design Problems

A single solution to a true design problem should not be possible. In a few of the design problems offered in the text, only one reasonable answer is possible. However in most cases the student may be able to select from a wide variety of assumptions and materials. Consequently this section provides sample solutions for one or, in some cases, several sets of assumptions. In many cases, only a discussion of the approach to the design problem or the design criteria that must be satisfied is provided. In some cases, the student may conclude that, at least with the information provided, no practical solution is possible.

As much as possible, the suggested solutions use data that appears in the text. One notable exception to this is Problem 12.37, which was written before the necessary figure was cut from the text in an effort to save space. This figure appears on page 269 of this manual and should be provided to the student. In many other cases, better designs could be obtained if the student is asked to seek outside references; certainly instructors are encouraged to do this.

Chapter 2 Atomic Structure

Suggested Solutions to Design Problems

2.25 You wish to introduce ceramic fibres into a metal matrix to produce a composite material, which is subjected to high forces and large temperature changes. What design parameters might you consider to assure that the fibres will remain intact and provide strength to the matrix? What problems might occur?

Both the modulus of elasticity and the coefficient of thermal expansion might be important design criteria in matching the fibres and the matrix.

a. if the fibres have a low coefficient of thermal expansion compared to the matrix, then a high tensile stress may develop in the fibres as the metal matrix heats and expands. If the stresses are high enough, the fibres may fracture; even if the fibres do not fracture, the high stresses produced thermally may limit the external forces that are applied during service. Due to the stronger bonding in the ceramic, we would expect that the ceramic will have a lower coefficient of expansion.

b. if the fibres have a low modulus of elasticity compared to the matrix and if the fibres and matrix are firmly bonded to one another, then the high force will try to cause the matrix to stretch more than the fibres and disturb the bonding between the fibre and the matrix. Due to the stronger bonding in the ceramic, we would expect that the ceramic fibre will have a higher modulus of elasticity than the metal matrix.

c. other factors also are important; we need to assure that good bonding is achieved between the fibres and the matrix; we need to be sure that the bonding remains at high temperatures; we need to assure that no adverse reaction occurs between the fibre and the matrix, which might destroy the bonding, embrittle the matrix, or even dissolve the fibre; we need to assure that the matrix will not oxidize at elevated temperatures.

2.26 A turbine blade made of nickel may corrode in a jet engine. What design parameters would you consider in selecting a coating that would protect the blade at the high operating temperatures, yet not break off when the blade cools to room temperature? What problems might occur? What types of materials might you select for the coating?

As in the previous problem, the modulus of elasticity and coefficient of thermal expansion are important. If the coefficient of thermal expansion of the coating is smaller than that of the nickel, the coating may crack when the turbine blade is heated, permitting the environment to attack the underlying metal. If the coefficient of thermal expansion of the coating is larger than that of the nickel, compressive stresses will be introduced into the coating and may cause it to buckle or even flake off the metal. If the coating is made of a protective ceramic material, or perhaps an intermetallic compound, the coating will have a low coefficient of thermal expansion, will be brittle, and will likely crack.

If the modulus of elasticity is different, differences in the strain produced during operation may cause the coating to debond from the nickel, leading to failure.

Again, the coating must be designed so that it has good high temperature properties and has good corrosion resistance at high temperatures in the environment inside the engine. This would suggest that materials such as ceramics or intermetallic compounds, which have good corrosion resistance and good high temperature properties, be used, provided that they can be engineered to give a coefficient of thermal expansion similar to that of the nickel.

2.27 An extrinsic semiconductor can be produced by introducing "impurity", or donor, elements into pure silicon. By doing so, additional electrons beyond those needed to participate in the bonding mechanism become part of the structure and can move. Design an alloy system that will cause this extrinsic semiconduction in silicon.

In order for the additional electrons to be introduced, the impurity, or donor, elements must have a higher valence than the silicon. Silicon has a valence of 4; therefore elements having a valence of 5 or greater might be possibilities. We could look at the elements in column VA of the periodic table, for example, to determine possible donors for the silicon. Nitrogen, phosphorus, arsenic, antimony, and bismuth are all possibilities. For each silicon atom replaced by a phosphorus atom, one extra electron is introduced into the semiconductor material.

To make the design problem more involved, the instructor might suggest that the student determine how much of a particular donor element might be required to produce a given number of electrons in one gram of silicon.

Chapter 3 Atomic Arrangement

Suggested Solutions to Design Problems

3.54 You would like to design a purification cell. The cell is to be composed of two sizes of spheres, with the smaller ones fitting into the holes between the large ones. A wide variety of sizes are available, but the largest is 1 cm in diameter. A contaminated gas will flow through the cell, with the contaminants being absorbed at the surfaces of the spheres. We have found that we need a total surface area of 10,000 cm² to accomplish the purification. Design such a cell.

There are as many possible solutions as there are students willing to attempt the design. The instructor may wish to provide information concerning what size spheres are available, or may wish to specify a certain overall volume that the cell should not exceed, or provide a density for the sphere and specify a weight limit for the cell. One possible solution might be as follows:

We assume that we will use the 1 cm diameter spheres to produce a FCC arrangement. We will then introduce smaller spheres that just fit into the tetrahedral interstitial sites in the FCC arrangement. The size of the smaller spheres must therefore be

$$r_{\text{small}} = 0.225r_{\text{large}} = (0.225)(0.5 \text{ cm}) = 0.1125 \text{ cm}$$

There are 8 unique tetrahedral sites in the FCC cube, which will have a "lattice" parameter of

$$a_{\text{FCC cell}} = 4r_{\text{large}} / \sqrt{2} = (4)(0.5) / \sqrt{2} = 1.414 \text{ cm}$$

The total surface area in the FCC "cell" is

$$A = (\text{number of large spheres/cell})(\text{area/large sphere}) + (\text{number of small spheres/cell})(\text{area/small spheres})$$

$$A = (4)[4\pi(r_{\text{large}})^2] + (8)[4\pi(r_{\text{small}})^2]$$

$$A = (4)(4\pi)(0.5)^2 + (8)(4\pi)(0.1125)^2 = 13.84 \text{ cm}^2$$

The number of cells needed to produce 10,000 cm² of surface area is

$$\text{cells} = 10,000 / 13.84 = 722.5 \text{ cells}$$

Therefore we need: 4 x 722.5 = 2890 spheres with a 1-cm diameter

$$8 \times 722.5 = 5780 \text{ spheres with a } 0.225\text{-cm diameter}$$

The volume occupied by the FCC "cell" is $V = (1.414)^3 = 2.827 \text{ cm}^3$; the volume of the purification cell must be $V_{\text{cell}} = 2.827 \times 722.5 = 2043 \text{ cm}^3$. If the cell were a cube, the cube must be 12.7 cm on each side.

Certainly a much smaller cell could be produced by using other combinations of sphere sizes.

3.55 You would like to sort iron specimens, some of which are FCC and others, BCC. Design a X-ray diffraction method by which this can be accomplished.

We might produce powders of both FCC and BCC iron and perform an X-ray diffraction analysis for both powders. We would expect that, when we index the diffracted peaks, we would obtain a different sequence of diffracting planes. We would have to select a particular wavelength of X-rays to use. We could use the X-rays with $\lambda = 0.07107$ nm as in Example 3.17, or the instructor could recommend a different wavelength or ask the student to determine what wavelength should be used. If we use $\lambda = 0.07107$, then we would expect to find the following 2θ values for the two different irons:

For the FCC iron
 $a_0 = 0.3589$ nm

$2\theta = 19.69$
 22.84
 32.52
 38.29
 40.12
 46.66

For the BCC iron
 $a_0 = 0.2866$ nm

$2\theta = 20.16$
 28.72
 35.40
 41.10
 46.16
 50.94
 55.29

By determining the 2θ pattern of peaks observed from the diffractometer, we can distinguish between the two forms of iron. Instead of obtaining a complete diffraction analysis, we may just wish to set the diffractometer at a particular 2θ angle, say 28.72° . If we get an indication of a peak, then the sample must be BCC; if no indication is observed, then the sample must be FCC.

Chapter 4 Imperfections in the Atomic Arrangement Suggested Solutions to Design Problems

4.37 The density of pure aluminium calculated from crystallographic data is expected to be $2.69955 \text{ Mg.m}^{-3}$. Suggest a suitable alloying element and calculate the atomic percentage of this element to produce an aluminium alloy that has a density of 2.6450 Mg.m^{-3} .

To change the density of the pure aluminium, we need to add lattice imperfections. To produce a lower density, we might introduce vacancies or we might introduce atoms having a lower atomic mass than aluminium ($26.981 \text{ g.mol}^{-1}$). One of the problems in doing this design, however, is that the effect of the lattice imperfections on the lattice parameter are not known; the student should be encouraged to either state that the lattice parameter will not change or to attempt to estimate a new lattice parameter based on how many lattice defects are added. Perhaps, for example, a lattice parameter could be estimated based on differences in atomic radii between aluminium and a substitutional atom.

As an example, we might decide to produce an alloy with a density of 2.6450 Mg.m^{-3} by adding lithium substitutional atoms, assuming that the lattice parameter of the aluminium ($a_0 = 0.404958 \text{ nm}$) does not change. We might let "x" be the number of lithium atoms introduced into each unit cell. Note that Al has a fcc structure.

$$2.6450 \times 10^6 = \frac{(4 - x)(26.981) + (x)(6.94)}{(4.04958 \times 10^{-10} \text{ m})^3 (6.02 \times 10^{23})}$$

$$x = 0.1088 \text{ Li atoms/cell}$$

or the atomic percent Li in the Al-Li alloy is

$$\text{at\% Li} = 0.1088 / 4 \times 100 = 2.7$$

4.38 You would like a metal plate with good weldability. During the welding process, the metal next to the weld is heated almost to the melting temperature and, depending on the welding parameters, may remain hot for some period of time. Design an alloy that will minimise the loss of strength in this "heat affected area" during the welding process.

The important conclusion that the student should reach is that whatever strengthening mechanism is used for the alloy, the strengthening mechanism should not be affected by high temperatures. Thus deciding to introduce large numbers of dislocations would not be effective, since the number of dislocations can be reduced by the "heat treatment" of the metal in the HAZ. Producing a very fine grain size may not be effective, since grain growth may occur in the HAZ. However, point defects such as interstitial and substitutional atoms are relatively stable -- these defects, once introduced into the lattice of the metal, remain even as the temperature changes.

Chapter 5 Atom Movement in Materials

Suggested Solutions to Design Problems

5.31 Design a spherical tank, with a wall thickness of 20 mm, that will assure that no more than 50 kg of hydrogen will be lost per year. The tank, which will operate at 500°C, can be made of nickel, aluminium, copper, and iron. A constant concentration of H in the tank of 2×10^{26} atom.m⁻³ will be maintained. The diffusion coefficient of hydrogen and the cost per kilogram for each available material is:

Material	Diffusion Data		
	D_0 m ² .s ⁻¹	Q J.mol ⁻¹	Cost £.kg ⁻¹
Nickel	0.0055×10^{-4}	37250	4.10
Aluminium	0.16×10^{-4}	43280	0.60
Copper	0.011×10^{-4}	39260	1.10
Iron	0.0012×10^{-4}	15150	0.60

Note that the diffusion data for hydrogen in iron is given for BCC iron, the form of iron at temperatures of 500°C and lower.

For this problem, we want to determine the diameter of a tank made of each of the materials that provides a flux that will prevent the 50 kg loss of hydrogen per year. The hydrogen atoms diffuse through the material as atoms, not molecules, so we find that the maximum number of hydrogen atoms that can diffuse per year is

$$\begin{aligned} \text{number of H atoms} &= \frac{(50,000 \text{ g})(6.02 \times 10^{23} \text{ atoms.mol}^{-1})}{(1.00797 \text{ g.mol}^{-1})} \\ &= 2.986 \times 10^{26} \end{aligned}$$

The flux of atoms J is atoms.m² s⁻¹. For our case, the number of seconds per year, the concentration gradient, and the area of the tank through which the atoms can diffuse can be calculated. We can assume that the concentration of the hydrogen outside the tank is zero.

$$t = 31.54 \times 10^6 \text{ s.yr}^{-1}$$

$$\Delta c/\Delta x = (0 - 2 \times 10^{26} \text{ H atoms.m}^3)/(20 \times 10^{-3} \text{ m}) = -1 \times 10^{28} \text{ H atoms.m}^3 \text{ m}^{-1}$$

$$A = 4\pi r^2 \quad \text{where } r \text{ is the radius of the tank}$$

Thus we can write J in two ways:

$$J = 2.986 \times 10^{26} \text{ atoms} / (31.54 \times 10^6 \text{ s})(4\pi r^2) = 7.53 \times 10^{17} / r^2$$

$$\text{and } J = -D \Delta c/\Delta x = 1 \times 10^{28} D$$

We can find the diffusion coefficient for each material at 500°C = 773 K:

$$\begin{aligned} D_{\text{nickel}} &= 1.67 \times 10^{-9} \text{ m}^2.\text{s}^{-1} \\ D_{\text{aluminium}} &= 1.903 \times 10^{-8} \\ D_{\text{copper}} &= 2.45 \times 10^{-9} \\ D_{\text{iron}} &= 1.152 \times 10^{-8} \end{aligned}$$

We can then calculate the maximum allowable tank radius for each material.

$$r = \sqrt{7.53 \times 10^{17} / 1 \times 10^{28} D} = 8.68 \times 10^{-6} \sqrt{D}$$

For the four materials,

$$r_{\text{nickel}} = 0.212 \text{ m} = 212 \text{ mm}$$

$$r_{\text{aluminium}} = 0.0629 \text{ m} = 62.9 \text{ mm}$$

$$r_{\text{copper}} = 0.0175 \text{ m} = 17.5 \text{ mm}$$

$$r_{\text{iron}} = 0.0081 \text{ m} = 8.1 \text{ mm}$$

If we are interested in having a large capacity of hydrogen, we wish to have a large diameter. The nickel will give the largest capacity, since it permits the largest radius. However nickel is expensive and also has a high density. Copper also has a large capacity, and is less expensive than aluminium. Aluminium is light weight, relatively inexpensive, but has a small capacity.

5.32 A steel gear initially containing 0.10% C is to be carburized so that the carbon content at a depth of 1.27 mm is 0.50% C. We can generate a carburizing gas at the surface that contains anywhere from 0.95% C to 1.15% C. Design an appropriate carburizing heat treatment.

We can use Fick's second law to help in our design. Only $c_o = 0.10\%$ C and $c_x = 0.50\%$ C at $x = 1.27$ mm are fixed. The surface carbon content c_s can vary from 0.95 to 1.15% C and the temperature can also vary. In carburizing, the steel is heated to produce FCC iron; to be safe, then, the minimum carburizing temperature should be 912°C. While there are many solutions, let's pick $c_s = 1.00\%$ C and find some possible combinations of temperatures and times.

$$\frac{1.00 - 0.50}{1.00 - 0.10} = \text{erf}(0.00127 / \sqrt{2Dt})$$

$$0.555 = \text{erf}(0.0635/\sqrt{Dt})$$

$$0.57 = 5.355 \times 10^{-4} / \sqrt{Dt}$$

$$Dt = 1.24 \times 10^{-6}$$

The diffusion coefficient for C in FCC iron is

$$D = 2.3 \times 10^{-5} \exp(-137,700/8.314)(T)$$

Thus: $(t) \exp(16,558/T) = 0.0539$

For different temperatures,

$$T = 950^\circ\text{C} = 1223 \text{ K}, \quad t = 41,000 \text{ s} = 11.41 \text{ h}$$

$$T = 1000^\circ\text{C} = 1273 \text{ K}, \quad t = 24,000 \text{ s} = 6.7 \text{ h}$$

$$T = 1100^\circ\text{C} = 1373 \text{ K}, \quad t = 9,350 \text{ s} = 2.60 \text{ h}$$

Different combinations of temperature and time would be obtained for different carbon atmospheres.

5.33 When a valve casting containing copper and nickel solidifies under nonequilibrium conditions, we find that the composition of the alloy varies substantially over a distance of 0.05 mm. Usually we are able to eliminate this concentration difference by heating the alloy for 8 h at 1200°C; however, sometimes this treatment causes the alloy to begin to melt, destroying the part. Design a heat treatment that will permit elimination of the nonuniformity without danger of melting. Assume that the cost of

operating the furnace per hour doubles for each 100°C increase in temperature.

The activation energy for Cu in Ni is higher than that for Ni in Cu; let's assume then that the diffusion of Cu in Ni limits the rate at which the heat treatment can be accomplished. For Cu in Ni,

$$D_{Cu} = 6.5 \times 10^{-5} \exp(-257450/RT)$$

We know that the heat treatment, under favourable conditions, can be accomplished in 8 hours at 1200°C, where

$$Dt = [6.5 \times 10^{-5} \exp(-257450 / 8.314 \times 1473)] (8 \text{ h}) = 4.181 \times 10^{-11}$$

We can determine the times for other possible temperatures, since the term Dt should be a constant.

$$\begin{array}{ll} T = 1100^\circ\text{C} & t = 39.7 \text{ h} \\ T = 1000^\circ\text{C} & t = 233 \text{ h} \\ T = 900^\circ\text{C} & t = 1859 \text{ h} \end{array}$$

Suppose that it costs £10 to operate the furnace for 1 h at 1200°C; then:

$$\begin{array}{l} \text{for } T = 1200^\circ\text{C, cost} = (\text{£}10/\text{h}) (8 \text{ h}) = \text{£}80 \\ \text{for } T = 1100^\circ\text{C, cost} = (\text{£}5/\text{h}) (39.7 \text{ h}) = \text{£}198.50 \\ \text{for } T = 1000^\circ\text{C, cost} = (\text{£}2.50/\text{h}) (233 \text{ h}) = \text{£}582.50 \\ \text{for } T = 900^\circ\text{C, cost} = (\text{£}1.25/\text{h}) (1859 \text{ h}) = \text{£}2324 \end{array}$$

It appears that we should use the highest possible temperature without melting the castings; perhaps we should drop the temperature to 1150°C and see if that will work.

Chapter 6 Mechanical Testing and Properties

Suggested Solutions to Design Problems

6.52 A hook for hoisting containers of ore in a mine is to be designed using a nonferrous material. The hook must support a load of 111 kN, and a factor of safety of 50% should be used. We have determined that the cross-section labeled "?" is the most critical area; the rest of the device is already well over-designed. Determine the design requirements for this device and, based on the mechanical property data given in Chapters 13 and 14 and the metal prices given in Table 13.1, design the hook and select an economical material for the hook.

We almost surely will not select any of the ceramics discussed in Chapter 14; the ceramics will be too brittle and likely will have too low of a fracture toughness for us to consider. Therefore we can limit our design to the nonferrous alloys in Chapter 13.

In order to have a factor of safety of 50%, we should design this section of the hook to support a load of 222 kN without exceeding the yield strength of the material that we select. Therefore we can calculate the cross-sectional area and diameter of the hook if we know the yield strength for the material of choice. We could select a variety of materials from the tables in Chapter 13. As a couple of examples, let's pick a 7075-T6 aluminium alloy from Table 13.5; this alloy has a yield strength of 505 MPa. Let's compare it to an age-hardened Cu-2% Be alloy in Table 13.7; its yield strength is 1205 MPa.

$$A_{Al} = F/\sigma = 222 \text{ kN} / 505 \text{ MPa} = 440 \text{ mm}^2 \quad d_{Al} = 23.6 \text{ mm}$$

$$A_{Cu} = F/\sigma = 222 \text{ kN} / 1205 \text{ MPa} = 184 \text{ mm}^2 \quad d_{Cu} = 15.3 \text{ mm}$$

The Cu-Be alloy will permit a smaller hook to be produced. However the weight of the hook will be larger and the cost of the alloy will be significantly higher.

6.53 A support rod for the landing gear of a private airplane is subjected to a tensile load during landing. The loads are predicted to be as high as 178 kN. Because the rod is crucial and failure could lead to a loss of life, the rod is to be designed with a factor of safety of 75% (that is, designed so that the rod is capable of supporting loads four times as great as expected). Operation of the system also produces loads that may induce cracks in the rod. Our nondestructive testing equipment can detect any crack greater than 0.5 mm deep. Based on the materials given in Table 6.6, design the support rod and the material, and justify your answer.

The cross-sectional area of the rod must support a load of 712 kN, when the factor of safety is included. The polymers listed in Table 6.6 would be eliminated based on their low yield strength, while the ceramics would likely be rejected based on poor toughness as well as on difficulties in fabrication and assembly of the part to the rest of the landing gear. The Si_3N_4 -SiC composite requires more

consideration; it has a high yield strength and good fracture toughness; however producing the ceramic-ceramic composite is likely to be very expensive, particularly for a private aircraft. Based on these considerations, it is likely that we would select one of the three metal alloys.

Based only on yield strength, the cross-sectional areas of the rod that would permit a load of 712 kN to be supported would be:

Al-Cu (high strength): $A = 455 = 1565 \text{ mm}^2$
 (low strength): $A = 325 = 2191 \text{ mm}^2$
 Ti (high strength): $A = 900 = 791 \text{ mm}^2$
 (low strength): $A = 860 = 828 \text{ mm}^2$
 Steel (high strength): $A = 1640 = 434 \text{ mm}^2$
 (low strength): $A = 1420 = 501 \text{ mm}^2$

Even considering the low density of aluminium, titanium or steel would more likely provide the lowest weight part. In addition the steel, even though alloyed with nickel and chromium, will likely be less expensive than titanium and will weigh about the same. The steel would then be our choice.

But the fracture toughness is also important, since we might expect to have flaws as large as 0.5 mm deep that we would not be able to detect by our inspection methods. If we just consider the steel, and assume that $f = 1$ in the fracture toughness equation $K = f\sigma\sqrt{\pi a}$

high strength steel: $\sigma = K/f\sqrt{\pi a} = 50/(1)\sqrt{0.0005\pi} = 1262 \text{ mPa}$
 $A = 564 \text{ mm}^2$

low strength steel: $\sigma = K/f\sqrt{\pi a} = 88/(1)\sqrt{0.0005\pi} = 2220 \text{ mPa}$
 $A = 321 \text{ mm}^2$

To account for fracture toughness as well as yield strength, the high strength steel must have a minimum cross-sectional area of 564 mm^2 (governed by fracture toughness), while for the low strength steel, the minimum cross-sectional area is 501 mm^2 (governed by yield strength/safety factor considerations). The same exercise might be repeated for the titanium alloy.

6.54 A lightweight rotating shaft for a pump on the national aerospace plane is to be designed to support a cyclical load of 66.7 kN during service. The maximum stress is the same in both tension and compression. The endurance limits or fatigue strengths for several candidate materials are shown below. Design the shaft, including an appropriate material, and justify your solution.

Al-Mn alloy	110 MPa fatigue strength
Al-Mg-Zn alloy	225 MPa fatigue strength
Cu-Be alloy	295 MPa fatigue strength
Mg-Mn alloy	80 MPa fatigue strength
Be alloy	180 MPa fatigue strength
Tungsten alloy	320 MPa fatigue strength

Let's assume that Equation 6.16 is appropriate.

$$\sigma = 10.18\ell F/d^3 \quad \text{or} \quad d^3/\ell = 10.18F/\sigma$$

The maximum allowable stress is the endurance limit, or fatigue strength. Neither the shaft length nor diameter are specified.

$$F = 15,000 \text{ lb} = 66,729 \text{ N}$$

For Al-Mn: $d^3/\ell = (10.18)(66,729 \text{ N})/(110 \text{ MPa}) = 6,175 \text{ mm}^2$

Thus for the Al-Mn alloy, the shaft might have the dimensions:

$$\begin{array}{lll} \ell = 100 \text{ mm} = 10 \text{ cm} & d = 85 \text{ mm} = 8.5 \text{ cm} & V = 567 \text{ cm}^3 \\ \ell = 200 \text{ mm} = 20 \text{ cm} & d = 107 \text{ mm} = 10.7 \text{ mm} & V = 1800 \text{ cm}^3 \end{array}$$

As we increase the length of the shaft, the diameter must also increase, thereby increasing the weight and cost. Therefore the student is forced to assume a reasonable length for the shaft, then determine the diameter that will match up with this length. From the volume, we can estimate a weight and, if we use Table 13.1 as a guide, estimate the cost of the aluminium alloy. We can do the same for the other alloys in the table, perhaps for the same assumed length, and then select the alloy that provides the assumed dimensions, a reasonable weight, and a reasonable cost. These calculations are left to the student, since the possibilities are endless.

6.55 A ductile cast iron bar is to support a load of 178 kN in a heat-treating furnace used to make malleable cast iron. The bar is located in a spot that is continuously exposed to 500°C. Design the bar so that it can operate for a least 10 years without failing.

We might use the Larson-Miller parameter, Figure 6-25(b), to help us in this design. The bar is to operate for 10 years = 87,600 h.

$$\text{Larson-Miller parameter} = (773/1000)[36 + 0.78 \ln(87,600)] = 34.7$$

From the graph, the maximum allowable stress is about 50 MPa. Thus the size of the bar must be

$$A = F/\sigma = 178 \text{ kN} / 500 \text{ MPa} = 3560 \text{ mm}^2$$

If the bar has a square cross-section, the dimensions would be 59 mm. If the bar has a round cross-section, the dimensions would be 67 mm dia.

Chapter 7 Strain Hardening and Annealing Suggested Solutions to Design Problems

7.26 Determine, using one of the processes shown in Figure 7.5, a method to produce each of the following products. Should the process include hot working, cold working, annealing, or some combination of these? Explain your decisions.

a. paper clips: *These are likely produced by cold working -- bending a wire into the proper shape. Cold working would be good since the parts are small, large forces are not needed, and the cold working provides good stiffness to the parts.*

b. I-beams that will be welded to produce a portion of a bridge: *This large section may require hot working to reduce the forces and equipment size needed. A hot rolling process might be used.*

c. copper tubing that will connect a water tap to the main copper plumbing: *Hard copper tubing might be produced by cold working, perhaps by an extrusion process. If we wanted the tubing to be more flexible, we might use a hot extrusion process.*

d. the steel tape in a tape measure: *We'd like the steel tape to be stiff; a cold rolling process might be used.*

e. a head for a carpenter's hammer formed from a round rod: *The hammer head is relatively complex. A closed die forging process might be used; several dies might be required to shape the steel into the final shape. To help assure production of the head without producing cracks, we would do this by hot forging.*

7.27 We plan to join two sheets of cold-worked copper by soldering. (Soldering involves heating the metal to a high enough temperature that a filler material melts and is drawn into the joint.) Describe a soldering process that will not soften the copper. Explain. Could we use higher soldering temperatures if the sheet material were a Cu-30% Zn alloy? Explain.

The recrystallisation temperature of copper is about $(1085 + 273)(0.4) - 273 = 270^{\circ}\text{C}$. If we solder at a temperature above 270°C , we run the risk of softening the area around the joint by recrystallisation; thus we should try to use a low melting point solder and produce the joint as quickly as possible.

Alloying elements are expected to increase the recrystallisation temperature, permitting us to perform the soldering process at higher temperatures. However, as we will see in later chapters, adding zinc also reduces the melting temperature compared to pure copper.

7.28 We wish to produce a 1-mm-diameter copper wire having a minimum yield strength of 400 Mn.m^{-2} and a minimum %elongation of 5%. We start with a 20-mm-diameter rod. Design the process by which the wire can be drawn. Include all important details and explain.

The percent cold work should be on the order of 40% if we want to have a minimum yield strength of 400 Mn.m^{-2} and a minimum %elongation of 5%. This means that the final step in the process should be a cold drawing process from an intermediate diameter; the intermediate diameter is:

$$40\% = \frac{d_i^2 - d^2}{d_i^2} \times 100 = \frac{d_i^2 - (1)^2}{d_i^2} \times 100 \quad \text{or} \quad d_i = 1.29 \text{ mm}$$

We would pull the 20 mm rod through a die, say with 80% cold work, to:

$$\frac{(20)^2 - d^2}{(20)^2} \times 100 = 80 \quad \text{or} \quad d = 8.94 \text{ mm}$$

We would then anneal and do another cold working step, anneal, cold work and continue until the rod is reduced to 1.29 mm. After a final anneal, the wire would be cold drawn to the final 1 mm diameter. We might use a final die that has a slightly smaller opening to compensate for any elastic springback that might occur.

We could reduce the number of steps by hot working the rod to an intermediate diameter; however we would not be able to use a hot drawing process, since drawing requires that strain hardening occur. We might, however, hot extrude the rod to a smaller diameter, then perform the final steps by cold drawing.

Chapter 8 Principles of Solidification Strengthening and Processing

Suggested Solutions to Design Problems

8.33 Aluminium is melted under conditions that give $0.6 \text{ mm}^3 \text{ H}_2$ per g of aluminium. We have found that we must have no more than $0.02 \text{ mm}^3 \text{ H}_2$ per g of aluminium in order to prevent the formation of hydrogen gas bubbles during solidification. Design a treatment process for the liquid aluminium that will assure that hydrogen porosity does not form.

Using Sievert's law, we could determine the quality of a vacuum required to remove the hydrogen. The aluminium was most likely melted at atmospheric pressure.

$$0.6 / 0.02 = K\sqrt{1 \text{ atm}} / K\sqrt{p_{\text{vacuum}}}$$

$$p_{\text{vacuum}} = 0.0011 \text{ atm}$$

We could melt the aluminium, place the aluminium in a vacuum chamber and reduce the pressure to 0.0011 atm . In order to establish equilibrium between the molten aluminium and the low pressure atmosphere, the hydrogen would escape from the molten metal.

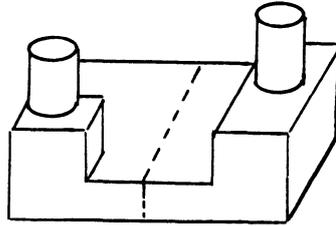
We could also help reduce hydrogen contents by melting the aluminium in a protective atmosphere, or we could assure that the aluminium was heated to only low superheats (minimising the amount of hydrogen gas picked up by the molten metal), or we could bubble an inert gas through the molten metal to remove the hydrogen.

8.34 When two 12 mm thick copper plates are joined using an arc-welding process, the fusion zone contains dendrites having a SDAS of 0.06 mm. However, this process produces large residual stresses in the weld. We have found that residual stresses are low when the welding conditions produce a SDAS of more than 0.2 mm. Design a process by which we can accomplish low residual stresses. Justify your design.

A SDAS of 0.06 mm corresponds to a solidification time of about 2 seconds, a reasonable time for a weld. We need to produce, however, a SDAS of 0.2 mm, which corresponds to a solidification time of about 200 seconds. The copper plate acts as a very good heat sink and consequently this long of a solidification time will be very difficult to achieve. We might recommend that the plates be preheated to a very high temperature before welding, and/or that postheat be applied immediately after welding and/or that a slow rate of heat input welding process such as oxyacetylene welding be used to reduce the rate at which heat is extracted from the weld and thus to produce the longer solidification time.

8.35 Design an efficient risering system for the casting in Figure 8.29. Be sure to include a sketch of the system, along with appropriate dimensions.

There appear to be two "hot spots" in the casting that will require risers. We might then have to split the casting into two sections, with one riser located at the 50 mm x 50 mm x 50 mm boss on the left and the second riser located at the 100 mm thick section on the right. We might split the casting as shown in the sketch:



For the left-hand side:

$$V_1 = (50)(50)(50) + (50)(162.5)(300) = 2.5625 \times 10^6 \text{ mm}^3$$

$$A_1 = [(300 \times 50) + (2)(162.5)(50) + (2)(162.5)(300) - (50)(50) + (5)(50)(50)] = 1.3875 \times 10^5 \text{ mm}^2$$

$$\frac{V_1}{A_1} = \frac{2.5625 \times 10^6}{1.3875 \times 10^5} = 18.5 \text{ mm}$$

We could set a cylindrical riser that has its height H equal to its diameter D . Then for the riser:

$$\frac{V_r}{A_r} = \frac{\pi D^2 H / 4}{[\pi D^2 / 4 + \pi D H]} = D / 5$$

The very minimum riser size is that which produces the same solidification time as the casting, or $V_r / A_r = V_1 / A_1$

$$D / 5 = 18.5 \text{ or } D = H = 92.5 \text{ mm, } V_r = 6.216 \times 10^5 \text{ mm}^3$$

For the right hand side:

$$V_2 = 3.5625 \times 10^6 \text{ mm}^3$$

$$A_2 = 1.8125 \times 10^5 \text{ mm}^2$$

$$\frac{V}{A} = 19.6 \text{ mm}$$

$$D / 5 = 19.6 \text{ or } D = H = 98 \text{ mm } V_r = 7.3921 \times 10^5 \text{ mm}^3$$

We would, in practice, make the riser larger than this minimum size.

8.36 Establish a process that will produce a steel casting having uniform properties and high strength. Be sure to include the microstructure features you wish to control and explain how you would do so.

We would like to assure that the liquid steel contains grain refiners that would encourage a large equiaxed zone and small equiaxed grains. We would like to encourage rapid solidification rates to help produce small secondary dendrite arm spacings in the casting; correct choice of a mould material might help us here. Of course, most of the properties of a steel casting will be developed later during heat treatment.

8.37 An aluminium casting is to be injected into a steel mould under pressure (die casting). The casting is essentially a 300 mm long, 50 mm diameter cylinder with a uniform wall thickness, and it must have a minimum tensile strength of 275 Mn.m². Based on the properties given in Figure 8.8, design the casting and process.

To obtain a tensile strength of 275 Mn.m^{-2} , we would need to produce a secondary dendrite arm spacing of less than 0.08 mm . For aluminium, the solidification time would then have to be less than 200 seconds . Thus the mould material and the thickness of the casting must be controlled to produce this solidification time.

$$V_c = [(\pi/4)(50)^2 - (\pi/4)d_{\text{inner}}^2](300)$$

$$A_c = [(\pi(50) + \pi(d_{\text{inner}})](300)$$

Note that the surface areas at each end of the casting have been omitted from this calculation.

We might consider a couple of different inner diameters and wall thicknesses.

$$d_{\text{inner}} = 25 \text{ mm}, \quad V = 4.418 \times 10^5 \text{ mm}^3, \quad A = 7.0686 \times 10^4 \text{ mm}^2$$

$$V/A = 6.25 \text{ mm}$$

$$d_{\text{inner}} = 37.5 \text{ mm}, \quad V = 2.577 \times 10^5 \text{ mm}^3, \quad A = 8.2467 \times 10^4 \text{ mm}^2$$

$$V/A = 3.125 \text{ mm}$$

If the inner diameter is 25 mm , or a wall thickness of 12.5 mm , then the mould constant required must be

$$t = 200 = B(6.25)^2 \quad \text{or} \quad B \leq 5.1 \text{ s.mm}^{-2}$$

If the inner diameter is 37.5 mm , or a wall thickness of 6.25 mm , then the mould constant required must be

$$t = 200 = B(3.125)^2 \quad \text{or} \quad B \leq 20.5 \text{ s.mm}^{-2}$$

The $B = 5.1 \text{ s.mm}^{-2}$ is typical for aluminium cast into a sand mould; this means that the cooling rate in the permanent mould should be more than sufficient to cause the casting to solidify within 200 s . For the thinner casting, which only requires a $B = 20.5 \text{ s.mm}^{-2}$, almost any casting process will produce cooling rates fast enough to produce a 200 s solidification time.

Chapter 9 Solid Solution Strengthening and Phase Equilibrium

Suggested Solutions to Design Problems

9.37 Homogenization of a slowly cooled Cu-Ni alloy having a secondary dendrite arm spacing of 0.025 cm requires 8 hours at 1000°C. Design a process to produce a homogeneous structure in a more rapidly cooled Cu-Ni alloy having a SDAS of 0.005 cm.

We can determine the constant c in Equation 9.6, assuming that the rate of diffusion, and thus homogenization, is controlled by copper in nickel ($Q = 257000 \text{ J.mol}^{-1}$)

$$t = c(\text{SDAS})^2/D_s \quad \text{or} \quad 8 \text{ h} = c (0.025 \text{ cm})^2/D_o \exp(-257000/8.314 \times 1273)$$

$$c = 3.53 \times 10^{-7} D_o$$

If we want to produce a SDAS of 0.005 cm, then

$$t = \frac{(3.53 \times 10^{-7} D_o)(0.005 \text{ cm})^2}{D_o \exp(-257000/8.314 * T)}$$

If $T = 1000^\circ\text{C}$, then $t = 0.32 \text{ h}$

If $T = 800^\circ\text{C}$, then $t = 29.7 \text{ h}$

Other combinations of homogenization temperature and time could also be calculated.

Therefore we could reduce either the homogenization time at 1000°C, or we could reduce the temperature required for homogenization. The student might make estimates as to what combination of temperature and time might be most economical.

9.38 Design a process to produce a NiO-60% MgO refractory whose structure is 40% glass phase at room temperature. Include all relevant temperatures.

When ceramic liquids cool, they may produce a glass rather than a crystalline phase. What we might wish to do is produce a structure that is 40% liquid, 60% solid, then rapidly cool this structure, permitting the liquid to become a glass. We therefore need to determine a temperature that will produce 40% liquid. From the phase diagram, at different temperatures,

$$\begin{aligned} 2500^\circ\text{C}: \quad \%L &= (74 - 60) / (74 - 50) = 58\% \\ 2480^\circ\text{C}: \quad \%L &= (72 - 60) / (72 - 46) = 46\% \\ 2460^\circ\text{C}: \quad \%L &= (70 - 60) / (70 - 44) = 38\% \\ 2440^\circ\text{C}: \quad \%L &= (67 - 60) / (67 - 42) = 28\% \end{aligned}$$

We could plot %L versus holding temperature, then find that we produce 40% liquid near 2470°C. If we heat the ceramic to this temperature, hold until 40% liquid forms, then cool rapidly, we may produce the desired structure.

9.39 Design a method by which glass beads (having a density of 2.3 Mg.m^{-3}) can be uniformly mixed and distributed in a Cu-20% Ni alloy (density of 8.91 Mg.m^{-3}).

If we simply stir the glass beads into the liquid alloy and pour the mixture into a mould, the glass beads will float and a nonuniform distribution will be obtained. However, if we allow the Cu-Ni alloy to cool to a temperature where perhaps half of the structure is liquid and the other half solid, then stir in the glass beads, the beads may be retained uniformly in the "mushy" Cu-Ni alloy. The "mushy" mixture of solid alloy, liquid alloy, and glass beads can then be injected under pressure into a mould. The temperature at which the glass beads is introduced would be somewhere between the liquidus (1200°C) and the solidus (1160°C). This is the essence of "compcasting".

9.40 Suppose that MgO contains 5 mol% NiO. Design a solidification purification method that will reduce the NiO to less than 1 mol% in the MgO.

We might take advantage of the fact that, when the ceramic liquid first starts to solidify, the first solid that forms is rich in the higher melting point material, in this case MgO. From the phase diagram, the first solid is near our goal of only 1 mol% NiO. Therefore we might develop a process in which the ceramic liquid is allowed to begin to solidify, and the solid that forms is then removed from the solidifying mixture. Perhaps a process could be designed in which the impure ceramic is continuously introduced, while solid is continuously removed. If the purification was not sufficient in the first step, the solid could be reprocessed; the liquidus temperature would be higher the second time, but the purity of the first solid to form would also be improved.

Chapter 10 Dispersion Strengthening by Solidification Suggested Solutions to Design Problems

10.34 Design a processing method that permits a Pb-15% Sn alloy solidified under nonequilibrium conditions to be hot-worked.

When the Pb-15% Sn alloy solidifies under nonequilibrium conditions, a nonequilibrium eutectic microconstituent may form. This will mean that the alloy will begin to melt when it is heated to 183°C, even though based on the equilibrium phase diagram the solidus temperature of the alloy is expected to be as high as 250°C. Therefore the maximum allowable hot working temperature for the nonequilibrium alloy should be on the order of 180°C.

10.35 Design a eutectic diffusion bonding process to join aluminium to silicon. Describe the changes in microstructure at the interface during the bonding process.

If we were to join aluminium and silicon by conventional fusion welding processes, the fusion zone will contain large percentages of both Al and Si. This could produce a hypereutectic alloy that might have poor ductility, causing the weld to be embrittled and crack. In addition, the large temperature gradients produced by the welding process in the brittle, covalently-bonded silicon might lead to thermal stresses that would crack the silicon in the heat affected area.

A different approach to joining the two materials might be eutectic diffusion bonding. In this approach, the aluminium and silicon are pressed together at a temperature below the melting points of both the pure aluminium and the pure silicon, but above the eutectic temperature (577°C) of the Al-Si system.

We might decide to heat the materials to 600°C. As the two surfaces are held in contact, a counterdiffusion process occurs, with aluminium diffusing into silicon while silicon diffuses into aluminium. This causes the composition of both the aluminium and silicon surfaces to change, forming some liquid at the interface, with the composition and amount of liquid varying depending on the location. The presence of the liquid assures that voids at the interface are filled and that there is good contact across the interface. At this point, the materials could be cooled below the eutectic temperature, the liquid would solidify, and the aluminium and silicon would be joined.

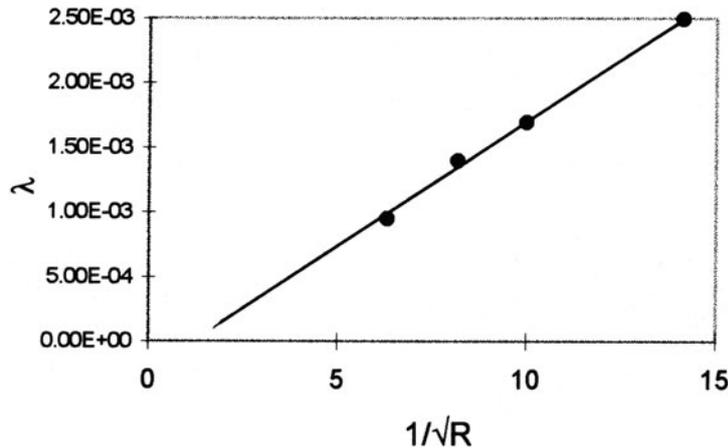
10.36 Design a directional solidification process that will give an interlamellar spacing of 0.0005 mm in a Pb-Sn eutectic alloy.

To obtain the desired interlamellar spacing of 5×10^{-4} mm in the eutectic, we need to determine the required growth, or solidification rate. From Figure 10.18, we find that the curve would have to be extrapolated to determine the necessary growth rate. We could try to get a better estimate of the growth rate by replotting the data

contained in Figure 10.18 as λ versus $1/\sqrt{R}$:

$\lambda = 2.5 \times 10^{-3} \text{ mm}$	$R = 0.005 \text{ mm.s}^{-1}$	$1/\sqrt{R} = 14.14$
$\lambda = 1.7 \times 10^{-3} \text{ mm}$	$R = 0.010 \text{ mm.s}^{-1}$	$1/\sqrt{R} = 10.00$
$\lambda = 1.4 \times 10^{-3} \text{ mm}$	$R = 0.015 \text{ mm.s}^{-1}$	$1/\sqrt{R} = 8.165$
$\lambda = 0.95 \times 10^{-3} \text{ mm}$	$R = 0.025 \text{ mm.s}^{-1}$	$1/\sqrt{R} = 6.325$

The data is plotted in the graph below. From the graph, we find that to obtain $\lambda = 5 \times 10^{-4} \text{ mm}$, $1/\sqrt{R} = 3.63$ or $R = 0.0756 \text{ mm.s}^{-1}$. We might design a directional solidification set-up as described in Example 10.7. By withdrawing the mould from the furnace at 0.0756 mm.s^{-1} , we can produce the desired interlamellar spacing.



10.37 Design an Al-Si brazing alloy and process that will be successful in joining an Al-Mn alloy that has a liquidus of 659°C and a solidus of 656°C .

Brazing requires that the filler material melt but that the base materials (in this case the Al-Mn alloy) does not melt. In order to braze the Al-Mn alloy, we must therefore select a brazing alloy that has a liquidus temperature below the solidus (656°C) of Al-Mn. From the Al-Si phase diagram, we find that Al-Si alloys containing between approximately 3% Si and 16% Si have a liquidus temperature below 656°C (it's hard to tell the exact cutoff compositions due to the scale of Figure 10.22).

We also want to be sure that we have a reasonable amount of superheat in the melted brazing filler material to assure that the liquid filler will flow into the joint, and have a little safety factor so that the Al-Mn alloy doesn't melt. If we assume that 50°C superheat is needed, and wish to stay at least 6°C below the Al-Mn solidus (we probably should use more of a safety factor than this), then the maximum liquidus might be about 600°C ; this will further restrict the silicon content of the filler to about 10 to 14% Si.

Our final design might be to select an Al-12% Si brazing material. We would heat the Al-Mn alloy parts to 650°C , being sure to maintain very careful control of the temperature. The brazing material would be applied to the joint between the Al-Mn parts, which should be separated by only a very small clearance to facilitate capillary filling of the joint. A flux might be required to help condition the surfaces of the Al-Mn parts to permit good wetting and joint filling. The brazing material will melt, be drawn into the joint, and, after the heat source is removed, will solidify and produce a bond.

10.38 Your company would like to produce light-weight aluminium parts that have excellent hardness and wear-resistance. The parts must have a good combination of strength, ductility, and internal integrity. Design the

process flow from the start of melting to the time that the liquid metal enters the mould cavity.

For excellent hardness and wear-resistance, we might select a hypereutectic Al-Si alloy; perhaps Al-17% Si would be appropriate. This would permit $(17-12.6) / (100 - 12.6) \times 100 = 5.7\%$ β (or nearly pure Si) to precipitate as a primary microconstituent during solidification. It is this primary β that we would rely on for hardness and wear-resistance.

To assure overall high quality cast parts, we might include in our process the following steps:

a. Melt the liquid Al-17% Si alloy in a protective atmosphere or under a protective flux to assure that excess oxidation or hydrogen pick-up in the liquid metal is avoided.

b. Degass the liquid aluminium with nitrogen, chlorine, or an inert gas to remove any hydrogen that might cause gas porosity in the final casting.

c. Add phosphorus (usually as a Cu-15% P alloy) to provide nucleation sites for the primary β ; this will help prevent gravity segregation, improve machinability, and improve surface finish.

d. Add strontium or sodium to modify the eutectic microconstituent to assure improved ductility and strength

e. Be sure that risers are used to compensate for solidification shrinkage.

Chapter 11 Dispersion Strengthening by Phase Transformation

Suggested Solutions to Design Problems

11.36 You wish to attach aluminium sheet to the frame on the 24th floor of a skyscraper. You plan to use rivets made of an age-hardenable aluminium, but the rivets must be soft and ductile in order to close. After the sheets are attached, the rivets must be very strong. Design a method for producing, using, and strengthening the rivets.

In order to satisfy the design requirements, we should use a natural aging alloy. The rivets would be produced by a cold forming process. We would solution treat the rivets to produce a soft, homogeneous structure, then quench the rivets to suppress the precipitation process. The rivets, after quenching, would be relatively soft and formable. The rivets would be stored in a refrigerated environment, below room temperature, to assure that the aging process would not begin. Only when the rivets are actually ready to be installed would they be removed from the refrigeration and used. Over a few hours time, aging would occur and the rivets would develop their strength. Thus our final material might be an Al-Cu naturally-aging alloy to be installed in the solution treated condition.

11.37 Design a process to produce a polypropylene polymer with a structure that is 75% crystalline. Figure 11.29 will provide appropriate data.

We could try to control the solidification of liquid polypropylene in such a way that 75% crystallisation would occur; however determining and controlling the cooling rate required might be very difficult. Instead, we might produce the amorphous structure, then heat treat the polymer to allow the crystalline structure to form. From Figure 11.29, we can estimate the time required for 75% transformation from the amorphous to the crystalline polypropylene to occur:

130°C: 12 min
140°C: 70 min
150°C: 500 min

From Table 15.2, we would find that the melting temperature for polypropylene is about 168 to 176°C, so that represents the maximum temperature.

We could make the problem easy by specifying that we will use one of the above temperatures; for example, produce a amorphous structure, heat the polymer to 140°C and hold for 70 min, then cool back to room temperature. Or, if we wished, we could determine an activation energy for the process, with the rate being the reciprocal of the time required for 75% transformation. A semi-log plot of reciprocal time versus the reciprocal of temperature should provide us with a straight line, whose slope is Q/R . With the Arrhenius relationship established, we could determine the transformation time required for any other temperature. By doing this, the equation will predict that the time required for transformation at room temperature would be very short; but in fact, below some temperature, the kinetics of the

transformation are expected to change and the time for transformation will actually begin to increase again.

11.38 An age-hardened Al-Cu bracket is used to hold a heavy electrical sensing device on the outside of a steel-making furnace. Temperatures may exceed 200°C. Is this a good choice? Explain. If it is not, design an appropriate bracket and explain why your choice is acceptable.

The design is almost surely a poor one. At 200°C, an Al-Cu alloy will overage rather rapidly, perhaps beginning within one hour, as Figure 11.11 indicates. As the alloy overages and its yield strength decreases, the bracket will be overloaded and will fail.

The bracket must be made of a material that will not lose its properties at 200°C -- controlling the age hardening process to produce higher strengths will not be of help. Selecting a cold worked aluminium alloy would be of little help either, since the recrystallisation temperature of the aluminium is expected to be near 100°C.

Consequently, if the bracket is to be made of aluminium, we would have to select a solid solution strengthened alloy that doesn't lose its strength catastrophically at 200°C and probably redesign the bracket, making it larger to account for the lower strength of the solid solution alloys compared to the age hardened alloys.

We might also use a dispersion strengthened aluminium alloy, as will be discussed in Chapters 13 and 16, which may maintain good strengths at elevated temperatures. Or we might be forced to switch to a different alloy system altogether.

11.39 You use an arc-welding process to join an eutectoid steel. Cooling rates may be very high following the joining process. Describe what happens in the heat-affected area of the weld and discuss the problems that might occur. Suggest a joining process that may minimise these problems.

During the welding process, the steel close to the fusion zone of the weld will heat above the eutectoid temperature, forming all austenite, or γ . This area is the heat affected zone of the steel weld. When the heat source, or arc, is removed, the surrounding base metal, still near room temperature, will act as a powerful heat sink, rapidly cooling the austenite in the heat affected area. With such high cooling rates, the austenite may transform to martensite and, due to temperature gradients associated with the welding process and thermal stresses induced both by the volume change caused by the transformation and by differences in thermal expansion and contraction of the metal, the weld may actually crack through the martensite in the heat affected area.

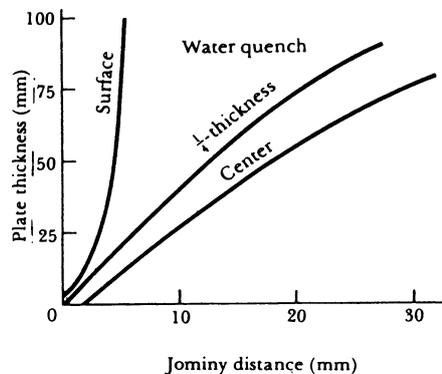
We might try to minimise the formation of martensite and the problems that it causes by several actions. For example, we might preheat the base metals to several hundred degrees prior to welding; this provides smaller temperature gradients, more uniform expansion and contraction of the metal, and, perhaps most importantly, may reduce the cooling rate sufficiently to permit the austenite to transform to pearlite rather than to martensite. Or we might immediately postheat the weld area: if the postheat is supplied immediately, the cooling rate will be reduced so that martensite does not form; if applied a bit later, any martensite that does form might be tempered, thus restoring some ductility to the weld area. Finally, we might use a welding process (such as oxyacetylene welding) that introduces the heat at a low rate, causing the base materials to heat to higher temperatures and also to then cool the heat affected zone at a slower rate.

Chapter 12 Ferrous Alloys

Suggested Solutions to Design Problems

12.37 We would like to produce a 50 mm thick steel wear-plate for a rock-crushing unit. To avoid frequent replacement of the wear plate, the hardness should exceed HRC 38 within 6 mm of the steel surface. However, the centre of the plate should have a hardness of no more than HRC 32 to assure some toughness. We have only a water quench available to us. Design the plate, assuming that we only have the steels given in Figure 12.23 available to us.

The graph that would be particularly helpful in creating this design was cut from the text; the author forgot to change the problem to reflect this cut. The student should be provided with the following graph, which provides the Jominy distance equivalents at various locations within steel plates for a water quench:



If we plan to do produce the plate by quenching the steel, we must consider the hardenability of the steel and the hardness that is produced at the two locations in the steel plate. At the centre of the 50 mm thick wear plate, the Jominy distance is about 18 mm. The hardness must be less than HRC 32 at this location. From Figure 12.23, only the 4320 and 1050 steels have a hardness below this value.

The quarter-thickness for the 50 mm plate is 12.5 mm; we are interested in the Jominy distance at 6 mm. From the graph, we can estimate that the Jominy distance at 6 mm (about half-way between the surface and 1/4-thickness curves) is 8 mm. The hardness must be greater than HRC 38 at this location. From Figure 12.23, the 4320 steel has a hardness of about 42, but the 1050 steel gives a hardness less than 38.

Therefore, perhaps our most reasonable selection would be the 4320 steel. We would austenitise the steel above the A_3 temperature (about 830°C) and quench the steel in water to produce the correct hardness profile. We might do a little tempering to help relieve stresses produced during quenching; however if any substantial tempering occurs, we will likely reduce the hardness near the surface below HRC 38 and ruin the part.

Perhaps an alternative method for producing the part would be to perform a surface hardening treatment; carburising is not a likely choice because the case depth would have to be 6 mm, which would

require a very long-time treatment. However, if we could selectively heat only the outer 6 mm of an annealed or normalised steel plate above the A_3 temperature to produce austenite, then quench, only the surface will harden while the interior of the plate would remain at its original hardness.

12.38 A quenched and tempered 1.1% C steel is found to have surface cracks that cause the heat-treated part to be rejected by the customer. Why did the cracks form? Design a heat treatment, including appropriate temperatures and times, that will minimise these problems.

The cracks likely formed as a consequence of the stresses caused by the transformation of the austenite to martensite; the steel may have been quenched too rapidly; the surface transformed to martensite first, and when the centre transformed to martensite, high tensile stresses developed at the brittle surface and produced the crack. We might consider using a slower quench, if this would still permit us to produce all martensite in the steel.

A better way might be to perform a martempering heat treatment. In this case, the steel would be heated above about 900°C (the A_{cm} temperature) to produce all austenite. The steel would then be rapidly quenched in a liquid bath held at about 250°C (above the M_s temperature, Figure 12.8(b)). The steel would be held at 250°C for no more than 1000 seconds, thus preventing the formation of any bainite. By this time, the temperature should be uniform throughout the steel part. Then the steel can be quenched to room temperature or below to transform the austenite to martensite. Because most of the steel is transforming at the same time, the stresses due to the transformation will be more uniformly distributed and the quench cracks will not form.

12.39 Design a corrosion-resistant steel to use for a pump that transports liquid helium at 4 K in a superconducting magnet.

For corrosion resistance, we might wish to use a stainless steel. However, we must carefully select the appropriate stainless steel so that it will have the necessary properties at 4 K. Any kind of a shock that might occur in a BCC structure at near-absolute zero temperatures could cause a BCC stainless steel to fracture, since the transition temperature for the steel is likely closer to room temperature. This would then rule out typical ferritic and martensitic stainless steel grades. However, the austenitic stainless steels don't display a transition temperature and may have some impact resistance even at this low temperature. Therefore we might select an austenitic stainless steel such as 304 or 321.

12.40 Design a heat treatment for a hook made of a 25 mm diameter steel rod having a microstructure containing a mixture of ferrite, bainite, and martensite after quenching. Estimate the mechanical properties of your hook.

One way that we might try to produce a mixed microstructure such as this would be to select a hypoeutectoid steel (perhaps 0.5% C) and perform an "interrupted" isothermal heat treatment. If we use the 0.5% C steel, whose TTT diagram is given in Figure 12.8(a), we might:

- a. Austenitise at 830°C for 30 to 60 minutes
- b. Quench to a salt bath at 700°C and hold for 100 seconds. The steel passes the F_s line, producing ferrite, but not the P_s line.
- c. Quench to 400°C and hold for 30 seconds. The remaining austenite passes the B_s line, but not the B_f line.

d. Quench to room temperature, allowing any remaining austenite to form martensite.

We might also be able to produce this structure by a controlled quench. For example, if we select the low-alloy 0.2% C steel, Figure 12-17, we might control the cooling rate between 20 and 100°C/s. The steel would then transform to a mixture of ferrite, bainite, and martensite. These cooling rates correspond to a Jominy distance range of about 3 mm to 11 mm. At the centre of the bar, this Jominy distance range could be achieved in a 25 mm diameter bar by any quenchant with a H coefficient of about 0.4 to 4.

The mechanical properties would be very difficult to estimate, due to the mixed structure.

12.41 Design an annealing treatment for a 0.5%C steel. Be sure to include details of temperatures, cooling rates, microstructures, and properties.

The 0.5%C steel has an A_3 temperature of about 760°C. Therefore the austenitising temperature should be about 30°C higher than this, or about 790°C. We would then hold the steel at the austenitising temperature until all austenite forms; the austenitising time will depend on the thickness of the steel. Approximately 1 hour per 25 mm of thickness might be a rough guide for austenitising. If the time is to be long, or if the surface condition of the steel part is critical, the heat treatment might be done in a protective atmosphere to assure that excessive decarburisation of the surface does not occur.

The steel must then be cooled slowly enough to produce coarse pearlite. We expect that a furnace cool would be sufficient. From the CCT curve, Figure 12.16(a), we would find that, provided the furnace cool gives cooling rates of less than about 5°C, we should expect to obtain an annealed steel with primary ferrite and coarse pearlite.

From the lever law, we expect that the amount of coarse pearlite is $(0.77 - 0.50) / (0.77 - 0.0218) \times 100 = 36\%$. The properties of the steel should be (using Figure 12.5):

tensile strength = 600 MN.m⁻²
yield strength = 375 MN.m⁻²
%elongation = 25%
impact energy = 20 J

12.42 Design a process to produce a 5 mm-diameter steel shaft having excellent toughness, yet excellent wear and fatigue resistance. The surface hardness should be at least HRC 60, and the hardness 0.1 mm beneath the surface should be approximately HRC 50. Describe the process, including details of the heat-treating atmosphere, the composition of the steel, temperatures, and times.

This application looks like a very good candidate for a surface hardening treatment such as carburising or nitriding. Let's consider as an example a possible carburising treatment. We could select a 0.1%C steel, assuring primarily a ferrite matrix which would provide the good toughness to the overall part.

The surface hardness must be at least HRC 60; from Figure 11.24, we will need to have at least 0.4% carbon at the surface, while to produce a hardness of HRC 50, we would need a martensite having about 0.2% C. Therefore, one solution to our design would be to carburise a 0.1%C steel so that we produce a minimum carbon content of 0.4% at

the surface and 0.2% C at a depth of 0.1 mm. We could then quench the carburised part to produce all martensite at these locations. Let's see how this might be done:

$$c_s = 0.4\% \text{ C} \quad c_o = 0.1\% \text{ C} \quad c_x = 0.2\% \text{ C at } x = 0.1 \text{ mm}$$

$$\frac{0.4 - 0.2}{0.4 - 0.1} = \text{erf}(0.1 \times 10^{-3} / 2\sqrt{Dt})$$

Carburising is done in the all-austenite portion of the phase diagram. Thus the 1010 steel part must be heated above its A_3 temperature of about 875°C. Then carbon diffuses in the FCC iron; for this system, $D_o = 2.3 \times 10^{-5} \text{ m}^2 \cdot \text{s}^{-1}$ and $Q = 137,700 \text{ J} \cdot \text{mol}^{-1}$.

$$0.667 = \text{erf}(0.1 \times 10^{-3} / 2\sqrt{Dt})$$

$$0.1 \times 10^{-3} / 2\sqrt{Dt} = 0.7 \quad (\text{from Table 5.3})$$

$$\text{but } Dt = 5.1 \times 10^{-9}$$

$$2.3 \times 10^{-5} \exp(-137700/8.314 \cdot T) [t] = 5.1 \times 10^{-9}$$

$$\exp(-137700/8.314 \cdot T) [t] = 2.22 \times 10^{-4}$$

The minimum carburising temperature is 875°C; let's pick a carburising temperature of 900°C. Then the carburising time needed is

$$\exp(-137700/8.314 \cdot 1173) [t] = 7.38 \times 10^{-7} [t] = 2.22 \times 10^{-4}$$

$$t = 301 \text{ s} = 5 \text{ min}$$

Thus one possible design would be to select a 0.1%C steel, carburise the steel at 900°C for 5 minutes in an atmosphere that produces 0.4% C at the surface. The steel would then be quenched to cause the surface to transform to martensite.

Actually, however, we would probably try to do a better job than this. We probably would like to temper the steel after quenching; thus the hardness (as well as the carbon content) at the surface should initially be higher than we used in the above example. We also wish to have excellent wear resistance and fatigue resistance; we might be able to do even better than our example if we introduce a higher carbon content at the surface.

Chapter 13 Nonferrous Alloys

Suggested Solutions to Design Problems

13.31 A part for an engine mount for a private aircraft must occupy a volume of 0.06 litres with a minimum thickness of 5 mm and a minimum width of 40 mm. The load on the part during service may be as much as 75000 N. The part is expected to remain below 100°C during service. Design a material and its treatment that will perform satisfactorily in this application.

If the part must withstand a load of 75000 N, we can determine the dimensions of the part if we know the yield strengths for some candidate materials. Since this is a part for a private aircraft, we would like to concentrate on light weight materials and, to keep costs down, probably on relatively inexpensive materials. Titanium and beryllium are likely ruled out due to their cost; aluminium and magnesium alloys would appear to be the most likely candidates. Since the temperature will remain below 100°C, we don't have to worry about overageing or recrystallisation; therefore cold worked or age hardened aluminium or magnesium alloys would be possible choices. As examples, let's consider a 7075-T6 aluminium alloy (yield strength of 505 MN.m⁻²) and a HK31A-H24 magnesium alloy (yield strength of 205 MN.m⁻²)/

Aluminium: $A = F/\sigma = 75000 \text{ N} / 505 \text{ MN.m}^{-2} = 1.49 \times 10^{-4} \text{ m}^2$

The minimum thickness and width give a minimum cross-sectional area of $2 \times 10^{-4} \text{ m}^2$; consequently the aluminium alloy will be more than adequate. The part might then be $60 \times 10^{-6} \text{ m}^3 / 2 \times 10^{-4} \text{ m}^2 = 30 \times 10^{-2} \text{ m}$ long. The part would be fabricated to the dimensions of 5 mm x 40 mm x 300 mm, solution treated, quenched, and artificially aged, probably near 200°C.

Magnesium: $A = F/\sigma = 75000 \text{ N} / 205 \text{ MN.m}^{-2} = 3.66 \times 10^{-4} \text{ m}^2$.

The minimum thickness and width again require a minimum cross-sectional area of $2 \times 10^{-4} \text{ m}^2$; in the case of the magnesium, however, we will need to use a minimum cross-sectional area of $3.66 \times 10^{-4} \text{ m}^2$ in order to withstand the applied load. The part then might be $60 \times 10^{-6} \text{ m}^3 / 3.66 \times 10^{-4} \text{ m}^2 = 0.1639 \text{ m}$. The part would be fabricated by cold working to the dimensions of perhaps 91.5 mm x 40 mm x 164 mm; the exact amount of cold working required can't be determined without knowing the relationship between tensile strength and percent cold work, but will be on the order of 30 to 40%. The cold worked part will then be partly annealed (due to the H24 temper specification).

13.32 You wish to design the rung on a ladder. The ladder should be light in weight so that it can easily be transported and used. The rungs on the ladder should be 6 mm x 25 mm and are 300 mm long. Select a suitable material and its processing for the rungs.

The rung on a ladder is supported by the two side rails; when a person steps on the ladder, the loading is somewhat similar to the 3-point bend test described in Chapter 6. We might equate the flexural

stress that causes yielding in the 3-point bend test to the yield strength of the metal. We would like to design the ladder so that it is safe; we might assume that, on occasions, a 135 kg person might climb the ladder. Thus we might design the rung to withstand a 1324 N load. The supports for our "bend" test are 300 mm apart; the width $w = 25$ mm and height $h = 6$ mm. Therefore the flexural stress that develops is:

$$\sigma = 3FL/2wh^2 = (3)(1324 \text{ N})(300 \times 10^{-3}\text{m}) / (2)(25 \times 10^{-3}\text{m})(6 \times 10^{-3}\text{m})^2 = 662 \text{ MN.m}^{-2}$$

If our assumptions were reasonable, we would need to select a material that has a yield strength of 662 MN.m⁻². We also want a material that is light weight and that is too expensive. While we can find alloys of steel, titanium, and other materials with this yield strength, these materials are too heavy to easily be handled, or are too expensive. Unfortunately, magnesium alloys have yield strengths well below the requirement, and even the best aluminium alloys barely approach this minimum strength.

In this case, we would probably decide to redesign the ladder; if we make the rungs a little thicker, perhaps we can find a suitable alloy. In doing the redesign, we would probably focus on an aluminium alloy and, in order to help in the fabrication of the ladder, we would focus on strain hardened aluminium alloys, thus eliminating the necessity for heat treating the material. If we selected, as an example, the 5182-H19 alloy, which has a yield strength of 395 MN.m⁻², then

$$395 \times 10^6 \text{ N.m}^{-2} = (3)(1324 \text{ N})(300 \times 10^{-3}\text{m}) / (2)(25 \times 10^{-3}\text{m})(h)^2$$

$$h = 7.8 \times 10^{-3} \text{ m} = 7.8 \text{ mm}$$

By making the rung a little thicker, we can use a conventional, light-weight aluminium alloy. The rungs can be formed into the right shape by a cold working process, simultaneously providing the required strength.

13.33 We have determined that we need an alloy having a density of $2.3 \pm 0.05 \text{ Mg.m}^{-3}$ that must be strong, yet still have some ductility. Design a material and its processing that might meet these requirements.

It might be tempting to select an advanced material, such as Al-Li, to produce this material. However, according to Figure 13.5, we would need more than 5% Li to achieve this low density; we wouldn't be able to add this amount of lithium and still be able to control its structure and properties without very expensive processing, such as rapid solidification processing.

Magnesium, on the other hand, is too light! In order to raise the density of the magnesium from 1.74 Mg.m^{-3} to 2.3 Mg.m^{-3} , we would have to add large amounts of alloying elements. Without a lot more information, we wouldn't be able to achieve this design based on magnesium. Beryllium starts out a little closer to the correct density, but at \$660 per kg, beryllium is rather unattractive.

We might consider a Mg-Al alloy; perhaps about 40% Mg in Al would produce the desired density. However the phase diagrams in Figure 13.3 and 13.8 indicate that this composition will lead to the presence of large amounts of a brittle intermetallic compound. This does not sound like a good solution, since one of our requirements is at least some ductility.

In this case, perhaps our best solution is to produce a composite material. For example, we might introduce relatively dense ceramic particles into magnesium; Al_2O_3 , for example, has a density of about 3.98 Mg.m^{-3} (Table 14.4). We could determine the amount of the Al_2O_3 needed to raise the density to about 2.3 Mg.m^{-3} .

Or, as another example, we might add a light-weight material to aluminium. If we introduced hollow glass beads, having a very low density, into the aluminium, we might be able to produce the correct density.

In either of the latter two designs, it is important that the nonmetallic constituents be uniformly distributed throughout the final part. Perhaps the thixocasting process described in Example 13.4 would be a suitable processing technique. By dispersing the nonmetallics uniformly throughout the part, they would detract only a little from the strength of the part. The surrounding matrix would be aluminium or magnesium, which will provide at least some ductility; the matrix could be an age hardenable alloy, providing the capability for good strength. The amount of such materials to be added is left to the student.

13.34 We wish to design a mounting device that will position and aim a laser for precision cutting of a composite material. What design requirements might be important? Select a suitable material and its processing that might meet these requirements.

We need a material that will be unusually stable -- neither changes in temperature nor any change in the stress acting on the mounting device should produce much of a change in the size or shape of the part. This means that we would like a material that has a low coefficient of thermal expansion, in case the temperature varies, as well as a very high modulus of elasticity, in case any changes in the stress on the part occurs. Materials having both of these properties usually are those that possess very strong bonding.

Of the material discussed in this chapter, beryllium and perhaps tungsten would be possibilities. Beryllium has a high modulus of elasticity (over 290 GN.m^{-2}) and is expected to have a low coefficient of thermal expansion. Tungsten has a modulus of elasticity of about 408 GN.m^{-2} and an unusually low coefficient of thermal expansion (Table 21.2). Either might be suitable choices.

Both Be and W have major disadvantages. Both are expensive, with Be costing about \$660 per kg and W costing about \$22 per kg. Tungsten is exceptionally heavy, particularly compared to the low-density beryllium. Both are difficult to fabricate. Beryllium is very reactive and has the HCP crystal structure; vacuum casting or special powder metallurgy processing may be required. Tungsten has a very high melting temperature, making casting difficult, oxidises at high temperatures, and tends to be brittle at low temperatures. Again fabrication is difficult. Hopefully the student will be able to make an appropriate choice.

Of course other materials might be considered. A fibre-reinforced composite, perhaps even based on a ceramic matrix, might provide good stiffness and thermal stability.

13.35 Design a nickel-titanium alloy that will produce 60 volume percent Ni_3Ti precipitate in a pure nickel matrix.

We need to determine the composition of a Ni-Ti alloy that will, after all of the titanium reacts with Ni, give the required 60 vol%

Ni_3Ti . In order to do this, we first need to find the densities of the two materials. The density of nickel is 8.902 Mg.m^{-3} . The density of Ni_3Ti isn't given in the textbook; the student might assume that the crystal structure of Ni_3Ti , when it forms the γ' phase, is the same as that of Ni_3Al . This structure is given in Figure 10.5(b). The Ni and Ti atoms would touch along a face diagonal, giving a lattice parameter of

$$a_o = (2r_{\text{Ni}} + 2r_{\text{Ti}}) / \sqrt{2} = [(2)(0.1243) + (2)(0.1475)] / \sqrt{2} = 0.3844 \text{ nm.}$$

The density of the compound would be

$$\rho = \frac{(3 \text{ Ni})(58.71) + (1 \text{ Ti})(47.9)}{(3.844 \times 10^{-9})^3 (6.02 \times 10^{23})} = 6.55 \times 10^{-3} \text{ Mg.m}^{-3}$$

The final structure should contain 60 vol% Ni_3Ti . In 1 m^3 of the final alloy, there will be 0.6 m^3 of the compound. The weight percent of Ni_3Ti is

$$\text{wt\% Ni}_3\text{Ti} = \frac{(0.6 \text{ m}^3)(6.55 \text{ Mg.m}^{-3})}{(0.6)(6.55) + (0.4)(8.902)} \times 100 = 52.46$$

Thus in 1 Mg of the final material, there is 0.5246 Mg of Ni_3Ti . The atomic percent Ti in the compound is 25%, or the weight percent Ti in Ni_3Ti is

$$\text{wt\% Ti in Ni}_3\text{Ti} = \frac{(0.25)(47.9 \text{ g.mol}^{-1})}{(0.25)(47.9) + (0.75)(58.71)} \times 100 = 21.38\%$$

In 1 Mg of the final material, the amount of titanium is $(0.5246 \text{ Mg})(0.2138) = 0.112 \text{ Mg Ti}$. Thus the original alloy should be a Ni-11.2 wt% Ti alloy.

13.36 An actuating lever in an electrical device must open and close almost instantly and carry a high current when closed. What design requirements would be important for this application? Design a material and its processing to meet these requirements.

The material must have good electrical conductivity to assure that the part does not overheat; the material must also have good wear resistance to assure good electrical conduct and prevent overheating or even arcing.

For good electrical conductivity, we want a pure metal such as copper or aluminium. However the pure metals have poor hardness and wear resistance. Methods such as solid solution strengthening and age hardening will improve the hardness but will dramatically reduce electrical conductivity. Strain hardening will increase the hardness with relatively little loss in conductivity; however if the contact does overheat, recrystallisation may occur and the wear resistance will decrease.

Perhaps the ideal solution is to introduce a strengthening material that does not dissolve in the pure metal. For example, we might add Al_2O_3 to pure copper. The alumina has no solubility in copper, so the copper remains pure and conductive; however the alumina, a ceramic, is very hard and provides good wear resistance. We might produce this structure by mixing powders of pure copper and alumina, consolidating them into a shape, sintering the powder compact, then extruding or forging the powder metallurgy part to close any remaining voids and produce the final shape of the lever.

13.37 A fan blade in a chemical plant must operate at temperatures as high

as 400°C under rather corrosive conditions. Occasionally, solid material is ingested and impacts the fan. What design requirements would be important? Select a suitable material and its processing for this application.

Obviously the environment requires that we have good corrosion resistance in whatever material we select. In addition, the material must have this good corrosion resistance at a relatively high temperature -- 400°C -- and it must have good strength and resistance to creep at this temperature. Finally, the material must have reasonable ductility, impact properties, and/or fracture toughness so that it doesn't catastrophically fail if a foreign material is ingested into the equipment.

Metals such as aluminium and magnesium will not survive at 400°C. Beryllium is likely to be too brittle, and certainly will be expensive. Ferrous metals will almost certainly corrode too rapidly. Tungsten and other refractory metals are relatively heavy and expensive. Ceramics may fit most of the requirements but they are brittle and hard to form into a fan blade. A number of the copper, nickel, and superalloys might be considered. They often have good corrosion resistance at elevated temperatures, can be formed reasonably easily into the shape of the fan, and may possess the mechanical properties of creep resistance and impact properties. On the other hand, they are relatively heavy; we might be able to minimise power requirements for the motor that drives the fan with a lighter-weight material.

Perhaps our best choice is a titanium alloy. The temperature is below the temperature at which the protective oxide film is disrupted; consequently the corrosion resistance should be adequate. The titanium alloys can maintain reasonable strength at elevated temperatures. Finally, we can design a titanium alloy, perhaps with a Widmanstatten microstructure, that has good fracture toughness. A very common titanium alloy is Ti-6Al-4V; it is an alpha-beta, heat-treatable alloy that will produce the desired microstructure and is readily available and formable. If the fan blade is extremely complex, the superplastic behavior of this titanium alloy might be used to produce the product.

Chapter 14 Ceramic Materials

Suggested Solutions to Design Problems

14.36 Using the data in Table 14.4, design a ceramic part containing flaws that are too small to propagate on their own but may prevent other major cracks from easily propagating. During use, the ceramic part will be loaded at 44,500 N in a manner similar to a three-point bend test, where the supports are 150 mm apart. To fit into the rest of the assembly, the part must be 30 mm wide.

We need to introduce flaws that are themselves too small to propagate but will blunt any larger flaws introduced during other processing steps or during service. The size of the flaws that we wish to introduce depends on the properties of the ceramic that we select. As examples, let's pick two materials -- Al₂O₃ and partially stabilised ZrO₂ -- from Table 14.4.

The alumina has a fracture toughness of 5.5 MN.^{-3/2}. Any flaw must not produce a stress that will cause the fracture toughness K_{Ic} to be exceeded. Let's consider several possibilities. Suppose that we are able to introduce flaws of different sizes by different techniques. If we assume that f = 1

$$5.5 = (1) (\sigma) \sqrt{(\pi a)}$$

we could create a number of scenarios:

$$\begin{aligned} a &= 0.025 \text{ mm} & \sigma &= 620 \text{ MN.m}^{-2} \\ a &= 0.254 \text{ mm} & \sigma &= 195 \text{ MN.m}^{-2} \\ a &= 2.54 \text{ mm} & \sigma &= 62 \text{ MN.m}^{-2} \end{aligned}$$

We could then design the size of the ceramic part, whose length is 150 mm and width is 30 mm.

$$\sigma = 3(44500 \text{ N})(150 \times 10^{-3} \text{ m}) / (2)(30 \times 10^{-3} \text{ m})h^2$$

Then, if

$$\begin{aligned} a &= 0.025 \text{ mm} & h &= 23.2 \text{ mm} \\ a &= 0.254 \text{ mm} & h &= 41.4 \text{ mm} \\ a &= 2.54 \text{ mm} & h &= 73.4 \text{ mm} \end{aligned}$$

We could repeat these calculations for the partially stabilised zirconia, which has a fracture toughness of 11 MN.m^{-3/2}

$$\begin{aligned} a &= 0.025 \text{ mm} & \sigma &= 1241 \text{ MN.m}^{-2} & h &= 16.4 \text{ mm} \\ a &= 0.254 \text{ mm} & \sigma &= 389 \text{ MN.m}^{-2} & h &= 29.3 \text{ mm} \\ a &= 2.54 \text{ mm} & \sigma &= 123 \text{ MN.m}^{-2} & h &= 52.1 \text{ mm} \end{aligned}$$

We now have several ways in which to design our part. We may introduce a network of tiny cracks 0.025 mm long, using zirconia having a thickness of perhaps 20 mm (allowing a bit of safety factor). These cracks should not themselves propagate, but may block other larger cracks from doing so.

14.37 Design a silica soda-lime glass that can be cast at 1300°C and that will have a O:Si ratio of less than 2.3 to assure good glass-forming tendencies. To assure adequate viscosity, the casting temperature should be at least 100°C above the liquidus temperature.

We could select a variety of compositions of soda and lime added to the silica. The first priority, perhaps, is to assure that the glass will have a liquidus temperature of no more than 1200°C. We also want to be sure that we have a reasonable amount of CaO present so that the solubility of the glass in water is not high. From Figure 14.31, we might determine several compositions that provide a liquidus temperature of 1200°C. Two examples are:

Glass 1: SiO₂ - 14 wt% Na₂O - 10 wt% CaO
 Glass 2: SiO₂ - 9 wt% Na₂O - 20 wt% CaO

For Glass 1,

$$f_{\text{soda}} = \frac{14 / 61.98 \text{ g.mol}^{-1}}{(14 / 61.98) + (10 / 56.08) + (76 / 60.08)} = 0.135$$

$$f_{\text{lime}} = \frac{10 / 56.08 \text{ g.mol}^{-1}}{(14 / 61.98) + (10 / 56.08) + (76 / 60.08)} = 0.107$$

$$O/Si = \frac{(1 \text{ O/soda})(0.135) + (1 \text{ O/lime})(0.107) + (2 \text{ O/silica})(0.758)}{(1 \text{ Si/silica})(0.758)}$$

O/Si = 2.32 This doesn't quite meet our requirements.

Similarly, for Glass 2 we would find

$$f_{\text{soda}} = 0.086 \quad f_{\text{lime}} = 0.212 \quad O/Si = 2.42$$

Neither Glass 1 nor Glass 2 meets the requirement for O/Si < 2.3. If we select a third glass, SiO₂ - 18 wt% Na₂O, we would find that $f_{\text{soda}} = 0.175$ and O/Si = 2.21. Somewhere between Glass 1 and the CaO-free glass should be a composition that gives the required O/Si ratio with at least some CaO present for moisture resistance.

14.38 Design a ceramic structure that will not fail when a tensile load of 25,000 N is applied, assuming that our processing can only assure that cracks are shorter than 0.7 mm.

Let's assume that the geometry factor $f = 1$.

$$K = f\sigma\sqrt{\pi a} = (1)(25,000 \text{ N} / A)\sqrt{\pi(0.7 \times 10^{-3} \text{ m})}$$

If we selected Al₂O₃, with a fracture toughness of 5.5 MN.m^{-3/2} then

$$A = (1)(25,000)\sqrt{\pi(0.7 \times 10^{-3})} / 5.495 = 2.13 \times 10^{-4} \text{ m}^2$$

If we selected transformation toughened zirconia, with a fracture toughness of 12 MN.m^{-3/2}, then

$$A = (1)(25,000)\sqrt{\pi(0.7 \times 10^{-3})} / 12 = 9.8 \times 10^{-5} \text{ m}^2$$

We can determine the cross-sectional area of the ceramic part needed to assure that the flaw will not propagate; for alumina, the cross-sectional area may need to be as large as 2.13 x 10⁻⁴m², while for transformation toughened zirconia, a much smaller part can be produced.

14.39 Design a 5-cm-long glass rod so that when a load of 10,000 N is applied at 1000°C, the rod will still be no longer than 5.002 cm after one year.

The total permitted strain is $(5.002 \times 10^{-2} - 5 \times 10^{-2}) / 5 \times 10^{-2} = 4 \times 10^{-4} \text{ m.m}^{-1}$. One year is 31.54 x 10⁶ s. Thus the maximum creep rate is

$$d\epsilon/dt = 4 \times 10^{-4} \text{ m.m}^{-1} / 31.54 \times 10^6 \text{ s} = 1.27 \times 10^{-11} \text{ m.m}^{-1}\text{s}^{-1}$$

The creep rate might also be given by

$$d\epsilon/dt = \sigma/\eta \quad \text{where } \sigma = F/A = 10,000 \text{ N} / A \quad \text{in } \text{N.m}^{-2}$$

At 1000°C, the viscosity of two glasses is (from Figure 14.25)

$$\text{fused silica: } \eta = 10^{14} \text{ Ns.m}^{-1}$$

$$\text{borosilicate: } \eta = 10^4 \text{ Ns.m}^{-1}$$

Thus the creep rate for fused silica is

$$d\epsilon/dt = (10,000 / A \text{ N.m}^{-2}) / 10^{14} \text{ Ns.m}^{-1} = 1.27 \times 10^{-11} \text{ m.m}^{-1}\text{s}^{-1}$$

$$A = 7.87 \text{ m}^2$$

For the borosilicate,

$$d\epsilon/dt = (10,000 / A \text{ N.m}^{-2}) / 10^4 \text{ Ns.m}^{-1} = 1.27 \times 10^{-11} \text{ m.m}^{-1}\text{s}^{-1}$$

$$A = 7.87 \times 10^{10} \text{ m}^2$$

Obviously this is impractical; none of the glasses are capable of preventing the creep at 1000°C. Instead, we will have to either greatly reduce the applied load (virtually to nothing) or we will have to reduce the temperature acting on the glass rod. Further calculations might give the maximum allowable temperature that several glasses can withstand at the current load and creep rate limitations.

14.40 We wish to produce a complex silicon nitride impeller; the strength of the part is relatively unimportant, but the dimensional accuracy must be very good. Design a method for producing this part.

Several methods might be used to produce the part. Slip casting will produce a shape at a low temperature, but then the part must be dried and sintered; large dimensional changes are expected during these processes, resulting in very poor dimensional accuracy. Silicon nitride powders could be pressed at room temperature and then sintered at high temperature; again large dimensional changes are expected during sintering. Hot pressing of powders will reduce the dimensional changes, but still may not produce completely accurate parts. Reaction bonding, converting a silicon part into silicon nitride by reaction with nitrogen, can be done at lower temperatures than hot pressing and may result in the best dimensional accuracy. However the reaction bonding process will sacrifice strength, as indicated in Table 14.3.

Chapter 15 Polymers

Suggested Solutions to Design Problems

15.43 Figure 15.41 shows the behavior of polypropylene, polyethylene, and acetal at two temperatures. You would like to produce a 300 mm-long rod of a polymer that will operate at 40°C for 6 months under a constant load of 2225 N. Design the material and size of the rod such that no more than 5% elongation will occur by creep.

While we might use the data in Figure 15.41 to determine an activation energy for creep and then use that to determine the behaviour of the three polymers at 40°C, it doesn't appear that we would introduce much error simply by interpolating between the 23°C and 60°C curves. If we drew curves for each polymer between the two temperature curves, then determine the equation for those lines, we could calculate the maximum stress that gives no more than 5% strain after 1000 hours.

$$\begin{aligned} \text{acetal: } \sigma &= 12.4/1 = 12.4 \text{ MN.m}^{-2}/\% \text{strain or } \sigma_{\max} = 62 \text{ MN.m}^{-2} \\ \text{PP: } \sigma &= 9/2 = 4.5 \text{ MN.m}^{-2}/\% \text{strain or } \sigma_{\max} = 22.4 \text{ MN.m}^{-2} \\ \text{PE: } \sigma &= 2.1/2 = 1.05 \text{ MN.m}^{-2}/\% \text{strain or } \sigma_{\max} = 5.25 \text{ MN.m}^{-2} \end{aligned}$$

But we need to have the rod creep no more than 5% over a 6 month period, equal to 4380 hours, not 1000 hours. If we assume that the creep rate is linear for a given stress, then the values for maximum stress above should be adjusted by a factor of 1000 / 4380, or

$$\begin{aligned} \text{acetal: } \sigma_{\max} &= 62(1000/4380) = 14.2 \text{ MN.m}^{-2} \\ \text{PP: } \sigma_{\max} &= 22.4(1000/4380) = 5.1 \text{ MN.m}^{-2} \\ \text{PE: } \sigma_{\max} &= 5.25(1000/4380) = 1.2 \text{ MN.m}^{-2} \end{aligned}$$

For a constant load of 2225 N, the minimum diameter of the bars must be

$$\begin{aligned} \text{acetal: } d &= \sqrt{(4F/\pi\sigma_{\max})} = 14.1 \text{ mm} \\ \text{PP: } d &= 23.6 \text{ mm} \\ \text{PE: } d &= 48.6 \text{ mm} \end{aligned}$$

The volume, weight, and cost of each rod are

$$\begin{aligned} \text{acetal: } V &= (\pi/4) (14.1 \times 10^{-3} \text{m})^2 (300 \times 10^{-3} \text{m}) = 4.7 \times 10^{-5} \text{m}^3 \\ W &= (4.7 \times 10^{-5} \text{m}^3) (1.42 \text{ Mg.m}^{-3}) = 66.8 \text{ g} \\ \text{cost} &= (66.8 \times 10^{-6} \text{Mg}) (2700 \text{ £.Mg}^{-1}) = \text{£}0.18 \text{ per rod} \end{aligned}$$

$$\begin{aligned} \text{PP: } V &= (\pi/4) (23.6 \times 10^{-3})^2 (0.300) = 1.31 \times 10^{-4} \text{m}^3 \\ W &= (1.31 \times 10^{-4}) (0.90) = 118 \text{ g} \\ \text{cost} &= (118 \times 10^{-6} \text{Mg}) (730 \text{ £.Mg}^{-1}) = \text{£}0.086 \text{ per rod} \end{aligned}$$

$$\begin{aligned} \text{PE: } V &= (\pi/4) (48.6 \times 10^{-3})^2 (0.300) = 5.56 \times 10^{-4} \text{m}^3 \\ W &= (5.56 \times 10^{-4}) (0.96) = 534 \text{ g} \\ \text{cost} &= (534 \times 10^{-6} \text{Mg}) (584 \text{ £.Mg}^{-1}) = \text{£}0.31 \text{ per rod} \end{aligned}$$

In this case, the polypropylene rod would have an intermediate size but the lowest cost.

15.44 Design a polymer material that might be used to produce a 75 mm-diameter gear to be used to transfer energy from a low-power electric motor. What are the design requirements? What class of polymers

(thermoplastics, thermosets, elastomers) might be most appropriate? What particular polymer might you first consider? What additional information concerning the application and polymer properties do you need to know to complete your design?

The gear should have a high strength and reasonable wear resistance; some degree of toughness might also be appropriate. If any heat is generated by friction during use of the gear, then reasonable creep resistance, or a high deflection temperature, might be required. Perhaps a good modulus of elasticity would be helpful to assure good engagement of teeth without too much elastic deformation, and if the gear is frequently engaged in a violent manner, good impact properties would be desired.

An elastomer would not be a likely candidate. However thermosetting polymers such as a polyester or an epoxy might be good candidates. Table 15.12 shows that polyesters can develop a tensile strength of as much as 87 MN.m^{-2} and a modulus of elasticity up to 4 MN.m^{-2} ; epoxies can have strengths up to 100 MN.m^{-2} and a modulus of 3 MN.m^{-2} . The heat deflection temperature for epoxy is very high (290°C). (Note that the heat deflection temperature for the polyester in Table 15.4 is for a thermoplastic, not a thermosetting, grade of the polymer.) These thermosets are expected to have relatively poor impact properties, however, and forming may be difficult -- a transfer moulding process, for example, may be needed.

We might also consider a thermoplastic polymer such as polyamide, which has a tensile strength of 80 MN.m^{-2} , a modulus of elasticity of 3330 MN.m^{-2} , and some impact resistance. Its heat deflection temperature is 90°C , which may be sufficient, and it should be relatively easy to form. If impact properties are critical, we might select a polycarbonate, which has reasonable but not outstanding strength and stiffness but exceptional impact properties.

15.45 Design a polymer material and a forming process to produce the case for a personal computer. (a) What are the design and forming requirements? (b) What class of polymers might be most appropriate? (c) What particular polymer might you first consider? (d) What additional information do you need to know?

The case for a PC is relatively complex, requiring that a thin part covering a large area be produced. Small holes in the cooling vents must be introduced. The case has to be rugged enough so that it isn't easily damaged, although we don't expect that it needs to survive being dropped off the table very often.

Since formability is particularly important, a thermoplastic polymer that might easily be introduced into a die in injection moulding might be most appropriate.

15.46 Design a polymer part for which, under an applied stress of 7 MN.m^{-2} , no more than 0.025 mm.mm^{-1} creep strain will occur at room temperature in one year. Be sure to consider cost in your design.

Figure 15.21 might provide us with a couple of choices. The 0.025 mm.mm^{-1} strain is 0.25%. One year is 8760 hours. At 8760 hours, for the acrylic (PMMA), we find that

$$\begin{array}{ll} \sigma = 40 \text{ MN.m}^{-2} & \epsilon = 6\% \\ \sigma = 30 \text{ MN.m}^{-2} & \epsilon = 2.5\% \\ \sigma = 15 \text{ MN.m}^{-2} & \epsilon = 1\% \end{array}$$

We could plot this and determine the maximum stress that would

produce 0.25% strain. The graph below indicates that the acrylic will creep this amount for practically any applied stress. The polypropylene is expected to be even worse.

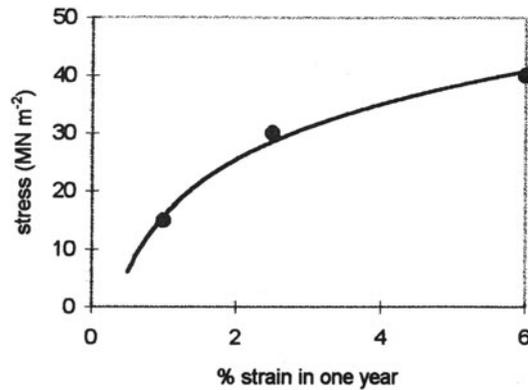


Figure 15.41 shows that an acetal polymer, at 23°C, can withstand a stress of perhaps 3.3 MN.m⁻² for 1000 hours. However, for an entire year, the maximum stress may be significantly lower than even this. In order to successfully design such a part, we need additional information for the creep behavior of polymers that creep at a slower rate than these relatively common thermoplastics. It appears that our design must specify a thermosetting polymer or one of the advanced thermoplastics.

Chapter 16 Composite Materials

Suggested Solutions to Design Problems

16.30 Select and calculate the constituents for an airplane wing that has an electrical conductivity of at least $200 \times 10^3 \text{ ohm}^{-1} \cdot \text{m}^{-1}$, a modulus of elasticity of at least $83 \text{ GN} \cdot \text{m}^{-2}$, and reasonable corrosion resistance.

We might do this in a variety of ways. For example, we might reinforce aluminium sheet material with a small volume fraction of carbon fibres. The electrical conductivity of the aluminium is well over $10^6 \text{ ohm}^{-1} \cdot \text{m}^{-1}$, so the carbon is unlikely to reduce the conductivity to the minimum level specified in the problem. The aluminium has good corrosion resistance. To increase the modulus from $70 \text{ GN} \cdot \text{m}^{-2}$ of a typical aluminium alloy to the required value of $83 \text{ GN} \cdot \text{m}^{-2}$ requires the following amount of high modulus carbon fibres, assuming that the fibres are parallel to the major stresses acting on the aluminium sheet.

$$E = 83 = f_c(531) + (1 - f_c)(70)$$

$$f_c = 0.03 \quad \text{or only 3 vol\% carbon fibres}$$

We could also produce a laminar composite, perhaps with a thin layer of a Kevlar fibre fabric impregnated with a phenolic resin that also adhesively bonds two sheets of aluminium together. The Kevlar has a modulus of $124 \text{ GN} \cdot \text{m}^{-2}$; if sufficient Kevlar is present in the intermediate layer, it will offset the low modulus of the phenolic resin and will still increase the modulus of the overall composite above $83 \text{ GN} \cdot \text{m}^{-2}$. At least in a direction parallel to the lamellae, the electrical conductivity will still be achieved, although this will not be the case perpendicular to the sheets. The good corrosion-resistant aluminium will still see whatever environment the airplane experiences.

16.31 Consider a Cu-15 wt% Sn alloy [Figure 13.10(b)]. Design a compocasting process that will permit the introduction of 40 vol% SiC. The density of the alloy is $8.5 \text{ Mg} \cdot \text{m}^{-3}$ and that of SiC is $3.0 \text{ Mg} \cdot \text{m}^{-3}$.

Compocasting involves stirring the solidifying liquid to break up the dendritic network, producing rounded solid particles of the primary phase surrounded by liquid. If we introduce the SiC at this point in the process, the SiC particles can be uniformly distributed throughout the partly liquid-partly solid mass and the SiC particles will not float, even though there is a big difference in their density compared with that of the Cu-Sn alloy.

The amounts of liquid and solid α as a function of temperature can be calculated using the lever law for the Cu-15% Sn alloy:

$$\begin{aligned} 925^\circ\text{C}: \quad \%L &= (15-4) / (17-4) = 85 \\ 900^\circ\text{C}: \quad \%L &= (15-5) / (18-5) = 77 \\ 875^\circ\text{C}: \quad \%L &= (15-6) / (20-6) = 64 \\ 850^\circ\text{C}: \quad \%L &= (15-8) / (22-8) = 50 \\ 825^\circ\text{C}: \quad \%L &= (15-9) / (23-9) = 43 \\ 800^\circ\text{C}: \quad \%L &= (15-13) / (24-13) = 18 \end{aligned}$$

We might decide that we would like to stir the solidifying alloy until it cools to 850°C, with 50% liquid and 50% α . We can then stir in the SiC until it is uniformly distributed. The material can then be allowed to solidify. If we started with 100 kg of the bronze, we would need to add

$$V_{\text{bronze}} = (100 \text{ kg}) / (8.5 \text{ Mg.m}^{-3}) = 0.0118 \text{ m}^3$$

$$f_{\text{SiC}} = x \text{ m}^3 \text{ SiC} / (x \text{ m}^3 \text{ SiC} + 0.0118 \text{ m}^3 \text{ bronze}) = 0.40$$

$$x = 0.0079 \text{ m}^3$$

$$W_{\text{SiC}} = (0.0079 \text{ m}^3) (3.0 \text{ Mg.m}^{-3}) = 0.0237 \text{ Mg} = 23.7 \text{ kg}$$

$$\text{wt\% SiC} = 23.7 / (23.7 + 100) \times 100 = 19\%$$

Later, we could cut the solid billet to the right volume, reheat the billet to 850°C, then inject the composite under pressure into a die (a die casting process). The billet will behave as a solid until the pressure is applied, at which time the composite will then behave as a liquid and flow into the die.

16.32 Design the materials and processing required to produce a discontinuous but aligned fibre-reinforced fibreglass composite that will form the hood of a sports car. The composite should provide a density of less than 1.6 Mg.m⁻³ and a strength of 140 MN.m⁻². Be sure to list all of the assumptions you make in creating your design.

The fibreglass might be an E-glass or an S-glass; as an example, let's use S-glass, having a density of 2.50 Mg.m⁻³ and a tensile strength of 4480 MN.m⁻² (Table 16.2). For the matrix, let's try a thermosetting polyester, Table 15.12, having a density of 1.28 Mg.m⁻³ and a strength of 87 MN.m⁻². In order to be sure that the density is less than 1.6 Mg.m⁻³, we need to determine the upper limit on the amount of S-glass fibres that can be added:

$$1.6 \text{ Mg.m}^{-3} = f_{\text{glass}} (2.5 \text{ Mg.m}^{-3}) + (1 - f_{\text{glass}}) (1.28 \text{ Mg.m}^{-3})$$

$$f_{\text{glass}} = 0.262 = \text{maximum amount of S-glass fibres}$$

Since the fibres are discontinuous, we might need to use Equations 16.9 and 16.10. A typical fibre diameter might be about 100 microns, or 10⁻⁴ m; we might assume that τ_i , or the stress at which the matrix begins to deform and debonding begins, is half the tensile strength of the polyester resin. If this were correct,

$$l_c = \sigma_f d / 2\tau_i = \frac{(87 \text{ MN.m}^{-2}) (10^{-4} \text{ m})}{(2) (87 / 2)} = 1 \times 10^{-4} \text{ m}$$

$$l_c = 0.1 \text{ mm}$$

We would like to have the fibres be about 15 times the critical fibre length, or

$$l = 15(1 \times 10^{-4} \text{ m}) = 1.5 \times 10^{-3} \text{ m}$$

If the stress σ_m on the matrix when the fibres begin to break is 87 MN.m⁻², the tensile strength of the polyester, then

$$140 \text{ MN.m}^{-2} = f_{\text{glass}} (4480 \text{ MN.m}^{-2}) [1 - (1 \times 10^{-4} / 2 \times 1.5 \times 10^{-3})] + (1 - f_{\text{glass}}) (87 \text{ MN.m}^{-2})$$

$$f_{\text{glass}} = 0.012$$

If our assumptions were correct, we need between 0.012 and 0.262 volume fraction glass fibres, each 10^{-4} m in diameter and about 1.5 mm long. The fibres are expected to cost about \$7 per kg, while the polyester might cost about \$4 per kg. Thus we might recommend that we use about 0.1 volume fraction of the fibres -- this will provide a compromise between cost and safety.

16.33 A 900 mm inside-diameter spherical tank is to be designed to store liquid Cl. The tank must have a modulus of elasticity in the tangential direction of at least 100 GN.m^{-2} , it should have a thermal conductivity in the radial direction of no more than $2.5 \text{ W.m}^{-1}.\text{K}^{-1}$, and it should weigh no more than 75 kg. Using only the materials listed in Table 21.3, formulate a material and determine a tank thickness that will be suitable. Estimate the cost of materials in your tank to assure that it is not prohibitively expensive.

Chlorine is a liquid between -34°C and -100°C . Therefore our design should assure that the tank doesn't become embrittled within this range.

The first step might be to recognise that we need to incorporate a low thermal conductivity material into the tank to assure that we will have the appropriate maximum thermal insulation. This may mean that we need to produce either (1) a laminar composite material, perhaps consisting of a metal and a low thermal conductivity coating or (2) a fibre reinforced polymer matrix composite containing high modulus fibres.

As an example of the first approach, we might use an austenitic stainless steel shell clad with a layer of polyethylene. The austenitic stainless steel would assure that no ductile-brittle transition temperature would make the tank susceptible to brittle failure in contact with the cold Cl. The modulus of elasticity of the stainless steel is about 195 GN.m^{-2} and its thermal conductivity is $30 \text{ W.m}^{-1}.\text{K}^{-1}$. Polyethylene has a modulus of elasticity of $270 \times 10^{-3} \text{ GN.m}^{-2}$ and a thermal conductivity of $0.33 \text{ W.m}^{-1}.\text{K}^{-1}$; the polyethylene has a glass transition temperature of -120°C and shouldn't become embrittled by the liquid chlorine.

The tangential direction of the tank is parallel to the laminar composite; thus the modulus of elasticity is given by:

$$E_{\text{composite}} = 100 = f_{\text{stainless steel}} (195) + (1-f_{\text{stainless steel}}) (270 \times 10^{-3})$$

$f_{\text{stainless steel}} \geq 0.512$ to assure the correct modulus of elasticity

The radial direction is perpendicular to the laminar composite; thus the thermal conductivity is given by:

$$1 / K_{\text{composite}} = 1 / 2.5 = f_{\text{stainless steel}} / 30 + (1-f_{\text{stainless steel}}) / 0.33$$

$f_{\text{stainless steel}} \leq 0.878$ to assure the correct conductivity

Any composite containing between 0.512 and 0.878 volume fraction stainless steel would work. The thickness of the composite will depend on the maximum allowable weight. Let's use a stainless steel volume fraction of 0.75, so $t = 3t_{\text{stainless}} + t_{\text{polymer}}$. In one m^3 of composite, there will be 0.25 m^3 of the polymer. The density of the polymer is about 0.92 Mg.m^{-3} and that of the stainless steel is about 7.91 Mg.m^{-3} . The density of the composite is

$$\rho_{\text{composite}} = (0.75) (7.91) + (0.25) (0.92) = 6.1625 \text{ Mg.m}^{-3}$$

$$V_{\text{tank}} = (75 \times 10^{-3} \text{ Mg}) / (6.1625 \text{ Mg.m}^{-3}) = 1.22 \times 10^{-2} \text{ m}^3$$

Therefore we can find the maximum thickness of the shell:

$$\text{Inner volume of tank} = (4\pi/3)(450 \times 10^{-3})^3 = 0.3817 \text{ m}^3$$

$$\text{Additional volume due to composite} = 1.22 \times 10^{-3} \text{ m}^3$$

$$\text{So, total tank volume} = 0.3939 \text{ m}^3$$

Radius of sphere of volume 0.3939 m^3 , given by

$$r = \sqrt[3]{((3 \times 0.3939)/(4\pi))} = 455 \times 10^{-3} \text{ m}$$

$$\therefore \text{Wall thickness is difference between outer and inner radii thickness} = 455 \times 10^{-3} - 450 \times 10^{-3} = 5 \times 10^{-3} \text{ m} = 5 \text{ mm}$$

Of course there are a tremendous number of other possibilities for the tank.

16.34 Select the constituents of an electrical contact material and describe a method for producing the material that will result in a density of no more than 6 Mg.m^{-3} , yet at least 50 vol% of the material will be conductive.

In this case, we need to select a material that will have good electrical conductivity, yet introduce a harder material that provides good wear resistance to the contact. If we selected aluminium as the contact material, we wouldn't have to worry about the density -- we could introduce up to 50 vol% of a hard material, such as alumina or silicon carbide, and still meet the design requirements.

However, if we select copper as our contact material, which we might due to its better conductivity and also higher melting temperature, then we will have to add a hard material that also has a lower density than copper (and lower than 6 Mg.m^{-3}). Suppose our design is to incorporate Al_2O_3 particles, with a density of 3.96 Mg.m^{-3} , into pure copper, with a density of 8.96 Mg.m^{-3} . To achieve a minimum density of 6 Mg.m^{-3} ,

$$\rho = 6 = f_{\text{alumina}}(3.96) + (1-f_{\text{alumina}})(8.96) \quad \text{or} \quad f_{\text{alumina}} \geq 0.592$$

But now the copper does not make up at least 50 vol% of the contact; thus the combination of alumina and copper will not work. We might consider some other hard material, other than alumina, as the reinforcing particles. Silicon carbide, for example, has a density of about 3 Mg.m^{-3} . Now, the amount of SiC needed for a density of 6 Mg.m^{-3} in the composite is:

$$\rho = 6 = f_{\text{SiC}}(3) + (1-f_{\text{SiC}})(8.96) \quad \text{or} \quad f_{\text{SiC}} \geq 0.497$$

Thus one possible design is to incorporate 50 vol% SiC particles into pure copper, providing a density of approximately 6 Mg.m^{-3} . We might produce the part by combining copper powders with silicon carbide powders, compacting them, and finally sintering them to produce the finished part.

Chapter 17 Construction Materials

Suggested Solutions to Design Problems

17.7 A wooden structure is functioning in an environment controlled at 65% humidity. Design a wood support column that is to hold a compressive load of 90 kN. The distance from the top to the bottom of the column should be 2400 ± 6 mm when the load is applied.

At 65% humidity, we would expect approximately 12% moisture in the wood; consequently the data in Tables 17.1 and 17.2 might be applicable. It is certain that the longitudinal grain will be aligned with the 2400 mm long column.

We might select any of the woods in the tables and design the column. For example, let's select oak. The modulus of elasticity of the oak is 12.4×10^3 MN.m⁻² and its compressive strength in the longitudinal direction is 43 MN.m⁻². In order to withstand a load of 90 kN, the cross-sectional area of the column must be

$$A = F/\sigma = 90 \times 10^3 / 43 \times 10^6 = 2.093 \times 10^{-3} \text{ m}^2$$

We can determine the strain if the column were loaded to its compressive strength:

$$\epsilon = \sigma / E = 43 \times 10^6 / 12.4 \times 10^9 = 3.47 \times 10^{-3} \text{ m.m}^{-1}$$

Over a 2400 m length, the total elastic deformation is $(3.47 \times 10^{-3})(2400) = 8.33$ mm. We may need to cut the oak column approximately 2406 mm long; when the load is applied, approximately 8 mm of elastic compression will occur, putting the column, under load, within the 2400 ± 6 mm specification.

17.8 Design a wood floor that will be 15250 x 15250 mm and will be in an environment in which humidity changes will cause a fluctuation of plus or minus 5% water in the wood. We want to minimize any buckling or gap-formation in the floor.

Let's assume that the flooring will be cut so that the radial direction of the cut represents the width of the boards. We can then use Table 17.4 to estimate the amount the the flooring will expand or contract as the humidity, and hence the moisture content of the wood, changes. The change in width is

$$\Delta x = x_0 [c(M_f - M_i)] = (15250) [c(10)]$$

where the "10" represents the maximum change, from highest to lowest moisture content, that is expected. Let's consider two woods -- cedar and oak. The dimensional coefficients in the radial direction are 0.00111 for cedar and 0.00183 for oak. Thus:

$$\text{Cedar: } \Delta x = (15250) [(0.00111)(10)] = 169 \text{ mm}$$

$$\text{Oak: } \Delta x = (15250) [(0.00183)(10)] = 279 \text{ mm}$$

With the cedar, the width of the floor may change by ± 85 mm when the humidity changes $\pm 0.5\%$; a greater variation occurs if oak flooring is used. This could also be expressed as ± 0.0056 mm.mm⁻¹. If the cedar boards were 150 mm wide, a gap of nearly 0.84 mm could open up between the boards.

17.9 We would like to produce a concrete that is suitable for use in building a large structure in a sulphate environment. For these situations, the maximum water-cement ratio should be 0.45 (by weight). The compressive strength of the concrete after 28 days should be at least 25 MN.m⁻². We have available coarse aggregate containing 2% moisture in a variety of sizes, and both fine and coarse sand containing 4% moisture. Design a concrete that will be suitable for this application.

Since the structure will be exposed to a sulphate environment, we would like to use a Type II or Type V cement; since the structure is large, perhaps the low rate of heat generation Type II would be our best choice, provided that the sulphate environment is not too aggressive.

We would like to have a 28-day compressive strength of at least 25 MN.m⁻²; this means, based on Figure 17.7, that the water-cement ratio should be less than about 0.56; since the maximum allowable water-cement ratio is given in the problem as 0.45, this is consistent. We also might want to assure that the water-cement ratio is at least 0.40 for workability. Let's pick a water-cement ratio of 0.42 by weight.

The structure is large; presumably the minimum thickness of any concrete wall will be several decimetres, but we don't know for sure. Since the maximum size of the aggregate should be about 20% of the wall thickness, perhaps we should select aggregate that is 37.5 mm thick. Since this is a large structure, a relatively small slump, let's say 75 mm, might be used. Based on the aggregate size and the slump, we would like to use (Figure 17.10) about 175 kg of water per cubic metre of concrete.

If we have 175 kg water per cubic metre of concrete, then based on the water-cement ratio of 0.42,

$$\text{amount of cement per m}^3 = 175 \text{ kg water} / 0.42 \text{ water/cement} = 417 \text{ kg}$$

Let's use the coarse sand, since the overall structure is large and the aggregate is rather coarse. From Figure 17.11, the volume ratio of aggregate to concrete would then be about 0.75. The volume ratio uses the bulk density of the aggregate, which is about 60% of the true density.

$$\text{volume of water/m}^3 \text{ concrete} = (175 \text{ kg}) / (1 \text{ Mg.m}^{-3}) = 175 / 1 \times 10^3 \text{ kg.m}^{-3} = 0.175 \text{ m}^3$$

$$\text{volume of cement/m}^3 \text{ concrete} = (417 \text{ kg}) / (1.75 \text{ Mg.m}^{-3}) = 417 \times 10^{-3} \text{ Mg} / 1.75 \text{ Mg.m}^{-3} = 0.238 \text{ m}^3$$

$$\text{volume of coarse aggregate/m}^3 = (0.75) (1 \text{ m}^3) (0.6) = 0.450 \text{ m}^3$$

Thus the volume of the coarse sand must be

$$\text{volume of coarse sand/m}^3 = 1 - 0.175 - 0.238 - 0.450 = 0.137 \text{ m}^3$$

The actual amount of coarse sand that must be added must be adjusted for the 4% moisture (by weight) in the sand:

$$\text{wet sand} = (1.04) (0.137 \text{ m}^3) (2.56 \text{ Mg.m}^{-3}) = 0.365 \text{ kg}$$

$$\text{water in wet sand} = 0.365 - (0.137 \text{ m}^3) (2.56 \text{ Mg.m}^{-3}) = 0.014 \text{ Mg} = 14 \text{ kg}$$

Likewise for the coarse aggregate, containing 2% moisture:

$$\text{wet aggregate} = (1.02) (0.450 \text{ m}^3) (2.72 \text{ Mg.m}^{-3}) = 1.248 \text{ Mg}$$

$$\text{water in wet aggregate} = 1.248 - (0.450 \text{ m}^3) (2.72 \text{ Mg.m}^{-3}) = 0.0245 \text{ Mg} = 24.5 \text{ kg}$$

Therefore our concrete mix per cubic metre might be:

sacks of concrete = 417 kg
 wet 37.5 mm aggregate = 1248 kg
 coarse wet sand = 365 kg
 water = $0.175 \text{ Mg} - 0.014 - 0.0245 = 0.1365 \text{ Mg} = 0.1365 \text{ Mg}/1 \text{ Mg.m}^{-3} = 0.1365 \text{ m}^3 = 136.5 \text{ dm}^3$

17.10 We would like to produce a concrete sculpture. The sculpture will be as thin as 75 mm in some areas and should be light in weight, but it must have a 28-day compressive strength of at least 12 MN.m^{-2} . Our available aggregate contains 1% moisture and our sands contain 5% moisture. Design a concrete that will be suitable for this application.

In this case, we would like to select a concrete mix that has a reasonable slump, since the sculpture is relatively thin and is likely to be rather complex. In addition, 20% of 75 mm gives a maximum aggregate size of about 15 mm. In this case, let's select a slump of 150 mm and a 12.5 mm aggregate. From Figure 17.10, the water in kg/m^{-3} is about 230.

We don't know of any unusual conditions to which the concrete will be exposed, and the structure isn't extraordinarily large, so let's recommend a general purpose Type I concrete. However we do want the concrete to be light in weight; perhaps we could use a lightweight slag, with a density of 1.28 Mg.m^{-3} , as our aggregate.

We could also include extra entrained air to reduce the weight, but for this example let's assume no extra entrained air. The concrete must have a strength of at least 12 MN.m^{-2} ; any water/cement ratio (by weight) less than about 0.8 would be satisfactory (Figure 17.9). A high water/cement ratio would also give exceptional workability. To provide a bit of a safety margin on the strength, let's select a water/cement ratio of 0.70.

Let's use a fine sand. For the 150 mm coarse aggregate, Figure 17.11 indicates that we need a volume ratio of aggregate to concrete of about 0.55, noting that this is based on a bulk density and must be adjusted by 60% in order to use the true density.

Now we should be able to calculate the mix:

water/ $\text{m}^3 = 230 \text{ kg} = 0.230 \text{ m}^3$
 cement/ $\text{m}^3 = 230 \text{ kg} / 0.7 = 328 \text{ kg}/1.75 \text{ Mg.m}^{-2} = 0.188 \text{ m}^3$
 aggregate/ $\text{m}^3 = (1 \text{ m}^3) (0.55) (0.6) = 0.33 \text{ m}^3$
 $= (0.33 \text{ m}^3) (1.28 \text{ Mg.m}^{-3}) = 422 \text{ kg}$
 sand/ $\text{m}^3 = 1 - 0.230 - 0.188 - 0.33 = 0.252 \text{ m}^3$
 $= (0.252 \text{ m}^3) (2.56 \text{ Mg.m}^{-3}) = 0.645 \text{ Mg} = 645 \text{ kg}$

But we must adjust for the moisture in the sand and aggregate.

wet aggregate = $(1.01) (422) = 426 \text{ kg}$ or 4 kg water
 wet sand = $(1.05) (645) = 677 \text{ kg}$ or 32 kg water
 added water = $230 - 4 - 32 = 194 \text{ kg} = 194 \text{ dm}^3$

Our final recommendation might be:

328 kg cement per m^3
 194 dm^3 of water per m^3
 422 kg wet light weight aggregate
 645 kg fine sand

17.11 The binder used in producing asphalt has a density of about 1.3 Mg.m^{-3} . Design an asphalt, including the weight and volume of each constituent, that might be suitable for use as pavement. Assume that the sands and aggregates are the same as those for a normal concrete.

We have relatively little information on which to base our design. The text indicates that we typically have about 5 to 10% of the binder in the asphalt and that voids of 2 to 5% should be present. We expect to use both a coarse aggregate and a finer sand. The bulk density of an aggregate is typically about 60% of the true density of the material. Thus, there should be about 40 volume percent void space between the coarse aggregate particles. In one cubic metre of asphalt, 0.6m³ should be coarse aggregate and 0.4m³ should contain sand, binder, and voids. If we want to have a final void space of say 5%, then 0.05 m³ of voids must be produced, or the sand and binder must occupy 0.35 m³. If we assume that there will be 5 vol% binder in the total asphalt, then the binder volume is 0.05 m³ and the sand volume is 0.35 - 0.05 = 0.30 m³. In terms of weight per m³.

aggregate = 0.6 m³ x 2.56 Mg.m⁻³ = 1.536 Mg
 void = 0.05 m³ x 0 Mg.m⁻³ = 0 Mg
 sand = 0.30 m³ x 2.72 Mg.m⁻³ = 0.816 Mg
 binder = 0.05 x 1.3 Mg.m⁻³ = 0.065 Mg

Chapter 18 Electrical Behaviour of Materials

Suggested Solutions to Design Problems

18.45 We would like to produce a 100Ω resistor using a thin wire of a material. Design such a device.

$$R = \ell/\sigma A = 100 \Omega \quad \text{or} \quad \ell/A = 100\sigma$$

In order to produce the resistor, we need to select an appropriate material, then select an appropriate length or diameter. Let's try three materials from Table 18.1 -- say magnesium, boron carbide, and alumina:

$$\text{Al}_2\text{O}_3: \ell/A = 100(10^{-12}) = 10^{-10}$$

If we use a 1 mm diameter wire, its length would be $\ell = 7.9 \times 10^{-14}$ mm. This is certainly too short to be a practical design.

$$\text{Mg}: \ell/A = 100(2.25 \times 10^7) = 2.25 \times 10^9$$

If we use a 1 mm diameter wire, its length would be $\ell = 1770$ m. This seems a little too long to be manageable.

$$\text{B}_4\text{C}: \ell/A = 100(200) = 200$$

If we use a 1 mm diameter wire, its length would be $\ell = 15.7$ mm. This might be a practical length and might therefore be one possible design.

18.46 Design a capacitor that is capable of storing $1 \mu\text{F}$ when 100 V is applied.

We might use Equation 18.33 to design a parallel plate capacitor:

$$C = \epsilon_0 \kappa (n-1)A/d = 1 \mu\text{F} = 1 \times 10^{-6} \text{ F}$$

There are a multitude of designs that we might use. First, let's narrow down the possible dielectrics in Table 18.9 that we might select. To make the capacitor relatively small, we might select a dielectric that has a relatively high dielectric constant κ , say alumina ($\kappa = 9$ at 60 Hz and 6.5 at 10^6 Hz) or barium titanate ($\kappa = 3000$ at 10^6 Hz). The permittivity $\epsilon_0 = 8.85 \times 10^{-12} \text{ F.m}^{-1}$. Let's design the capacitor for use in a 10^6 Hz circuit.

$$\text{Al}_2\text{O}_3: (8.85 \times 10^{-12})(6.5)(n-1)A/d = 10^{-6}$$

Now we can select a combination of number of plates, area of plates, and thickness of dielectric that might be suitable. We can use the dielectric strength of alumina ($6 \times 10^6 \text{ V.m}^{-1}$) to assure that the dielectric will not break down:

$$6 \times 10^6 = V/d = 100/d \quad \text{or} \quad d_{\text{minimum}} = 100/6 \times 10^6 = 16.7 \times 10^{-6} \text{ m} \\ = 16.7 \times 10^{-3} \text{ mm}$$

The alumina must be at least 0.0167 mm thick:

$$(n-1)A = (10^{-6})(16.7 \times 10^{-6}) / (8.85 \times 10^{-12})(6.5) = 2900$$

$$\begin{aligned} \text{If } A &= 1 \text{ cm}^2 & n &= 2901 \\ A &= 10 \text{ cm}^2 & n &= 291 \\ A &= 100 \text{ cm}^2 & n &= 30 \end{aligned}$$

A reasonable design might be to use a parallel plate capacitor with 30 aluminium conductor plates, 29 layers of alumina dielectric sheets each 0.0167 mm thick; the dimensions of the conductor and alumina plates would be 100 mm x 100 mm.

$$\text{BaTiO}_3: (8.85 \times 10^{-12}) (3000) (n-1)A/d = 10^{-6}$$

The dielectric strength of the titanate is $12 \times 10^6 \text{ V.m}^{-1}$

$$12 \times 10^6 = V/d = 100/d \text{ or } d_{\text{minimum}} = 100 / 12 \times 10^6 = 8.33 \times 10^{-6} \text{ m} \\ = 8.33 \times 10^{-3} \text{ mm}$$

$$(n-1)A = (10^{-6}) (8.33 \times 10^{-4}) / (8.85 \times 10^{-14}) (3000) = 3.14$$

$$\text{If } n = 5, A = 0.785 \times 10^{-4} \text{ m}^2$$

Thus a reasonable design might be to use a parallel plate capacitor with 5 aluminium conductor plates, 4 layers of barium titanate dielectric sheets each $8.33 \times 10^{-4} \text{ cm}$ thick; the dimensions of the conductor and barium titanate sheets could be circles of approximately 10 mm diameter.

18.47 Design an epoxy-matrix composite that has a modulus of elasticity of at least 24.1 GN.m^{-2} and an electrical conductivity of at least $1 \times 10^7 \Omega^{-1} \text{ m}^{-1}$.

Note: the student may have to use other reference materials to find the modulus and conductivity of some of the possible materials. For example, the conductivity of beryllium, tungsten, and carbon might be useful to have.

Both the electrical conductivity and the modulus of elasticity are very high. Only a few metals, such as beryllium and tungsten, have a modulus of more than 24 GN.m^{-2} ; the electrical conductivity of these metals are not included in Table 18.1, but they are relatively low. Yet we need to use a metal with a high conductivity, such as aluminium or copper, to obtain the needed conductivity. One approach might be to produce a composite material that is reinforced with enough of a stiff material, such as carbon or ceramic fibres, yet still have a sufficient volume fraction of conductive metal left to provide the electrical conductivity. For example, we might reinforce aluminium with boron carbide (B_4C) fibres.

$$\sigma_{\text{composite}} = f_{\text{Al}} \sigma_{\text{Al}} + (1 - f_{\text{Al}}) \sigma_{\text{boron carbide}} = 1 \times 10^5$$

$$f_{\text{Al}} (3.77 \times 10^5) + (1 - f_{\text{Al}}) (1.5) = 1 \times 10^5$$

$$f_{\text{Al}} \geq 0.265$$

$$E_{\text{composite}} = f_{\text{Al}} E_{\text{Al}} + (1 - f_{\text{Al}}) E_{\text{boron carbide}} = 35 \times 10^6$$

$$f_{\text{Al}} (10 \times 10^6) + (1 - f_{\text{Al}}) (70 \times 10^6) = 35 \times 10^6$$

$$f_{\text{Al}} \leq 0.583$$

Therefore any composite that contains between 0.265 and 0.583 volume fraction of aluminium strengthened by directionally aligned boron carbide fibres would give the appropriate properties, at least in one direction of the contact material.

18.48 Design a semiconductor thermistor device that will activate a cooling system when the ambient temperature reaches 500°C.

We need a material that will perhaps change its electrical conductivity as the temperature changes. When the material reaches 500°C, it will then achieve an electrical conductivity that will permit an electrical circuit to be activated (or perhaps deactivated).

One approach might be to use a metal wire. As the temperature increases, the electrical conductivity decreases. When the conductivity falls below a critical level, at 500°C, an electrical circuit might be broken. We could determine the conductivity at 500°C using Table 18.3. For example, if we used a nickel wire as our device,

$$\rho_{500} = \rho_r(1 + a\Delta T) = (6.84 \times 10^{-8}) [1 + (0.0069)(500 - 25)] \\ = 2.93 \times 10^{-7} \Omega \cdot m$$

$$\sigma_{500} = 3.41 \times 10^6 \Omega^{-1} m^{-1}$$

A second approach might be to use an intrinsic semiconductor; as the temperature increases, the electrical conductivity also increases. When the conductivity reaches a critical level at 500°C, an electrical circuit might be completed. For example, we might select germanium.

$$\sigma_{500} = n_o q (\mu_e + \mu_h) \exp(-E_g/kT)$$

From Example 18.6, $n_o = 1.01 \times 10^{25}$ carriers/m³

$$\sigma_{500} = (1.01 \times 10^{25}) (1.6 \times 10^{-19}) (0.38 + 0.18) \exp[-0.67 / (8.63 \times 10^{-5}) (773)] \\ = 39.3 \Omega^{-1} m^{-1}$$

If the student has some background in circuits, perhaps the design can be further extended.

18.49 Design a dielectric device that will detect whether sand is at a particular level in a sand storage tank.

One method that might work would be to produce a simple parallel plate capacitor. Two conductor plates would be separated by a distance, say 5 mm. This separation would be sufficient to allow sand to fill the space between the plates if the level of the sand is high enough to have reached that particular location in the tank. Silica is a dielectric material (note that data for fused, or glassy silica, is provided in Table 18.9; sand usually is quartz, however, and the student may have to use other reference materials to find the dielectric constant for the crystalline material). The capacitance measured by an electrical circuit will be different if sand fills the space between the conductors compared to when no sand is present.

18.50 Design a piezoelectric part that will produce 25,000 V when subjected to a stress of 35 kN.m⁻².

The electric field must be $\xi = V/\text{thickness} = g\sigma$

where $V = 25,000$ V and $\sigma = 35$ kN.m⁻². Thus

$$\text{thickness} = V/g\sigma = 25,000/35 \times 10^3 g = 0.714 g$$

The constant $g = 1/Ed$, where values for d are given in Table 18.10.

$$\text{thickness} = 0.714Ed$$

Suppose that we make the part using quartz, which has a modulus of elasticity of approximately 73 GN.m^{-2} and $d = 2.3 \times 10^{-12} \text{ m.V}^{-1}$

$$\text{thickness} = (0.714) (73 \times 10^9) (2.3 \times 10^{-12}) = 120 \text{ mm}$$

Perhaps a quartz rod about 120 mm long, exposed to a stress of 35 kN.m^{-2} will produce this voltage.

Chapter 19 Magnetic Behavior of Materials

Suggested Solutions to Design Problems

19.17 Design a solenoid no longer than 10 mm that will produce an inductance of 0.3 tesla.

From the equations in the text, we can determine the relationship between the inductance, the magnetic properties of the material, and the characteristics of the coil that we want to design to produce the solenoid:

$$H = nI/\ell = B/\mu = B/\mu_r\mu_o$$

If we use SI units then $\mu_o = 4\pi \times 10^{-7} \text{T.m.A}^{-1}$. $B = 0.3$ tesla. The length ℓ must be no longer than 10 mm so let it equal 10 cm = 0.01 m.

$$nI/0.01 = 0.3/\mu_r \quad \text{or} \quad nI = 0.003/(\mu_r \times 4\pi \times 10^{-7})$$

Now we need to come up with a combination of material (μ_r), current, and number of turns that will satisfy the equation. Let's first try 45 Permalloy (Table 19.3), which has a relative permeability of 25,000.

$$nI = 0.003 / (25000 \times 4\pi \times 10^{-7}) = 0.095$$

*If $I = 1 \text{ mA} = 0.001 \text{ A}$, then $n = 95$ turns
 $I = 10 \text{ mA} = 0.01 \text{ A}$, then $n = 9.5$ turns*

We could also try 99.95% iron (Table 19-3) with a relative permeability of 5000. Then

$$nI = 0.003 / (5000 \times 4\pi \times 10^{-7}) = 0.477$$

*If $I = 1 \text{ mA} = 0.001 \text{ A}$, then $n = 477$ turns
 $I = 10 \text{ mA} = 0.01 \text{ A}$, then $n = 47.7$ turns*

The level of current will clearly influence the selection based on the above designs.

19.18 Design a permanent magnet that will have a remanence of at least 0.5 tesla, that will not be demagnetised if exposed to a temperature of 400°C or to a magnetic field of 80000 A.m⁻¹, and that has good magnetic power.

Let's first look at the materials in Table 19.5. Gadolinium, nickel, and Nd₂Fe₁₂B have Curie temperatures below 400°C and thus would not be suitable; others in the table appear to meet the temperature requirement.

For a remanence of at least 0.5 tesla, Table 19.4 only rules out SrO 6Fe₂O₃ and BaO 6Fe₂O₃.

For a coercive field of at least 80000 A.m⁻¹, Table 19.4 indicates that Co₅Sm, as well as the two ceramic ferrites and the NdFeB would work. But the latter three were eliminated by other factors. As it turns out, only Co₅Sm meets all of the requirements. It also has an exceptionally high power (200,000 T.A.m⁻¹).

19.19 Design a spinel structure that will produce a total magnetic moment per cubic metre of $5.6 \times 10^5 \text{ A.m}^{-1}$.

Based on Example 19.6, we expect that, for magnetite, a total magnetic moment per cubic metre would be $5.1 \times 10^5 \text{ A.m}^2 \text{ per m}^3$. To produce a larger total magnetic moment, we would need to replace Fe^{2+} ions with ions having more Bohr magnetons per atom. Fe^{2+} has 4 Bohr magnetons; let's try adding Mn^{2+} , with 5 Bohr Magnetons.

In Example 19.6, we find that the volume of the spinel unit cell is $5.86 \times 10^{-28} \text{ m}^3$. If we let "x" be the fraction of Mn^{2+} ions that replace the Fe^{2+} ions, then

$$\text{moment} = \frac{(8 \text{ subcells}) [(x) (5 \text{ magnetons}) + (1-x) (4 \text{ magnetons})] (9.27 \times 10^{-24})}{5.86 \times 10^{-28}}$$

$$5.6 \times 10^5 = [40x + 32 - 32x] (9.27 \times 10^{-24}) / 5.86 \times 10^{-28}$$

$$8x + 32 = 35.4 \quad \text{or} \quad x = 0.425$$

Thus 42.5% of the Fe^{2+} ions would have to be replaced by the manganese ions to produce the required magnetic moment.

19.20 Design a spinel structure that will produce a total magnetic moment per cubic metre of $4.1 \times 10^5 \text{ A.m}^{-1}$.

In this case, we need to replace the Fe^{2+} ions with ions having fewer than 4 Bohr magnetons per ion. Possibilities might include

Ni^{2+} - 2 Bohr magnetons
 Cu^{2+} - 1 Bohr magneton
 Zn^{2+} - 0 Bohr magneton

If we assume that the volume of the spinel unit cell remains as $5.86 \times 10^{-28} \text{ m}^3$ when the iron ions are replaced, we can determine the ion fraction "x" of the replacement ions required.

$$\text{Ni}^{2+}: 4.1 \times 10^5 = \frac{(8 \text{ subcells}) [x_{\text{Ni}} (2 \text{ mag.}) + (1-x_{\text{Ni}}) (4 \text{ mag.})] (9.27 \times 10^{-24})}{5.86 \times 10^{-28}}$$

$$4.1 \times 10^5 = [16x_{\text{Ni}} + 32 - 32x_{\text{Ni}}] (9.27 \times 10^{-24}) / 5.86 \times 10^{-28}$$

$$16x_{\text{Ni}} = 32 - 25.9 = 6.1 \quad \text{or} \quad x_{\text{Ni}} = 0.381$$

$$\text{Cu}^{2+}: 4.1 \times 10^5 = \frac{(8 \text{ subcells}) [x_{\text{Cu}} (1 \text{ mag.}) + (1-x_{\text{Cu}}) (4 \text{ mag.})] (9.27 \times 10^{-24})}{5.86 \times 10^{-28}}$$

$$24x_{\text{Cu}} = 32 - 25.9 = 6.1 \quad \text{or} \quad x_{\text{Cu}} = 0.254$$

$$\text{Zn}^{2+}: 4.1 \times 10^5 = \frac{(8 \text{ subcells}) [x_{\text{Zn}} (0 \text{ mag.}) + (1-x_{\text{Zn}}) (4 \text{ mag.})] (9.27 \times 10^{-24})}{5.86 \times 10^{-28}}$$

$$32x_{\text{Zn}} = 32 - 25.9 = 6.1 \quad \text{or} \quad x_{\text{Zn}} = 0.191$$

The amount of the Fe^{2+} ions that must be replaced depends on the type of ions that are substituted. Of course, we could also substitute some combination of other ions as well.

Chapter 20 Optical Behaviour of Materials

Suggested Solutions to Design Problems

20.24 Nickel X-rays are to be generated inside a container, with the X-rays being emitted from the container through only a small slot. Design a container that will assure that no more than 0.01% of the K_α nickel X-rays escape through the rest of the container walls, yet 95% of the K_α nickel X-rays pass through a thin window covering the slot. The following data give the mass absorption coefficients of several metals for nickel K_α X-rays. The mass absorption coefficient μ_m is μ/ρ , where μ is the linear mass absorption coefficient and ρ is the density of the filter material.

Material	μ_m ($\text{m}^2 \cdot \text{Mg} \cdot \text{m}^{-1}$)	ρ ($\text{Mg} \cdot \text{m}^{-3}$)
Be	1.80×10^2	1.848
Al	5.84×10^3	2.699
Ti	2.47×10^4	4.507
Fe	3.54×10^4	7.87
Co	5.44×10^3	8.832
Cu	6.50×10^3	8.93
Sn	3.22×10^4	5.765
Ta	2.00×10^4	16.6
Pb	2.94×10^4	11.36

Most of the container must be designed so that less than 0.01%, or $I/I_0 \leq 0.0001$, of the X-rays escape. However the window in the slot must be designed so that more than 95%, or $I/I_0 \geq 0.95$, of the X-rays do escape.

We would also like to make the container using inexpensive materials and it would also be nice if it was not any heavier than necessary. Thus we might first try to make the main portion of the container from aluminium, iron, or copper:

$$I/I_0 = 0.0001 = \exp(-\mu_m \rho x)$$

$$\begin{aligned} \text{Al: } 0.0001 &= \exp[-(5.84 \times 10^3)(2.699)x] & x &= 0.58 \text{ mm} \\ \text{Fe: } 0.0001 &= \exp[-(3.54 \times 10^4)(7.87)x] & x &= 0.033 \text{ mm} \\ \text{Cu: } 0.0001 &= \exp[-(6.50 \times 10^3)(8.93)x] & x &= 0.15 \text{ mm} \end{aligned}$$

To assure that 95% of the X-rays pass through the slot, we might use a thin foil of a material that has a low absorption coefficient. Beryllium looks like a reasonable choice.

$$I/I_0 = 0.95 = \exp(-\mu_m \rho x) = \exp[-(1.80 \times 10^2)(1.848)x]$$

$$x = 0.154 \text{ mm}$$

One possible design might be to make the container from aluminium sheet material 1 mm thick (a little thicker than the minimum required thickness), with the opening covered by a beryllium foil 0.15 mm thick.

20.25 Design a method by which a photoconductive material might be used to measure the temperature of a material from the material's thermal emission.

We might use the fact that the photons produced thermally will cause electrons to enter the conduction band if their energy is high enough. As the temperature of the material increases, higher energy photons will be produced, causing a larger number of electrons to be excited into the conduction band and thus increasing the voltage or current in the photoconductive circuit. We might be able to correlate the voltage or current in the circuit with the temperature of the material emitting the photons; from this correlation (which might be different for different materials), we could then estimate the temperature.

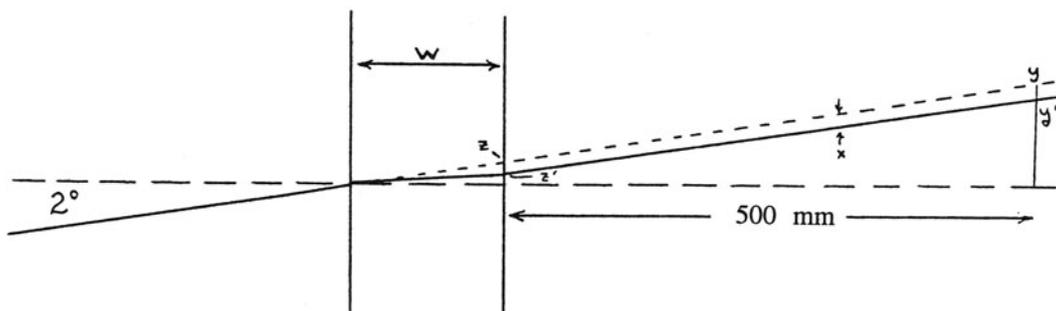
If we wish to be able to measure temperatures over a wide range, we might wish to select a photoconductive material that has a large energy gap, so that there is a continuous increase in the voltage or current as the temperature increases and produces photons of wavelength varying from 10 μm near room temperature to 0.1 μm at high temperatures. We could use the Equation $\lambda = hc/E_g$ to help us select an energy gap. If $\lambda = 10 \mu\text{m}$, the photons will possess an energy of

$$E = hc/\lambda = \frac{(6.62 \times 10^{-34} \text{ J s})(3 \times 10^8 \text{ m.s}^{-1})}{(10 \times 10^{-6} \text{ m})(1.6 \times 10^{-19} \text{ J/eV})} = 0.124 \text{ eV}$$

If $\lambda = 0.1 \mu\text{m}$, then $E = 12.4 \text{ eV}$. At low temperatures, the photons have a low energy (0.124 eV) and will excite few electrons into the conduction band; at high temperatures, the photons may have energies of 12.4 eV, sufficient to excite large numbers of electrons into the conduction band. The student may wish to select possible semiconducting materials from the tables in Chapter 18 and calculate the magnitude of the changes in electrical conductivity that are obtained in the different materials as the wavelength of the emitted photons changes.

20.26 Design a method, based on a material's refractive characteristics, that will cause a beam of photons originally at a 2° angle to the normal to the material to be displaced from its original path by 20 mm at a distance of 500 mm from the material.

The sketch below shows that the offset will be caused by two contributions: refraction of the beam and simply the 2° angle that the incident beam makes with the normal to the glass.



As an example, let's use silica glass, having an index of refraction of 1.46. Then

$$\sin\beta = (1/n)\sin 2^\circ = 0.0239 \quad \text{or} \quad \beta = 1.369^\circ$$

If no glass were present, the beam would reach the location of the back side of the glass at

$$z = w \tan 2^\circ = 0.0349w$$

But when the glass is present, the beam hits the back side of the glass at $z' = w \tan \beta$. The amount "x" by which the beam is deflected is

$$x = z - z' = (0.0349 - \tan 1.369)w = 0.011w$$

At a distance of 500 mm from the back of the glass, if no glass is present, the beam is offset "y" from the normal, or

$$y = (w + 500) \tan 2^\circ = 0.0349(w + 500)$$

But the beam is offset "x" by the glass, so at 500 mm the beam reaches a point y' from the normal. But y' is 20 mm. Thus

$$y' = y - s = 0.0349(w + 500) - 0.011w = 20 \text{ mm}$$

$$w = 106.69 \text{ mm}$$

We might try other materials to determine whether the refractive material could be thinner, and thus perhaps less expensive.

20.27 Design a 900 mm diameter satellite housing an infrared detector that can be placed into a low Earth orbit and that will not be detected by radar.

The satellite housing should, of course, be light in weight so it can be delivered into orbit at a low cost. This may rule out the use of certain types of materials, such as ceramic ferrites, that can sometimes be used to absorb radar but have a high density.

We would like the housing to be transparent to infrared radiation (or wavelengths of about 100 to 1 μm). This would permit the detectors inside the satellite to perform their function. However, we would like the longer radiation (microwaves and radio waves) typical of radar to be either absorbed within the satellites structure, transmitted through the structure, or reflected at an angle that would not permit the satellite to be observed.

We might design the satellite housing so that it does not contain any flat surfaces that might permit reflection of the radar signal -- using only curved surfaces or using many flat but differently oriented surfaces might be useful.

We might make the housing of a dielectric or semiconducting material that will absorb wavelengths in the radar spectrum. This might be applied as a coating to the underlying housing so as to minimize the amount of the material needed.

We might make the housing from a material that is transparent to photons in the radar range; however this may not be effective if the radar signal then is reflected from the electronics within the satellite.

Chapter 21 Thermal Properties of Materials

Suggested Solutions to Design Problems

21.20 A chemical reaction vessel contains liquids at a temperature of 680°C. The wall of the vessel must be constructed so that the outside wall operates at a temperature of 35°C or less. Design the vessel wall and appropriate materials if $Q_{\text{maximum}} = 25000 \text{ W}$.

The required temperature difference $\Delta T = 680 - 35 = 645^\circ\text{C}$.

$$Q/A = K\Delta T/\Delta x$$

If $Q = 25000 \text{ W} = 25000 \text{ J}\cdot\text{s}^{-1}$, then:

$$AK/\Delta x = Q/\Delta T = 25000 \text{ J}\cdot\text{s}^{-1} / 645 \text{ K} = 38.76 \text{ J}\cdot\text{s}^{-1}\text{K}^{-1}$$

Let's assume that we produce the wall of the vessel from Al_2O_3 , which has a thermal conductivity of $16 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$. Then

$$A/\Delta x = 2.422 \text{ m}$$

If the vessel is 0.1 m in diameter and 5 m long, then

$$A = \pi DL = \pi(0.1)(5.0) = 1.57 \text{ m}^2 \quad \text{and} \quad \Delta x = 0.65 \text{ m thick}$$

We can repeat this for a variety of refractory materials and vessel geometries.

21.21 Liquid copper is held in a silicon nitride vessel. The inside diameter of the vessel is 75 mm. The outside of the vessel is in contact with copper that contains cooling channels through which water at 20°C flows at the rate of 50 litres per minute. The copper is to remain at a temperature below 25°C. Design a system that will accomplish this end.

The heat flux through the silicon nitride vessel will be given by

$$Q/A = K\Delta T/\Delta x$$

The inside diameter of the vessel is 75 mm = $7.5 \times 10^{-2} \text{ m}$, or the inside surface area, or contact between the liquid copper and the silicon nitride is

$$A = 4\pi r^2 = (4)(\pi)(7.5 \times 10^{-2}/2)^2 = 1.77 \times 10^{-2} \text{ m}^2$$

The thermal conductivity of silicon nitride is $15 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$. The change in temperature through the vessel is the temperature of the liquid copper minus 25°C; the melting temperature of copper is about 1085°C, so perhaps we could assume that the copper is being held at a temperature of 1100°C. Thus

$$\begin{aligned} Q &= AK\Delta T/\Delta x = (1.77 \times 10^{-2} \text{ m}^2)(15 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1})(1100 - 25)/\Delta x \\ &= 285/\Delta x \quad \text{W or J}\cdot\text{s}^{-1} \end{aligned}$$

If we were to assume that heat transfer from the outer wall of the vessel into the cooling water was perfect, the heat that can be removed from the system by the water is

$$Q = (c_{\text{water}}) (\text{density water}) (\Delta T) (\text{volume flow rate})$$

where the specific heat of water is $4186 \text{ J.kg}^{-1}.\text{K}^{-1}$, the density is 1 Mg.m^{-3} , and the temperature change is $25 - 20^\circ\text{C}$.

$$Q = (4186 \text{ J.kg}^{-1}.\text{K}^{-1}) (10^3 \text{ kg.m}^{-3}) (50 \times 10^{-3} / 60 \text{ m}^3.\text{s}^{-1}) (25-20 \text{ K})$$

$$Q = 17442 \text{ J.s}^{-1}$$

If the maximum rate at which we can remove heat through the coils is 17442 J.s^{-1} , then we can calculate the thickness of the silicon nitride:

$$17442 = 285/\Delta x \quad \text{or} \quad \Delta x = 0.0163 \text{ m or } 1.63 \text{ cm}$$

21.22 Design a metal panel coated with a glass enamel capable of thermal cycling between 20°C and 150°C . The glasses generally available are expected to have a tensile strength of 35MN.m^{-2} and a compressive strength of 350MN.m^{-2} .

Suppose that we apply a glass enamel to a metal panel at 20°C . When the coated panel is then heated to 150°C , the thermal stresses will be

$$\sigma_{\text{thermal}} = \alpha E \Delta T$$

where $\Delta T = 150 - 20 = 130^\circ\text{C}$

E = modulus of elasticity of the glass

α = difference in coefficient of thermal expansion between the glass and the metal

σ_{thermal} must be less than the strength of the enamel

We can design our panel with different metals and different glass enamels. Suppose that we use a soda-lime glass, with a coefficient of thermal expansion of $9 \times 10^{-6} \text{ K}^{-1}$ and a modulus of elasticity of about $8.3 \times 10^{10} \text{ Nm}^{-2}$. During heating, the glass is likely to expand less than almost any metal that we select. Therefore during heating, there is a good chance that tensile stresses of greater than 35 MN.m^{-2} may develop and the glass enamel will crack. We could calculate the coefficient of thermal expansion of the metal that would prevent this from happening.

$$\sigma_{\text{thermal}} \leq 35 \times 10^6 = (\alpha_{\text{metal}} - 9 \times 10^{-6}) (8.3 \times 10^{10}) (130)$$

$$\alpha_{\text{metal}} - 9 \times 10^{-6} = 3.2 \times 10^{-6} \quad \text{or} \quad \alpha_{\text{metal}} = 1.22 \times 10^{-5} \text{ K}^{-1}$$

If the coefficient of thermal expansion of the metal is greater than $1.22 \times 10^{-5} \text{ K}^{-1}$, then the thermal stresses in the soda-lime glass will be large enough to cause the coating to crack. Iron has a coefficient of thermal expansion of $1.2 \times 10^{-5} \text{ K}^{-1}$, so an iron panel and a soda-lime glass coating might be one possible design.

21.23 Design a turbine blade for a jet engine that may be capable of operating at higher temperatures.

Turbine blades are typically produced from nickel-base superalloys, often heavily alloyed with solid solution strengthening elements and typically strengthened by carbide dispersions and precipitation of γ' particles. They may be directionally solidified, or even produced as single crystals, in an effort to improve their high temperature resistance. In many cases, the blades contain cooling channels to help them survive at high temperatures.

However we can also help to improve the life of the blades at higher temperatures by using a thermal barrier on the surface of the superalloy blade. The thermal barriers are generally a ceramic or complex

intermetallic coating. Some of the design considerations would include:

- the coating must have a high melting temperature and must resist oxidation or corrosion in the engine environment
- the coating must have similar expansion and contraction characteristics as the underlying superalloy so that it does not crack or flake off
- the coating must be dense and nonporous so that corrosive gases cannot penetrate through the coating and attack the superalloy; diffusion rates of these gases through the coating must also be very slow
- the coating must be firmly bonded to the superalloy blade; often this may require interlayers that bond well to both the superalloy and the thermal barrier

Chapter 22 Corrosion and Wear

Suggested Solutions to Design Problems

22.32 A cylindrical steel tank 1 m in diameter and 2.5 m long is filled with water. We find that a current density of 150 A.m^{-2} acting on the steel is required to prevent corrosion. Design a sacrificial anode system that will protect the tank.

$$A_{\text{tank}} = \pi DL = (\pi) (1 \text{ m}) (2.5 \text{ m}) \\ = 7.854 \text{ m}^2$$

$$i = I/A \geq 150 \text{ A.m}^{-2} \quad \text{or} \quad I \geq (150 \text{ A.m}^{-2}) (7.854 \text{ m}^2) = 1178 \text{ A}$$

One method might be to introduce a direct current electrical signal of 1178 A into the steel tank, being sure that the electrons are flowing from the rectifier, or dc source, towards that steel tank.

A second method might be to use a sacrificial anode, such as zinc. The zinc would be attached to the inside of the tank, causing physical contact with the steel. The same current would be required in both the sacrificial anode and the steel tank. If we used a block of zinc having a surface area of 0.1 m^2 , then the rate of zinc loss per hour would be

$$W_{\text{Zn loss}} = ItM/nF = (1178 \text{ A}) (3600 \text{ s}) (65.38 \text{ g.mol}^{-1}) / (2) (96,500) = 1436 \text{ g.h}^{-1}$$

If we used a 10 kg block of zinc, it would have to be replaced every 7 hours. In this case, we would likely prefer to use the impressed current method rather than the sacrificial anode method for protecting the steel tank.

22.33 The drilling platforms for offshore oil rigs are supported on large steel columns resting on the bottom of the ocean. Design an approach to assure that corrosion of the supporting steel columns does not occur.

We could set up a generator on the oil rig and produce a direct current, with the electrons flowing into the oil rig, to provide an impressed voltage. An alternative method would be to use zinc, magnesium, or another metal as a sacrificial anode. These would be fixed onto the steel columns, underwater, at intervals sufficient to provide a high enough current to prevent the corrosion of the steel. In the case of the sacrificial anodes, divers would have to periodically replace the anodes as they are consumed.

22.34 A storage building is to be produced using steel sheet for the siding and roof. Design a corrosion-protection system for the steel.

We might approach this in several ways. Although we could use an impressed voltage or sacrificial anodes, we more likely would try to use an appropriate coating. For example, we might use a phosphate coating on the steel to produce a relatively rough surface to which layers of paint would then adhere. If we were afraid that the paint would not be permanent enough, or that it would be scratched and thus produce an aggressive composition cell, we might coat the steel with a more permanent coating. Tin coatings would have the same problems that a painted surface would have -- if the coating is scratched, localized corrosion might be very severe.

Thus we might use galvanized (zinc-coated) steel or cadmium-coated steel. In these cases, the coating would also act as the anode should the coating be disrupted. The zinc-coating is by far the more common and least expensive.

22.35 Design the materials for the scraper blade for a piece of earth-moving equipment.

In this case, the major design criteria is that the blade have excellent abrasive wear resistance; consequently the blade should be produced from a material that can develop a high hardness. On the other hand, the blade will surely be subjected to impact-type blows, so some reasonable impact properties should also be characteristic of the material that we select.

A variety of materials might be used; for example, we might use a quenched and tempered steel heat treated to provide good hardness yet still tempered sufficiently to provide toughness to the blade. We might make it from a manganese steel (not mentioned in the text) that would strain harden during use, yet because of its FCC structure retains reasonable toughness. We might produce the blade from a relatively soft steel, for toughness, but hard surface the blade with a high carbon or hardenable steel by a welding process, or even overlay the blade, by welding, with a white cast iron that produces high hardness. We probably would not use anything exotic, like the cobalt alloy Stellite, due to cost considerations.

Chapter 23 Failure - Origin, Detection, and Prevention Suggested Solutions to Design Problems

23.30 Design a nondestructive method to continuously monitor the thickness of lead foil during a cold-rolling process. The lead foil is intended to be 0.0025 mm thick.

One method might be to use an ultrasonic resonance test. The lead has an ultrasonic velocity of $1.96 \times 10^3 \text{ m.s}^{-1}$. Suppose we decide that we would like to produce 14 half-wavelengths, or 7 full wavelengths, in the lead foil (we could choose more or less). Then $\lambda = 0.0025 \text{ mm} / 7 = 0.000357 \text{ mm} = 3.571 \times 10^{-7} \text{ m}$. The frequency to which we would set our ultrasonic inspection unit would thus be

$$\nu = v/\lambda = 1.96 \times 10^3 \text{ m.s}^{-1} / 3.53 \times 10^{-7} \text{ m} = 5.49 \times 10^9 \text{ Hz}$$

We could then perhaps connect the output from the resonance unit to a computer that would monitor the number of half-wavelengths. If the number changes, then an indication that the thickness has varied might be made.

Or perhaps a radiographic technique could be used; X-rays might pass through the lead foil and be received on a radiation counter. If the foil is too thick, the intensity of the transmitted X-rays will increase.

23.31 Design a nondestructive ultrasonic inspection method that will allow us to separate white cast iron, grey cast iron, and ductile cast iron from one another. You may wish to review the section on cast irons in Chapter 12.

White cast iron contains no graphite in its structure, ductile cast iron contains spheroidal graphite particles, and grey cast iron contains graphite flakes. Graphite will attenuate an ultrasonic pulse, particularly when the graphite is the flake form. Therefore we might use a through-transmission ultrasonic inspection method to measure the relative amount of absorption of the ultrasonic pulse. Relatively little of the pulse will be attenuated by white cast iron; a moderate amount will be attenuated by the ductile iron; most of the pulse will be absorbed by the grey iron.

23.32 Design the shielding around a Co^{60} gamma ray source to assure that no more than 0.001% of the radiation escapes into the surrounding environment.

The mass absorption coefficient for cobalt gamma rays is about $5.5 \text{ m}^2.\text{Mg}^{-1}$ for most materials. Let's try two materials as possible shielding -- beryllium and lead -- and determine the thickness of the shielding that is needed.

$$\text{Be: } I/I_0 = 0.00001 = \exp[-(5.5 \text{ m}^2.\text{Mg}^{-1})(1.85 \text{ Mg.m}^{-3})x] \quad x = 1130 \text{ mm}$$

$$\text{W: } I/I_0 = 0.00001 = \exp[-(5.5 \text{ m}^2.\text{Mg}^{-1})(11.34 \text{ Mg.m}^{-3})x] \quad x = 184 \text{ mm}$$

To absorb most of the gamma rays, very thick shielding will be required.

23.33 The boom of a crane is subjected to large stresses when it lifts heavy loads. As a result, cracks sometimes initiate and, if not detected, cause the

boom to fail, with catastrophic results. Design an inspection method to detect and locate cracks before they become large enough to cause fracture.

A boom is a very large structure to try to inspect completely using most of the nondestructive tests; in order to determine whether there are any cracks that might be large enough to pose a threat, we might first try acoustic emission testing. We might connect receivers at several locations on the boom, then load the boom of the crane (perhaps by using the crane to lift a load near its maximum rating). If there are cracks present in the crane, the load may cause the cracks to propagate a short distance, releasing strain energy that could be detected by one or more of the receivers. By triangulation of the emission using several receivers, the general area of the crack may be identified. Once the area containing suspected cracks is narrowed down, then it may be cost-effective and practical to use other techniques to locate the precise location and even size of the crack and therefore permit repairs to be made.

Sample Homework, Study Guides, Quizzes

The following section includes samples of questions for most of the chapters in the text; these have been used at the University of Missouri-Rolla as homework assignments intended to help the student in reading assignments and in preparation for examinations. We grade these assignments, with the total assignment score counting a small fraction of the total semester grade. These could also, however, be used as quizzes or simply as study guides. In general, students have been very receptive to these assignments, believing that the assignments help them understand what important topics the instructor expects them to learn.

The author typically generates two sets of these questions each academic year. He would be pleased to provide these to any instructor who wishes to use them. Either a hard copy or a WordPerfect 5.1 file can be obtained simply by calling Don Askeland on (314) 341-4730.

1. Suppose you must select a material that will most effectively conduct electricity. You would select a
 - a. polymer
 - b. ceramic
 - c. metal
 - d. semiconductor
2. Suppose you must select a material that will serve as an electrical insulator, preventing arcing between two conductors, in a high-temperature environment. You would select a
 - a. polymer
 - b. ceramic
 - c. metal
 - d. semiconductor
3. In general, when the temperature increases, the strength of most materials will
 - a. increase
 - b. decrease
4. In general, if the only design requirement for a component is the ability to survive at very high temperatures, you would select a
 - a. polymer
 - b. ceramic
 - c. metal
 - d. semiconductor
5. Although an aluminium alloy has only half the strength of a copper alloy, designers of aerospace components often select the aluminium alloy because
 - a. the strength-to-weight ratio of the aluminium is higher than copper
 - b. aluminium bends, while copper behaves in a very brittle manner
 - c. aluminium has a much higher melting temperature than copper
6. Most of the important atomic bonding processes insure that the outermost energy level of each atom appears to
 - a. contain four electrons
 - b. contain either zero or eight electrons
 - c. equal the element's valence
7. Atoms that tend to easily give up the electrons in the outermost energy level are said to be
 - a. electronegative
 - b. electropositive
8. A material is most likely to be ionically bonded when
 - a. the atoms in the material have the same valence
 - b. the atoms in the material have very different electronegativities
 - c. all of the atoms in the material are of the same element (for example all sodium)
9. Materials tend to have good conductivity and ductility for
 - a. metallic bonding
 - b. covalent bonding
 - c. ionic bonding
10. Electrical charge is conducted by movement of entire atoms rather than just by movement of tiny electrons for
 - a. metallic bonding
 - b. covalent bonding
 - c. ionic bonding
11. The compound CuAl_2 will be important when we discuss how aluminium can be strengthened. Without doing any calculations, you would expect bonding between the Cu and Al atoms to be mostly
 - a. Van der Waals
 - b. covalent
 - c. ionic
 - d. metallic
12. Two different types of elements are required in order to produce

- a. covalent bonding b. metallic bonding c. ionic bonding
13. Bonds between atoms are highly directional, or have fixed bond angles, for
a. covalent bonding b. metallic bonding c. ionic bonding
14. For covalently bonded materials, such as ceramics and semiconductor materials, increasing the temperature
a. increases the strength of the material by causing the strength of the atomic bonding to increase
b. increases the electrical conductivity of the material by breaking bonds and freeing some electrons to carry a charge
15. When a force is applied, atoms or groups of atoms most easily move relative to other atoms or atom groups when bonding is
a. metallic b. covalent c. ionic
16. Based on atomic bonding, we generally expect ceramic materials to
a. have good conductivity but behave in a brittle manner
b. have good conductivity and good ductility
c. have poor conductivity but behave in a ductile manner
d. have poor conductivity but behave in a brittle manner
17. Based on atomic bonding, we generally expect metals to
a. have good conductivity but behave in a brittle manner
b. have good conductivity and good ductility
c. have poor conductivity but behave in a ductile manner
d. have poor conductivity but behave in a brittle manner
18. Bonding within the chains of polymer materials is predominantly
a. metallic b. ionic c. covalent d. Van der Waals
19. Bonding between the chains of thermoplastic polymer materials is
a. metallic b. ionic c. covalent d. Van der Waals
20. Materials are stiff (high modulus of elasticity) and have a low coefficient of thermal expansion when the binding energy is
a. high b. low

1. Calculate the atomic radius (in cm) for a BCC metal with $a_0 = 0.3294$ nm and one atom per lattice point.
2. Determine the crystal structure for a metal with $a_0 = 4.9489$ nm, $r = 1.75$ nm, and one atom per lattice point.
3. The density of thorium, which has the FCC structure and one atom per lattice point, is 11.72 Mg.m^{-3} . The atomic weight of thorium is 232 g.mol^{-1} . Calculate (a) the lattice parameter and (b) the atomic radius of thorium.
4. A metal having a cubic structure has a density of 2.6 Mg.m^{-3} , an atomic weight of 87.62 g.mol^{-1} , and a lattice parameter of 6.0849 nm. One atom is associated with each lattice point. Determine the crystal structure of the metal.
5. Bismuth has a hexagonal structure with $a_0 = 0.4546$ nm and $c_0 = 1.186$ nm. The density is 9.808 Mg.m^{-3} and the atomic weight is $208.98 \text{ g.mol}^{-1}$. Determine (a) the volume of the unit cell, (b) how many atoms are in each unit cell, and (c) the packing factor for the unit cell.
6. Above 882°C , titanium has a BCC crystal structure with $a_0 = 0.332$ nm. Below this temperature, titanium has a HCP structure with $a_0 = 0.2978$ nm and $c_0 = 0.4735$ nm. Determine the volume change when BCC titanium transforms to HCP titanium. Is this a contraction or expansion?
7. A typical paper clip weighs 0.59 g and consists of BCC iron. Calculate (a) the number of unit cells and (b) the number of iron atoms in the paper clip. See Appendix A for required data.

8. Calculate the distance between two adjacent (122) planes in a cubic unit cell having a lattice parameter of 4.0 nm.

9. Cobalt oxide (CoO) has the sodium chloride crystal structure. (a) How many cobalt ions and how many oxygen ions are uniquely present in each unit cell? (b) What is the lattice parameter for the CoO unit cell? (c) What is the packing factor for the CoO unit cell. Be sure to use ionic radii from Appendix B.

10. Materials that are _____ can exist in more than one type of crystal structure.

11. A material that displays _____ behaviour will have the same mechanical and physical properties in all crystallographic directions.

12. The packing factor in the BCC structure is about _____ and that in the FCC structure is about _____.

13. The coordination number in the FCC structure is _____, that in the BCC structure is _____, and that in the simple cubic structure is _____.

14. The Miller indices for close packed planes in FCC structures are _____ while the close packed planes such as (0001) in the HCP structure are called _____ planes.

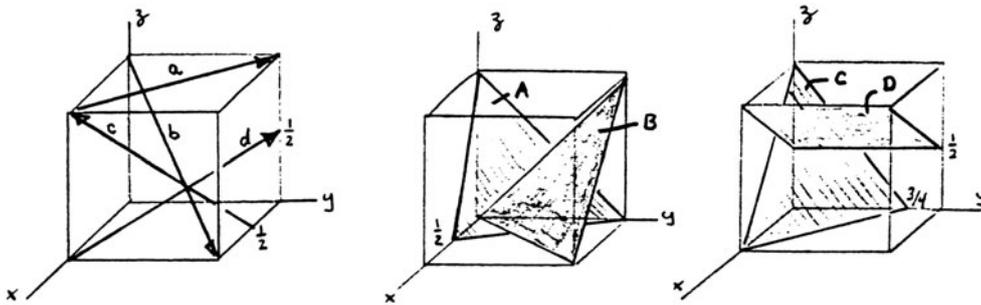
15. The stacking sequence for the face centred cubic structure is _____ while for the HCP structure the stacking sequence is _____. What is the stacking sequence for the BCC structure?

16. A (tetrahedral / octahedral) interstitial site has six nearest neighbors in a crystal structure.

17. We generally expect to have (higher / lower) packing factors in the covalently or ionically bonded ceramics than we do in metals.

18. The close packed directions in simple cubic are of the form _____, the close packed directions in BCC are of the form _____, and the close packed directions in FCC are of the form _____.

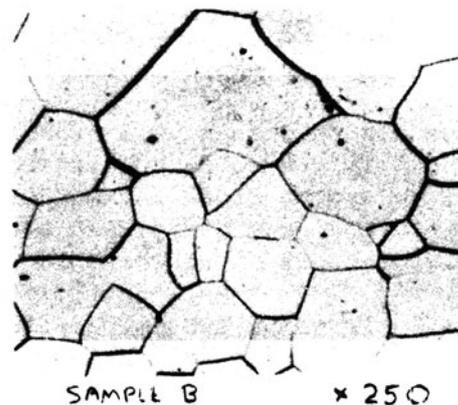
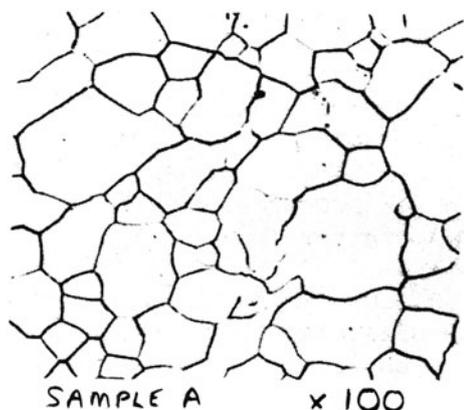
19. Determine the Miller indices for the directions and planes shown in the sketches below. Be sure to use the proper notation.



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1. The main reason metals have good ductility compared to ceramics is that _____ can easily move in metals but ionic repulsion or directionality of bonds prevents their movement in ceramics.
2. The most important methods for strengthening a metal rely on introducing lattice imperfections that interfere with the movement of _____.
3. The direction and distance that a dislocation moves in one step is given by the _____ vector.
4. The dislocation _____ is our usual measure of the number of dislocations in a material; 10^{10} m.m^{-3} is a small number of dislocations.
5. Ceramics have a large Burgers vector; this means that the stress required to cause a dislocation to move is (large / small) compared to metals, which have a small Burgers vector.
6. Increasing the number of dislocations will
 - a. (increase / decrease) the strength of a material
 - b. (increase / decrease) the ductility of a material.
7. The movement of dislocations is called _____.
8. The critical resolved shear stress is
 - a. a property or characteristic of the material
 - b. the external force applied to a material
 - c. independent of crystal structure
 - d. is the same for metals and crystalline ceramics
9. Materials that have a high critical resolved shear stress tend to have (high / low) strength.
10. HCP metals tend to have poor ductility compared to metals with a FCC or BCC structure because
 - a. the slip planes in HCP metals do not intersect
 - b. the critical resolved shear stress is too high
 - c. there are no close packed directions in the HCP structure
 - d. there are no close packed planes in the HCP structure
11. BCC metals tend to have a high critical resolved shear stress because
 - a. the slip planes do not intersect
 - b. there are no close packed directions in the BCC structure
 - c. there are no close packed planes in the BCC structure
 - d. dislocations cannot move in the BCC structure
12. The slip planes in FCC unit cells have Miller indices of the form _____. The slip directions in FCC unit cells have Miller indices of the form _____. The slip directions in BCC unit cells have Miller indices of _____.

13. When point and surface imperfections are introduced to a lattice, we find that (larger / smaller) forces are required to move dislocations.
14. When we increase the temperature of a material, we would NOT expect
- the number of vacancies to increase
 - the number of Frenkel defects to increase
 - the number of substitutional or interstitial defects to increase
15. When producing metallic alloys, we typically try to increase the number of
- Frenkel or Schottky defects
 - interstitial or substitutional defects
 - vacancies
16. When a tin atom is introduced into a copper lattice, we expect that a (tensile / compressive) strain field will distort the copper lattice (see Appendix B).
17. We expect that the strength of aluminium will (increase / decrease) when we increase the size of the grains.
18. A _____ is produced when the stacking sequence of close packed planes is disturbed in FCC metals.
19. The density of a sample of FCC palladium is 11.97 Mg.m^{-3} and its lattice parameter is 0.38902 nm . Calculate (a) the fraction of the lattice points that contain vacancies and (b) the total number of vacancies in a cubic centimeter of Pd.
20. The strength of titanium is found to be 450 MN.m^{-2} when the grain size is 0.0017 cm and 565 Mn.m^{-2} when the grain size is $0.8 \times 10^{-6} \text{ m}$. Assuming that the Hall-Petch equation is valid, calculate the strength of the titanium when the grain size is reduced to $0.2 \times 10^{-6} \text{ m}$.
21. The microstructures of two iron samples are shown below, but at different magnifications. Determine the ASTM grain size number for both samples and determine which sample -- A or B -- will have the higher strength.



1. Suppose we have a pure crystal of copper containing 10^{10} vacancies per m^3 . Which of the following statements is correct?
 - a. no diffusion of any kind will occur because there is no concentration gradient in the crystal
 - b. diffusion will occur only if the copper crystal is contaminated by gases dissolved into the crystal from the surrounding atmosphere
 - c. diffusion will occur, but no net flux will be observed
2. Copper and nickel atoms have nearly the same atomic radius. Therefore we would expect that zinc will diffuse in copper by a (interstitial / vacancy) diffusion mechanism.
3. Several pairs of conditions are described below. For each, circle the condition for which you expect to have the highest activation energy for diffusion.
 - a. interstitial diffusion versus vacancy (substitutional) diffusion
 - b. a crystal with a 0.68 packing factor versus a 0.74 packing factor
 - c. a metal with a high melting point versus a low melting point
 - d. diffusion of beryllium in copper versus barium in copper
 - e. an ionically bonded material versus a metallically bonded material
 - f. diffusion of Mg^{2+} ions in MgO versus O^{2-} ions in MgO
4. When we increase the temperature, the grain size in a metal (increases / decreases).
5. We expect that diffusion will occur more quickly (along grain boundaries / through the grains).
6. We would expect the electrical conductivity of an ionically bonded ceramic material to (increase / decrease) when we increase the temperature.
7. The sintering stage of a powder compaction process -- either in powder metallurgy or in production of ceramic components -- is expected to occur more rapidly when the temperature is (increased / decreased).
8. Suppose you have a steel container (like maybe some kind of a fuel cell) that has a wall thickness of 0.5 mm. You introduce hydrogen inside the container using a pressure that gives 5×10^{22} H atoms per m^3 . The container is in an evacuated chamber (like outer space) that contains no hydrogen atoms. Determine the concentration gradient through the wall of the container. Express your answer in units of atoms/ $m^3 \cdot m$.

9. Suppose the container is facing away from the sun, so its temperature is -50°C . The steel is essentially BCC iron. Calculate the flux of hydrogen atoms through the wall of the container.

10. Suppose the container is facing the sun, so its temperature is 450°C . The steel is still BCC iron. Recalculate the flux of hydrogen atoms through the wall of the container.

11. Will the flux in either question 9 or 10 remain constant with time? Explain briefly.

12. Suppose the grain size of the steel that makes up the container can be varied. Would you expect the loss of hydrogen pressure from the container to increase or decrease if the grain size is made much smaller? Explain briefly.

13. One method to fabricate complex aerospace components is diffusion bonding. We wish to join a nickel sheet, that is intended to rapidly transfer lightning strike energy away from critical locations, to a FCC iron (stainless steel) plate. We find that we can accomplish a good bond in 8 hours when the two metals are joined at 800°C . How long will it take to obtain an equally good bond if we decide to perform the bonding at 600°C ? (Assume that the quality of the bond is determined by the rate of diffusion of the nickel into the FCC iron).

1. The following data were collected from a standard 12.5 mm diameter test specimen of a copper alloy.

Load (lb)	Gauge Length (mm)
0	50.00000
13000	50.0422
26000	50.0845
32500	50.1059
39000	50.228
45500	51.016
52000	56.804
53300	62.7 (max load)
49000	75.9

After fracture, the gage length is 76.5 mm and the diameter is 9.5 mm.

- Determine the 0.2% offset yield strength
- Determine the tensile strength
- Determine the modulus of elasticity
- Determine the %elongation
- Determine the %reduction in area
- Determine the engineering stress at fracture
- Determine the true stress at fracture
- Why does the load decrease when the gage length increases from 62.7 mm to 75.9 mm?

2. A three-point bend test is performed on a block of ZrO_2 that is 200 mm long, 12.5 mm wide, and 6.25 mm thick and is resting on two supports 100 mm apart. When a force of 400 lb is applied, the specimen breaks. Calculate the flexural strength (modulus of rupture, MOR).

3. A ceramic matrix composite contains surface flaws introduced during the manufacturing process. Our nondestructive testing experts tell us that there are no cracks larger than 0.01 mm. The plane strain fracture toughness of the composite is 10,000 MPa \sqrt{m} . If $f = 1$, determine the minimum stress that can be applied before the composite is likely to fail.

4. A Brinell hardness test is performed on a heat treated steel and is found to be BHN 350. Estimate both the tensile strength and the endurance limit for the steel.

5. The stiffness of a material is most closely related to the _____ of the material.

6. The toughness of a material is most closely related to the _____ of the material.

7. In order to design the maximum loads that a material can withstand and still be able to perform its function, you would most likely be interested in the (tensile strength / yield strength / breaking strength).

8. You would expect the (yield strength / modulus of elasticity / %elongation) to increase when the temperature increases.

9. The service temperature at which a part is used should be
 - a. above the ductile-brittle transition temperature
 - b. below the ductile-brittle transition temperature
 - c. equal to the ductile-brittle transition temperature

10. We would expect a material to best resist failure due to an impact or shock load at -195°C if the material is
 - a. a metal with a BCC structure
 - b. a metal with a FCC structure
 - c. a metal with a HCP structure
 - d. a ceramic

11. The results from an impact test of a given material at a given temperature are
 - a. fixed values
 - b. highly dependent on the size and geometry of the specimen
 - c. usually easily incorporated into some type of an equation to permit the dimensions of a part to be designed

12. In most cases, materials and in particular metals fail where (tensile / compressive) forces act on the material

13. Suppose you want to design a part that will not fail within 250,000 applications of a cyclical load. The most important property of the material that will allow you to design this part is the (endurance limit / endurance ratio / fatigue strength / fatigue life).

14. Suppose you want to design a part that will never fail when a cyclical load is applied. The most important property of the material that will allow you to design this part is the (endurance limit / endurance ratio / fatigue strength / fatigue life).

15. The creep rate of a material will increase when
 - a. the temperature and/or applied stress increase
 - b. the temperature and/or applied stress decrease
 - c. the temperature increases or the applied stress decreases
 - d. the temperature decreases or the applied stress increases

16. The rupture time of materials will increase when
 - a. the temperature and/or applied stress increase
 - b. the number of substitutional point defects in the lattice increases
 - c. the melting temperature of the material decreases

17. Dislocation climb is particularly important in the (impact / tensile / creep / fatigue) test.

18. Fracture toughness is related to
 - a. the modulus of elasticity
 - b. the ease with which a crack initiates and propagates
 - c. the ability of dislocations to move in ceramics

19. The Charpy test is an example of a (impact / fatigue / hardness / creep) test.

20. The Rockwell test measures the _____ of a material.

1. A (high / low) strain hardening coefficient suggests that a material can be strain hardened a great deal.
2. Metals with the (FCC / BCC / HCP) crystal structure typically have a higher strain hardening coefficient.
3. The strain hardening coefficient is the slope of the stress-strain curve in the plastic region in a (true / engineering) stress-strain curve.
4. Strain hardening is really an important phenomenon in (ceramics / polymers / metals).
5. Strain hardening occurs because the number of (point defects / dislocations / grain boundaries) is increased dramatically.
6. When we cold work a material that originally has equiaxed, or randomly oriented, grains, we expect that the mechanical properties will become more (isotropic / anisotropic).
7. Cold working typically (increases / decreases) the level of residual stresses within a material.
8. Strain hardening is a particularly (effective / ineffective) method to improve the creep resistance of metals at elevated temperatures.
9. During the recovery stage of annealing of a cold worked metal,
 - a. the yield strength (increases / decreases / is unchanged)
 - b. the %elongation (increases / decreases / is unchanged)
 - c. the electrical conductivity (increases / decreases / is unchanged)
 - d. residual stresses (increase / decrease / are unchanged)
10. The grain size is typically reduced significantly during the (recovery / recrystallization / grain growth) step of annealing.
11. Recrystallization will occur at lower temperatures when
 - a. the amount of cold work is (30% / 80%)
 - b. the annealing time is (10 minutes / 10 hours)
 - c. the diameter of the original grains is (10 microns / 100 microns)
12. A stress relief anneal is another name for the (recovery / recrystallization / grain growth) stage of annealing.
13. When you weld a cold worked metal, then subject the welded assembly to a high stress, you expect the assembly to fail in the (fusion zone / heat affected zone / unaffected base metal).
14. The surface finish of a metal is expected to be better when the metal is shaped by (cold / hot) working.
15. The dimensional accuracy of a metal part is expected to be more precise when the metal is shaped by (cold / hot) working.

16. Metals with the HCP structure can be formed into more complex shapes by (cold / hot) working.
17. To obtain a small grain size, you should hot work a metal
- just below the recrystallization temperature
 - just above the recrystallization temperature
 - just above the grain growth temperature
18. Superplastic forming processes require
- metals with a (large / small) grain size
 - applying the forming force at a (high / low) rate
 - deforming the metal at (high / low) temperatures
19. Suppose that we cold work a 100 mm diameter 3105 aluminium alloy to a 50 mm diameter.
- the %cold work is _____
 - the yield strength is _____
 - the %elongation is _____
20. Suppose we cold work a 4-in. diameter 3105 aluminium alloy to a 7.5 mm diameter, take an hour off for lunch, and then cold work the alloy to a 50 mm diameter. What is the total %cold work in the final part? (Figure 7.22)
21. Suppose we would like to obtain a Cu-30%Zn plate that is 25 mm thick, has a yield strength of at least 60,000 psi, and a %elongation of at least 10%. How might we do this?

1. _____ nucleation requires that large numbers of atoms cluster together, usually at a large undercooling, in order to produce a spherical solid with a critical radius.
2. _____ nucleation is obtained when foreign substrates provide a location at which small numbers of atoms can cluster together, usually at a low undercooling, in order to produce a stable solid nucleus.
3. Which one of the following would NOT be a necessary or useful trait for an effective heterogeneous nucleating agent for solidification?
 - a. the agent should have a crystal structure that is similar to that of the solid to be nucleated
 - b. the agent should have a melting temperature above the melting temperature of the solid to be nucleated so that the agent is stable
 - c. the agent should have about the same density as the liquid into which it is introduced so that the agent neither floats nor sinks
 - d. the crystal structure of the nucleating agent should be very different from that of the solid to be nucleated, thus producing a large thermodynamic driving force for solidification
4. Heterogeneous nucleation in most aluminium alloys can be encouraged by introducing _____ and _____ (two elements).
5. (Planar / Dendritic) growth occurs in pure metals when a large undercooling is achieved.
6. _____ and _____ are the two terms used to describe our efforts to intentionally cause heterogeneous nucleation.
7. We would expect (planar / dendritic) growth to occur when we provide effective heterogeneous nuclei to a solidifying liquid metal.
8. We generally expect to control the grain size of a casting by controlling (nucleation / growth) phenomena.
9. We expect to control the substructure within a grain in a casting by controlling (nucleation / growth) phenomena.
10. The solidification time for several castings are shown below. Find the constants B and n in Chvorinov's rule for this metal.

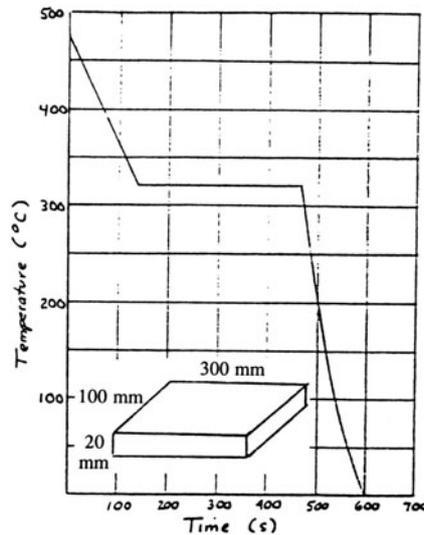
10 mm x 10 mm x 60 mm plate	28.58 s
20 mm x 40 mm x 40 mm plate	98.30 s
40 mm x 40 mm x 40 mm plate	155.89 s
80 mm x 60 mm x 50 mm plate	306.15 s

11. The secondary dendrite arm spacing was determined for several local solidification times in a casting. Find the constants c and m in the equation for the SDAS.

156 s	0.176 mm
282 s	0.216 mm
606 s	0.282 mm
1356 s	0.374 mm

12. A cooling curve is shown. Determine

- the pouring temperature
- the freezing temperature
- the superheat
- the cooling rate just before solidification begins
- the total solidification time
- the local solidification time
- the probable identity of the metal
- If the cooling curve was obtained at the centre of the casting sketched in the figure, determine the mould constant B , assuming $n = 2$.



13. Generally, if we want a casting with uniform properties, we want to encourage a large (columnar / equiaxed) zone in a casting.

14. We are best able to produce a large equiaxed zone by (inoculation / short solidification times).

15. Rapid solidification of a casting generally encourages a (large / small) secondary dendrite arm spacing in a casting.

16. We expect to obtain higher strengths in a casting when we encourage a (large / small) secondary dendrite arm spacing.

17. We expect to obtain higher strengths in a casting when we (increase / decrease) the amount of inoculant added to the liquid prior to pouring.

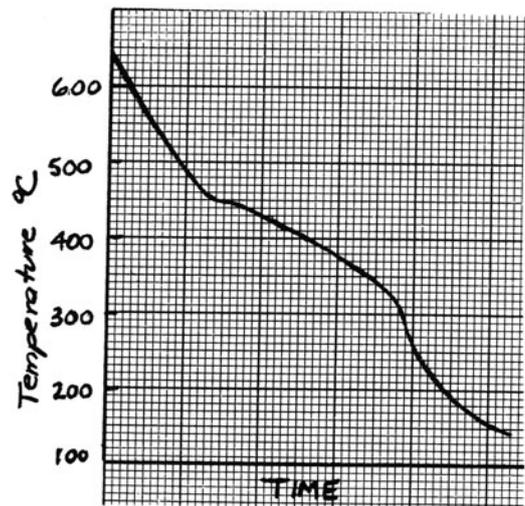
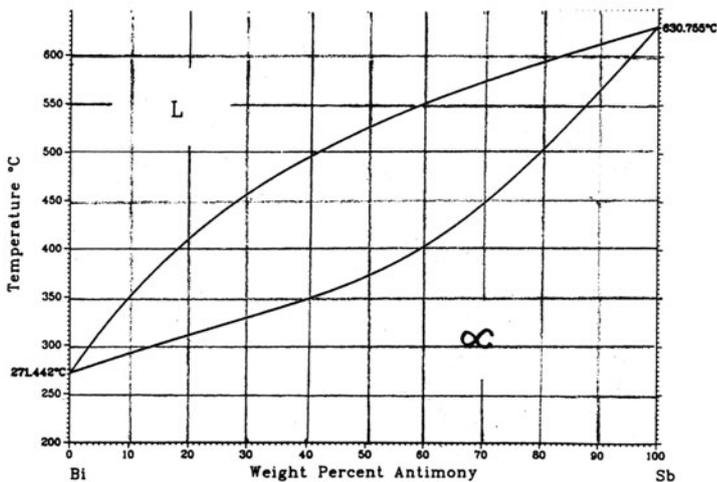
18. Explain why we prefer to obtain only columnar grains in a directionally solidified turbine blade casting.

1. Suppose you would like to strengthen gold wire. (See Appendix B)
 - a. Would you expect to be able to add unlimited amounts of silver to the gold without exceeding the solubility limit? Explain.
 - b. Would you expect to be able to add unlimited amounts of aluminium to gold without exceeding the solubility limit? Explain.
 - c. Would you expect to be able to add unlimited amounts of sodium to gold without exceeding the solubility limit? Explain.
 - d. If you could add 1% of silver, aluminium, or sodium to gold, which one would you expect to give the greatest strengthening effect? Explain.
2. Figure 9.7, showing the effect of zinc on the properties of copper, is correct. However the effect of zinc on one of the properties is not typical for solid solution strengthening. Which property is unusual?
3. You would expect that (substitutional / interstitial) alloying elements would ordinarily have the higher solubility in a material.
4. Would you expect MgO and FeO, both of which have the sodium chloride structure, to display complete solid solubility in one another? Explain.
5. If you wanted to design a metallic alloy for prolonged use at high temperatures, you would base most of the strengthening on (solid solution strengthening / strain hardening / grain size strengthening).
6. _____ is the name for the heat treatment used specifically to reduce segregation that occurs during solidification of alloys.
7. You would most likely be successful in eliminating macrosegregation in an alloy ingot if you use (cold working / hot working / heat treatment).
8. We expect the higher melting point element in an alloy to be located (at the centre of a dendrite / in the interdendritic regions) in a casting.

An isomorphous phase diagram and a cooling curve are shown below.

1. Consider an alloy containing 60% Bi.

- a. the liquidus temperature is _____
- b. the solidus temperature is _____
- c. the freezing range is _____
- d. the composition of each phase in equilibrium at 600°C is:
- e. the amount of each phase in equilibrium at 600°C is:
- f. the composition of each phase in equilibrium at 500°C is:
- g. the amount of each phase in equilibrium at 500°C is:
- h. the composition of each phase in equilibrium at 300°C is:
- i. the amount of each phase in equilibrium at 300°C is:
- j. if the alloy cooled rapidly rather than under equilibrium conditions, the apparent solidus temperature would be (higher / lower) than given by the phase diagram.
- k. if the alloy cooled rapidly rather than under equilibrium conditions, the amount of solid present at 400°C would be (higher / lower) than that calculated in part f above.
- l. the alloy for which the cooling curve was obtained most likely contained about _____% Sb



1. Almost always, we would like the (precipitate / matrix) to be hard and the (precipitate / matrix) to be continuous.
2. (Stoichiometric / Nonstoichiometric) intermetallic compounds have a fixed composition.
3. Most of the important intermetallic compounds considered for aerospace applications have a (random / ordered) crystal structure, thus giving (large / small) Burgers vectors and difficult slip.
4. Parts produced from intermetallic compounds
 - a. tend to have good ductility and impact properties
 - b. can easily be manufactured by casting or forming techniques
 - c. typically will have good high temperature strength and creep resistance
5. To obtain the best dispersion strengthening in a metallic alloy, we should create a (large / small) total amount of interface between the matrix and the precipitate.
6. A miscibility gap is found when the _____ three phase reaction occurs.
7. Only the _____ and _____ three phase reactions tend to produce good dispersion strengthened alloys.
8. Although the _____ and _____ reactions are intended to produce a single phase alloy, the slow rate of transformation normally produces very high degrees of segregation.
9. Some type of heat treatment to control the _____ reaction will provide effective dispersion strengthening.
10. Good control of the _____ reaction during the solidification process may provide effective dispersion strengthening.
11. Complete solid solubility (is / is never) observed when an eutectic is present in a phase diagram.
12. Consider the silver-cadmium phase diagram:
 - a. Identify by name the three phase reactions at:

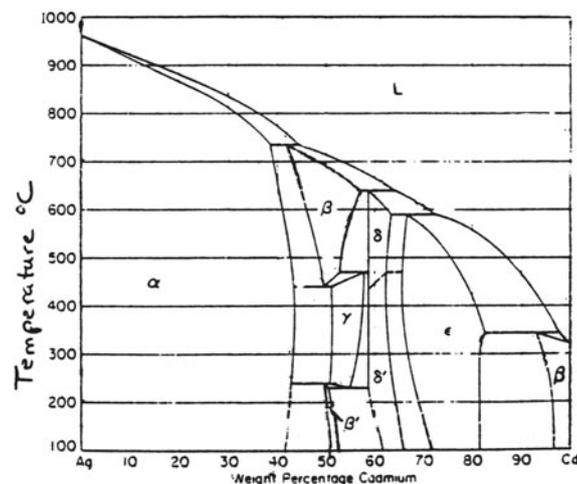
736°C _____	640°C _____
590°C _____	470°C _____
440°C _____	343°C _____
240°C _____	230°C _____

b. The β phase is a (stoichiometric intermetallic compound / nonstoichiometric intermetallic compound / solid solution).

c. The α phase is a (stoichiometric intermetallic compound / nonstoichiometric intermetallic compound / solid solution).

d. In an Ag-70%Cd alloy, determine the composition and amount of each phase present under equilibrium conditions at 300°C.

e. In an Ag-90%Cd alloy, determine the composition and amount of each phase present under equilibrium conditions at 400°C.



13. We can obtain dispersion strengthening in an alloy if we (exceed / do not exceed) the solubility limit of the alloying element in the "host" material.

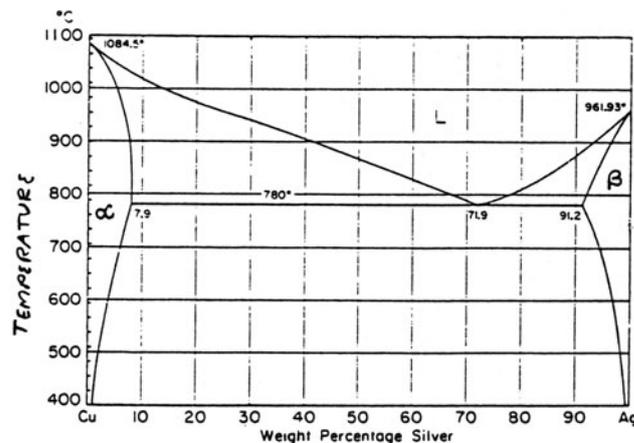
14. A microconstituent can be
- a. a mixture of two or more phases
 - b. one phase, having a distinct appearance, in a multiple-phase material
 - c. both a and b can describe a microconstituent

15. In an eutectic system, the (primary microconstituent / eutectic microconstituent) sometimes has the same appearance and characteristics as a metallic composite material.

16. The best eutectic microconstituent might be composed of
- a. two solid solutions based on FCC metals
 - b. a mixture of 75% solid solution based on an FCC metal and 25% intermetallic compound
 - c. a mixture of 25% solid solution based on an FCC metal and 75% intermetallic compound

17. We can improve the strength of an alloy in which the eutectic reaction occurs by
- (increasing / decreasing) the cooling rate during solidification
 - (increasing / decreasing) the local solidification time of the alloy
 - (increasing / decreasing) the interlamellar spacing in the eutectic microconstituent
 - (increasing / decreasing) the amount of the eutectic that forms in the alloy by adjusting the original alloy composition
 - (increasing / decreasing) the eutectic grain, or colony, size
 - (increasing / decreasing) the effectiveness of the inoculant
 - (increasing / decreasing) the size of any primary microconstituent
 - (increasing / decreasing) the rate at which diffusion of atoms occurs in the liquid at the solid-liquid interface
 - (increasing / decreasing) the amount of strontium or sodium added to an aluminium-silicon eutectic alloy
18. _____ of an Al-Si eutectic alloy means changing the silicon, or β phase, from a needle-shape to a rod-shape
19. When non-equilibrium freezing occurs in an eutectic alloy, the amount of eutectic microconstituent (increases / decreases).
20. The (eutectic / primary phase) is generally the continuous microconstituent in a hypo- or hypereutectic alloy.

Consider the Cu-Ag phase diagram below:



- A Cu-40%Ag alloy is (hypoeutectic / hypereutectic).
- The liquidus temperature for a Cu-40%Ag alloy is _____.
- Calculate the total amount of α at 400°C in a Cu-40%Ag alloy.
- Calculate the amount of the eutectic microconstituent in a Cu-40%Ag alloy.
- Calculate the amount of the primary β in a Cu-80%Ag alloy.
- The maximum solubility of Ag in copper is _____ %.

27. The composition of the α in a Cu-40%Ag alloy at 900°C is _____.
28. Calculate the amount of α in the eutectic microconstituent.
29. You would expect that the (α / β) phase is the continuous phase in the eutectic microconstituent.
30. The composition of the eutectic microconstituent is _____.
31. Calculate the amount of eutectic microconstituent in a Cu-15%Ag alloy.
32. A photograph of a Cu-80%Ag alloy is shown. The portion of the structure labeled "A" is (primary α / primary β / eutectic microconstituent). The white, dendritic-appearing portion must be (primary α / primary β / eutectic microconstituent).



1. Nucleation of a new phase in a solid usually occurs
 - a. homogeneously
 - b. heterogeneously at grain boundaries and other lattice defects
2. When precipitates form at grain boundaries, the alloy is brittle if
 - a. the precipitate is ductile and round
 - b. the precipitate is brittle and round
 - c. the precipitate is brittle and forms a grain boundary film
 - d. the precipitate is ductile and forms a grain boundary film
3. The age hardening treatment produces the highest strength in an age hardenable alloy when the precipitate particles are
 - a. coherent with the matrix
 - b. not coherent with the matrix
4. Age hardenable alloys are solution treated by holding them
 - a. just below the solidus temperature but above the eutectic
 - b. just below the liquidus temperature
 - c. just below the eutectic but above the solvus temperature
 - d. just below the solvus temperature
5. Quenching a solution treated alloy
 - a. retains the single phase produced by solution treating
 - b. produces a structure that is too strong and brittle
 - c. produces good dispersion strengthening in the alloy
6. We obtain the highest strength in an age hardenable alloy when we
 - a. age just below the solidus temperature
 - b. age just below the solvus temperature
 - c. age at the lowest possible temperature that permits precipitation
7. Age hardenable alloys are solution treated to
 - a. produce a single phase by dissolving precipitate particles
 - b. produce a soft material that can be cold worked easily
 - c. strengthen the alloy by reducing the grain size
8. We normally expect to obtain the highest strength when we
 - a. artificially age
 - b. naturally age
9. When an age hardened alloy overages, the strength decreases because
 - a. precipitate particles redissolve and disappear
 - b. precipitate particles lose coherency with the matrix
 - c. the alloy recrystallises

10. The only one of the following requirements that an alloy need NOT meet to be age hardenable is:
- the solvus line must show decreasing solubility with decreasing temperature
 - the precipitate must be soft and ductile, while the matrix is hard and brittle
 - the matrix and precipitate must be coherent
 - the alloy must be capable of being quenched to retain a single phase after solution treatment
11. Rapid solidification processing may increase the strength attained in a particular alloy system by
- avoiding the softening effect of solution treatment
 - reducing the ageing time
 - increasing the total volume of precipitate that can be produced during ageing
12. Widmanstatten precipitates in an alloy structure
- generally give the alloy unusually good ductility
 - generally make it more difficult for a crack to nucleate
 - generally make it more difficult for a crack to grow
 - always produce alloys with very poor fracture toughness
13. Increasing the ageing temperature
- increases the time required to reach the maximum strength
 - decreases the maximum strength that can be obtained
14. Suppose an Al-4%Cu alloy that has been age hardened to the maximum strength is put into service at 540°C. (see Figure 11.5)
- the strength will decrease because of overageing
 - the strength will decrease because the precipitates will dissolve (i.e. solution treating will begin again)
 - the strength will decrease because the alloy will begin to melt
 - the strength will increase
15. We would prefer that the precipitate that forms during ageing is
- a solid solution based on a face centred cubic metal
 - an intermetallic compound
16. Aluminium age hardenable alloys are usually quenched in hot water instead of cold water to
- assure the formation of round rather than angular precipitates
 - assure that grain boundary precipitation will occur
 - minimise residual stresses associated with quenching

1. The solubility of carbon is higher in (ferrite / austenite / δ -ferrite).
2. The FCC crystal structure is found in (ferrite / austenite / cementite).
3. The composition of pearlite is _____%carbon.
4. The primary microconstituent in a hypereutectoid steel is (ferrite / cementite / austenite / pearlite).
5. Pearlite is a mixture of two phases -- _____ and _____.
6. Only (pearlite / spheroidite / bainite / martensite / tempered martensite) is NOT a microconstituent containing ferrite and cementite.
7. Because of hot rolling and/or heat treatment, the (eutectoid / peritectic) reaction in steels is NOT usually an important reaction.
8. Calculate the amount of pearlite that forms in a 0.6%C steel.
9. The continuous microconstituent in a 0.3%C steel is (ferrite / cementite / pearlite).
10. Increasing the amount of cementite in a steel (increases / decreases) the strength of the steel, at least up to 0.77%C.
11. (Martensite / Pearlite / Bainite) forms as a result of a diffusionless, athermal reaction that depends only on temperature.
12. The first step in most heat treatments of a steel is called (quenching / austenitising / austempering / spheroidising).
13. Increasing the cooling rate of the steel from austenite (increases / decreases) the spacing of the lamellae in pearlite.
14. Increasing the isothermal transformation temperature of a steel (increases / decreases) the spacing of the lamellae in pearlite.
15. We obtain (higher / lower) strength in a steel for a finer pearlite.
16. If we quench a 0.8%C steel from 850°C to 650°C and hold for 10 hours before cooling to room temperature, the heat treatment is called (austempering / annealing / normalising / isothermal annealing) AND the expected microstructure is (pearlite / bainite / tempered martensite).
17. If we quench a 1077 steel from 850°C to 450°C and hold for 10 hours before cooling to room temperature, the heat treatment is called (austempering / annealing / quench and tempering / isothermal annealing) AND the expected microstructure is (pearlite / bainite / tempered martensite).
18. The hardness of martensite depends primarily on the (amount of carbon in the steel / the amount of alloying elements added to the steel).

19. Cementite particles are the most round in (pearlite / bainite / tempered martensite).
20. Increasing the tempering temperature from 300°C to 600°C (increases / decreases) the strength of tempered martensite.
21. The carbon content of martensite is (greater than / less than / equal to) the carbon content of the austenite from which it is produced.
22. Furnace cooling a 0.5%C steel from above the A_3 temperature to produce a coarse pearlite and ferrite structure is called (normalising / spheroidising / annealing / austempering).
23. Air cooling a 1030 steel from above the A_3 temperature to produce a fine pearlite and ferrite structure is called (normalising / austempering / annealing / spheroidising).
24. Holding a steel at 700°C for long times to produce round cementite in a ferrite matrix is called (normalising / spheroidising / annealing / austempering).
25. Bainite is produced as a result of (annealing / normalising / austempering / spheroidising / quench and tempering).
26. Increasing the amount of alloying elements such as nickel in a steel
- (increases / decreases) the time required for isothermal annealing or austempering
 - requires (faster / slower) cooling rates to produce a completely martensitic structure
 - (increases / decreases) the rate at which martensite will temper when heated to 500°C
 - (encourages / discourages) the formation of martensite during quenching
 - permits a steel to be used at (higher / lower) service temperatures without losing its strength
 - permits you to use an (oil / water) quench to obtain martensite
27. Quenching in (water / oil) is less likely to cause quench cracks in a steel.
28. _____ is a process by which carbon is diffused into the surface of a low carbon steel to produce an excellent combination of strength, fatigue resistance, toughness, and wear resistance.
29. (Induction hardening / Carburising) produces the deeper case of hardened steel but also gives the lower overall toughness and hardness.
30. Stainless steels must include at least 12% _____.
31. When large amounts of nickel are added to a steel, we find that (ferrite / austenite) is stable at room temperature.

1. Characteristics of aluminium and its alloys include:
 - a. (good / poor) resistance to fatigue, creep, and wear
 - b. (good / poor) resistance to failure by oxidation
 - c. (good / poor) resistance to corrosion in most environments
 - d. (good / poor) specific strength and specific strength compared to plain carbon steels
 - e. a density of about _____ Mg.m^{-3} compared to about 7.8 Mg.m^{-3} for steels
 - f. a modulus of elasticity of _____ GN.m^{-2} compared to 205 GN.m^{-2} for steels
 - g. a maximum strength of about (345 / 550 / 825) MN.m^{-2}
 - h. a melting temperature of about (1500°C / 650°C / 200°C)
 - i. the crystal structure is (BCC / HCP / FCC)

2. Aluminium and its alloys can be strengthened by all of the following but (check two of the following)
 - a. grain size strengthening
 - b. strain hardening
 - c. martensitic transformations
 - d. eutectoid reactions
 - e. age hardening
 - f. eutectic reactions
 - g. solid solution strengthening

3. You would like to order an Al-Cu aluminium alloy that has been heated to about 550°C , quenched in hot water, and reheated to 180°C . You might ask your friendly metal supplier for
 - a. a 3004-H18 alloy
 - b. a 7075-T4 alloy
 - c. a 1100-O alloy
 - d. a 2024-T6 alloy

4. A 356-T6 alloy contains about 7%Si and 0.3%Mg. This means that
 - a. the metal is a wrought alloy that has been cold worked
 - b. the metal is a wrought alloy that has been age hardened
 - c. the metal is a casting alloy that has been cold worked
 - d. the metal is a casting alloy that has been age hardened

5. We would expect to have better corrosion resistance for
 - a. 1100 alloy
 - b. 5160 alloy
 - c. 2024 alloy
 - d. 7075 alloy

6. Aluminium-lithium alloys typically
 - a. have a (higher / lower) density than conventional aluminium alloys
 - b. have a (higher / lower) modulus than conventional aluminium alloys
 - c. have a (higher / lower) specific strength and specific modulus than conventional aluminium alloys
 - d. (can / cannot) be strengthened by age hardening
 - e. are (more expensive / less expensive) than conventional aluminium alloys
 - f. are (easier / more difficult) to produced, process, and fabricate than conventional alloys

7. The maximum service temperature for aluminium alloys is approximately (150°C / 300°C / 450°C).

8. A common skin material for wings is composed of a thick sheet of 2024-T6 alloy sandwiched between two thin layers of 1100-O alloy. The name for this material is _____.

9. Some typical characteristics of magnesium and its alloys are

- a. the density is about _____ Mg.m^{-3}
- b. maximum strengths are on the order of (140 / 345 / 690) MN.m^{-2}
- c. the crystal structure is (BCC / HCP / FCC)
- d. the modulus of elasticity is about _____ GN.m^{-2}
- e. the resistance to creep, fatigue, and wear is (good / poor)
- f. the resistance to oxidation is generally rather (good / poor)
- g. the melting temperature is (much lower / much higher / about the same) as that for aluminium and its alloys
- h. the specific strength and specific modulus are (much lower / much higher / about the same) as that for aluminium and its alloys
- i. the maximum temperature at which magnesium can be used is (a little higher / a little lower) than that for aluminium

10. Wrought magnesium is a little harder to form than wrought aluminium alloys. Why?

11. More precautions are required when melting and pouring magnesium casting alloys than when processing aluminium casting alloys. Why?

12. We normally would prefer to use aluminium parts over magnesium parts for use in a satellite. Why?

13. Some of the characteristics of beryllium include

- a. having a (BCC / HCP / FCC) crystal structure
- b. having a melting temperature that is considerably (lower / higher) than for aluminium and magnesium alloys
- c. having a (higher / lower) maximum service temperature compared to aluminium and magnesium
- d. a density of about _____ Mg.m^{-3}
- e. a modulus of elasticity of about _____ GN.m^{-2}

14. Why are beryllium and its alloys more difficult to process and fabricate than aluminium and magnesium alloys?

15. Suppose you are designing a mechanism that includes a part that must move back and forth rapidly. For maximum response of the part, you would most likely select a (aluminium / magnesium / beryllium) alloy.
16. You would like to produce a frame for a complex mirror that will most accurately reflect and focus a laser beam, even when some small vibrations occur. You would most likely select a (aluminium / magnesium / beryllium) alloy to produce the framework that holds the mirror in place.
17. Some of the characteristics of copper and its alloys include:
- a density that is (higher / lower) than that of steels
 - a (BCC / HCP / FCC) crystal structure
 - generally (good / poor) resistance to corrosion
 - (better / worse) resistance to creep, fatigue and wear failures
18. We obtain the highest strength in copper alloys when we
- quench and temper an aluminium bronze
 - solid solution strengthen by zinc additions to produce brass
 - cold work commercially pure copper
 - age harden a beryllium copper alloy
19. The two metals that typically are used as the basis for superalloys are _____ and _____. Of these two, _____ often provides exceptional resistance to wear due to its high hardness.
20. The density of superalloys is (greater / smaller) than the density of steels.
21. Superalloys generally have a melting temperature that is (higher / lower) than that of steel.
22. In addition to good corrosion resistance, superalloys generally are capable of operating at high temperatures for long times. The three mechanisms by which a nickel-base superalloy obtains its high temperature strength and creep resistance are:
- -
 -
23. The coherent precipitates that form in nickel-base superalloys and provide the age hardening effect are _____.
24. The precipitates that form during ageing of nickel-base superalloys are (more / less) resistant to overageing than the precipitates that form in aluminium and magnesium alloys.
25. At room temperature, the specific strength of superalloys generally is (higher / lower) than that found in the strongest aluminium alloys.

26. Some of the characteristics of titanium and its alloys are
- At room temperature, pure titanium has a (BCC / HCP / FCC) crystal structure but above about 882°C, the crystal structure is (BCC / HCP / FCC).
 - The density of pure titanium is about _____ Mg.m⁻³
 - At room temperature, titanium alloys are considered to have (good / poor) oxidation and corrosion resistance.
 - at 1000°C, titanium alloys are considered to have (good / poor) oxidation and corrosion resistance
 - The protective TiO₂ film on titanium and its alloys breaks down at approximately _____°C
27. Among the alloys of titanium, the (alpha / beta / alpha-beta) alloys generally have good corrosion and oxidation resistance but relatively low strengths.
28. Very large amounts of solid solution alloying elements are normally required to produce a (alpha / beta / alpha-beta) titanium alloy.
29. The most commonly used aerospace titanium alloy, which also displays superplastic behaviour, is
- commercially pure titanium
 - Ti-5%Al-2.5%Sn
 - Ti-13%V-11%Cr-3%Al
 - Ti-6%Al-4%V
30. Generally, a larger amount of alloying element in an alpha-beta titanium alloy is required in order to produce (age hardening / the martensite reaction).
31. Titanium martensite is generally (weaker / stronger) than tempered titanium martensite.
32. During tempering of titanium martensite, the strength (increases / decreases), unlike the case for steel martensite.
33. In tempered titanium martensite, the microstructure is expected to be
- alpha precipitates in a beta matrix
 - beta precipitates in an alpha matrix
34. The formation of a Widmanstatten, or basket-weave, precipitate in titanium alloys tends to improve fracture toughness by
- making it more difficult for cracks to nucleate
 - providing a more tortuous path for crack growth
35. The specific strength of titanium alloys is generally (higher / lower) than that of aluminium and magnesium alloys.
36. The specific strength of refractory metals is generally (higher / lower) than that of aluminium and magnesium alloys.
37. Why aren't refractory metals, with their exceptionally high melting temperatures, used more often as a thermal protection system for aerospace vehicles that must re-enter earth's atmosphere?

1. The atoms in the crystal structures of ceramic materials generally are bonded by _____ or _____ bonding.
2. In some ceramic materials such as mica, clay, and talc, strong "sheets" of the crystal structure are bonded to one another by _____ bonds.
3. Because of the bonding in ceramic materials, you typically expect ceramics to have (compared to most metals):
 - a. (high / low) melting temperatures
 - b. a (high / low) modulus of elasticity
 - c. a (high / low) electrical conductivity
 - d. a (high / low) thermal conductivity
 - e. (ductile / brittle) behaviour
 - f. (good / poor) fracture toughness
4. Most ceramic crystal structures contain (one / several) atoms or ions per lattice point.
5. We expect (crystalline / glassy) ceramics to have better resistance to creep at elevated temperatures.
6. Suppose we remove 6 Al^{3+} ions from the Al_2O_3 crystal and replace them with Mg^{2+} ions. Will we create vacancies or interstitial point defects? How many will we create?
7. A metallurgist would expect that the point defects described in the preceding question would strengthen the material. Is this true? Explain.
8. Vitrification means that a (glassy / crystalline) phase has formed in a ceramic material.
9. _____ is the temperature at which localised movement of chains or groups of atoms or ions no longer occurs when a glass cools.
10. Increasing the cooling rate of a molten ceramic material (increases / decreases) the likelihood of producing a glass.
11. Modifying oxides in a glass generally
 - a. (increase / decrease) the melting temperature of glass
 - b. (increase / decrease) the formability or workability of the glass
 - c. (increase / decrease) the rate at which the material must be cooled in order to produce a glass.
12. In a ceramic, the tensile strength is generally (twice as great as the yield strength / 50% greater than the yield strength / equal to the yield strength).

13. In ceramics, we expect better success in improving the fracture toughness if we do something to
- prevent the formation of any potential cracks
 - make it more difficult for any potential cracks to propagate
14. A ceramic-ceramic composite such as SiC fibres in a SiC matrix has better toughness than a single phase ceramic. Describe two mechanisms that might be responsible for this improvement?
15. What is a cermet?
16. Tempered glass has better toughness than untempered glass because
- small crystallites precipitate and block dislocation movement
 - small crystallites precipitate and block potential cracks
 - compressive residual stresses are introduced at the surface
17. Briefly explain how "microcrazing" can improve the toughness of a ceramic. Would the introduction of fine porosity do the same thing?
18. Stabilised zirconia and transformation toughened zirconia both may have better toughness than pure ZrO₂ cooled to room temperature under equilibrium conditions. Explain how toughening is obtained in each type.
- stabilised zirconia:
- transformation toughened zirconia:
19. Crystalline SiO₂ is expected to have (better / poorer) resistance to thermal shock than fused SiO₂.
20. You would expect stabilised zirconia to have (better / poorer) resistance to thermal shock than pure zirconia.
21. Cracking due to thermal shock is more likely if
- the coefficient of thermal expansion (increases / decreases)
 - the thermal conductivity (increases / decreases)
 - the ceramic is (polymorphic / single phase at all temperatures)
22. In diffusion, a high activation energy means that diffusion will be slow. In ceramic glasses, a high activation energy for viscous flow means
- the viscosity will be (high / low)
 - the melting temperature of the glass will be (high / low)
 - the creep resistance of the glass will be (high / low)

23. During sintering of a ceramic body, the density may increase and porosity may decrease by what TWO physical phenomena?

- a.
- b.

24. When the amount of porosity is reduced, the thermal conductivity of the ceramic is expected to (increase / decrease). How does this relate to the thermal protection system in the space shuttle?

25. We would most likely produce the highest strength in a ceramic part by

- a. reaction bonding
- b. conventional pressing and sintering
- c. hot isostatic pressing

26. A glass ceramic is

- a. a glass tempered to produce excellent thermal shock resistance
- b. a glass in which a crystalline phase is precipitated
- c. a crystalline ceramic which is allowed to vitrify

27. Partially stabilised ZrO_2 has a tensile strength of about 450 MN.m^{-2} . A microcrack at the surface is 0.01 mm deep with a radius of 100 nm. Is this crack likely to propagate when an applied stress of 20 MN.m^{-2} is applied? Show by suitable calculations. (see Table 14.4)

28. Very pure ceramic parts generally have better high temperature properties than impure ceramic parts. Why?

29. We can, by using the appropriate raw materials and manufacturing processes, produce very strong ceramic materials. Why aren't ceramics therefore used more often in critical structural applications?

1. The density of polymers is (higher / lower) than that of metals.
2. The density of polymers is generally (higher / lower) than ceramics.
3. The tensile strength of polymers is (higher / lower) than that of metals and ceramics.
4. The modulus of elasticity of polymers is (higher / lower) than that of metals and ceramics.
5. The specific strength and specific modulus of polymers are generally (higher / lower) than that of most metals and ceramics.
6. The maximum temperature at which a typical polymer can be used is (higher / lower) than that for most metals and ceramics.
7. You would expect the electrical conductivity of a typical polymer to be closer to that for a typical (metal / ceramic).
8. We expect the largest amount of plastic deformation from a (thermoplastic polymer / thermosetting polymer / elastomer).
9. We expect the largest amount of elastic deformation from a (thermoplastic polymer / thermosetting polymer / elastomer).
10. When we increase the rate at which a load is applied to a thermoplastic polymer, we expect
 - a. the tensile strength to (increase / decrease)
 - b. the impact properties, or toughness, to (increase / decrease)
11. When a monomer contains an unsaturated bond between the carbon atoms, we expect to have polymerization occurring by the (addition / condensation) mechanism.
12. When a by-product is produced as a result of polymerization, we expect that the (addition / condensation) mechanism was involved.
13. At least some degree of cross-linking of polymer chains is required for all BUT (thermoplastic polymers / thermosetting polymers / elastomers).
14. When we increase the load on an elastomer, we expect the nonlinear modulus of elasticity to (increase / decrease).
15. Monomers with a functionality of (two / three) are required to produce a thermosetting polymer.
16. At least two different monomers is REQUIRED to produce a (thermoplastic polymer / thermosetting polymer / elastomer).
17. When we increase the amount of initiator, such as hydrogen peroxide, to ethylene, the length of the polyethylene chains is expected to (increase / decrease).

18. In order to shape a thermoplastic polymer, we should apply a stress at a rate that (is less than / exceeds) the rate at which chains can slide past one another in a viscous manner.
19. We expect the strength of a thermoplastic polymer to (increase / decrease) as the viscosity of the polymer increases.
20. To minimize problems with room temperature creep, we would want to select a thermoplastic polymer which has a glass transition temperature (above / below) room temperature.
21. We would expect a thermoplastic polymer to have a higher strength when it is in the (crystalline / rubbery / leathery) condition.
22. We would expect a thermoplastic polymer to have a higher modulus of elasticity when it is in the (glassy / rubbery / leathery) condition.
23. We would more likely be able to form a thermoplastic into a complicated shape at a high rate if the polymer is in the (glassy / crystalline / rubbery / leathery) condition.
24. At room temperature, polyethylene is in the (glassy / rubbery / leathery) condition.
25. Crystalline polymers are (100% crystalline / less than 100% crystalline).
26. Crystallization of a thermoplastic polymer is more likely when the liquid polymer is cooled at (1°C per minute / 100°C per minute).
27. Crystallization of a thermoplastic polymer is more likely when the degree of polymerization is (2000 / 10,000).
28. We would expect that (polyethylene / PMMA) will more likely crystallize during cooling.
29. Increasing the degree of polymerization (increases / decreases) the strength of a thermoplastic polymer.
30. Polyvinyl chloride and polystyrene are stronger than polyethylene because
- they always have a higher degree of polymerization
 - the bonds within the individual chains are much stronger
 - the bonds between the chains are stronger
31. Adding side groups to the thermoplastic polymers, as in producing polyvinyl chloride rather than polyethylene, is expected to (increase / decrease) the glass transition temperature AND (increase / decrease) the ability of the polymer to crystallize. Which one of these two effects will tend to cause higher strength in the polymer?
32. Introducing S, O, N, or benzene rings in the backbone of a thermoplastic polymer will generally (increase / decrease) the polymer strength.

33. Kevlar® is an example of a (thermoplastic elastomer / interpenetrating polymer network / liquid-crystalline polymer).
34. More extensive branching of a thermoplastic polymer will (increase / decrease) the density AND (increase / decrease) the polymer's strength.
35. An important example of an aromatic polyamide, or aramid, polymer is (polyimide / PEEK / Kevlar).
36. High density polyethylene is (more / less) extensively branched than low density polyethylene.
37. Crystallization occurs (more / less) easily when the polymer is highly branched.
38. Producing "copolymers" affects the properties of a thermoplastic in a manner similar to (solid solution strengthening / dispersion strengthening) of a metal.
39. Introducing small regions of an elastomer into a thermoplastic matrix, as in ABS, is "alloying" and in some respects is analogous to (solid solution strengthening / dispersion strengthening) in a metal.
40. Introducing sulphur cross-linking atoms into an elastomer is called _____.
41. Increasing the amount of sulphur added to an elastomer will (increase / decrease) the modulus of elasticity of the polymer AND will (increase / decrease) the total amount of elastic strain that can be obtained from the elastomer.
42. Elastomers (can / cannot) generally be reheated and subsequently be reformed easily into a different shape.
43. The "cross-linking" that occurs in producing a thermoplastic elastomer is due to (addition of sulfur atoms / strong van der Waal's type bonding at selected locations).
44. The BS thermoplastic elastomers represent an example of a (copolymer / polymer alloy).
45. Certain polymers may be capable of very large amounts of elastic deformation at low temperatures and very large amounts of plastic deformation at higher temperatures. These polymers are:
- | | |
|-----------------------------|---------------|
| a. thermoplastics | b. thermosets |
| c. thermoplastic elastomers | d. elastomers |
46. Ring scission of a C-O-C group helps permit polymerization in (polyesters / amines / epoxies / phenolics).
47. Polymers based on O - Si - O chains rather than C - C - C chains are the basis for
- | | |
|--------------|-----------------------------|
| a. urethanes | b. acrylics |
| c. silicones | d. thermoplastic elastomers |

1. Particulate composites tend to display (isotropic / anisotropic) behaviour.
2. The dispersion strengthened "composites" such as SAP perform better than conventional aluminium alloys
 - a. at high temperatures (above about 300 to 400°C)
 - b. at low temperatures (below about 300 to 400°C)
 - c. at all temperatures
3. The dispersed phase in (true particulate composites / dispersion strengthened "composites") is normally intended to provide strengthening by effectively blocking dislocation movement.
4. "Internal oxidation" is a technique used to produce (true particulate composites / dispersion strengthened "composites").
5. Cemented carbides, or cermets, are generally used
 - a. for their combination of high strength and corrosion resistance
 - b. for their combination of high hardness and electrical conductivity
 - c. for their combination of high hardness and toughness
6. The rule of mixtures does NOT work well in estimating
 - a. the modulus of elasticity of a composite
 - b. the electrical conductivity of a composite
 - c. the toughness of a composite
 - d. the density of a composite
7. An electrical contact material is produced by infiltrating liquid copper into a porous tungsten powder compact at 1093°F. The densities of copper and tungsten can be found in the appendix of the textbook. If the density of the final composite at room temperature is 15 Mg.m^{-3} , then
 - a. the volume fraction of copper in the composite = _____
 - b. the volume fraction of pores in the tungsten powder compact before copper infiltration = _____
 - c. the original density of the tungsten powder compact before copper infiltration = _____
8. Laminar composite materials display (isotropic / anisotropic) behaviour.
9. Roll bonding would be an effective manufacturing method for producing a laminar composite composed of two sheets of (ceramics / metals).
10. _____ is a clad laminar composite consisting of an age hardened aluminium alloy surrounded by commercially pure aluminium.

11. _____ is a clad laminar composite which, because of the different coefficients of thermal expansion of the materials, will alter its shape when heated.
12. _____ is a complex laminar composite composed of alternating layers of an aluminium alloy and a fibre-reinforced polymer.
13. We would obtain the very lowest density for
 a. an Arall-type laminar composite
 b. a titanium-clad stainless steel
 c. a "sandwich" of aluminium facing sheets and aluminium honeycomb
14. _____ and _____ are two manufacturing techniques by which metallic laminar composites might be produced in large sizes, such as 1.2 m x 2.4 m panels.
15. Many continuous-fibre reinforced composite structures could also be defined as (particulate / laminar) composites. T F
16. Tows are individual filaments of a metal, ceramic, or polymer. T F
17. A yarn is a bundle of thousands of filaments twisted together. T F
18. Rovings are bundles of thousands of nontwisted filaments. T F
19. Prepregs are tapes or fabrics of fibres impregnated with a polymer resin that are not completely polymerized. T F
20. Composites containing chopped fibres rather than continuous fibres are more likely to have isotropic behaviour. T F
21. Most high performance composites contain more than 90% fibres by volume. T F
22. Increasing the length of a fibre means that there is less of a chance that critically-sized flaws will be present on the fibre surface. T F
23. Fibres with a large aspect ratio typically give a higher strength composite. T F
24. Composites built up from tapes containing unidirectionally-aligned fibres typically perform well even when three-dimensional stresses are applied. T F
25. Delamination may occur when individual tape or fabric layers are poorly bonded. T F
26. Pull-out may occur when fibres are poorly bonded to the matrix. T F
27. Whiskers are nearly perfect single crystals having a relatively small aspect ratio. T F
28. Borsic fibres are really a composite fibre containing tungsten, boron, and silicon carbide. T F

29. We expect whiskers to have higher strengths than fibres. T F
30. Boron fibres are generally produced by a chemical vapor deposition technique. T F
31. PAN, or polyacrylonitrile, polymer fibres are a common precursor for carbon fibres. T F
32. When carbonising of a polymer fibre into a carbon fibre is done at higher temperatures, the strength increases but the modulus of elasticity decreases. T F
33. Carbon-carbon composites are rather unique in that they typically become stronger as the temperature increases. T F
34. Composite materials composed of continuous fibres are usually considered to be relatively brittle materials. T F
35. One of the functions of the matrix is to keep the fibres in alignment while transferring any applied load uniformly to the fibres. T F
36. Because the matrix, particularly in metal and polymer matrix composites, is more ductile than the fibres, any crack that forms in the fibres is arrested by the matrix and cannot propagate. T F
37. Polymer matrix composites are limited to lower temperature applications than are metal or ceramic matrix composites. T F
38. Kevlar fibres are often used to reinforce metal matrix composites. T F
39. Poor bonding between the fibres and the matrix makes it more difficult to transfer load to the fibres and causes poor strength in metal and polymer matrix composites. T F
40. Sizing of glass fibres means separating the fibres into groups according to their length and diameter. T F
41. Ceramic-Ceramic composites require particularly good bonding between the fibres and the matrix if good toughness is to be achieved. T F
42. Advanced composites containing Kevlar fibres have exceptionally good toughness. T F
43. In a hybrid composite, a fibre reinforces a matrix composed of more than one material. T F
44. One of the limitations of a metal matrix composite compared to a polymer matrix composite is that the metal matrix composites usually have a high density. T F

1. The number of charge carriers increases significantly when the temperature increases in
 - a. metals
 - b. semiconductors
 - c. ionic crystals
2. The mobility of charge carriers increases significantly when the temperature increases in
 - a. metals
 - b. semiconductors
 - c. ionic crystals
3. The mobility of charge carriers decreases significantly when the temperature increases in
 - a. metals
 - b. semiconductors
 - c. ionic crystals
4. The mobility of charge carriers decreases significantly when lattice imperfections are introduced into
 - a. metals
 - b. semiconductors
 - c. insulators
 - d. ionic crystals
5. The number of charge carriers increases significantly when appropriate lattice imperfections are introduced into
 - a. metals
 - b. semiconductors
 - c. ionic crystals
6. In electrical conduction, which of the following is analogous to the concentration gradient in diffusion?
 - a. the flux J
 - b. the energy gap E_g
 - c. the conductivity σ
 - d. the electric field ξ
7. The electrical conductivity of metals is on the order of
 - a. $10^5 \Omega^{-1} \text{ m}^{-1}$
 - b. $10^{-1} \Omega^{-1} \text{ m}^{-1}$
 - c. $10^{-15} \Omega^{-1} \text{ m}^{-1}$
8. The maximum number of charge carriers in a metal
 - a. is equal to the number of valence electrons
 - b. is equal to the total number of electrons
 - c. is significantly higher when the temperature increases
 - d. changes significantly when lattice imperfections are introduced
9. The mean free path for electrons in a metal is smallest when the metal is strengthened by
 - a. cold working
 - b. solid solution strengthening
 - c. age hardening
 - d. dispersion strengthening
 - e. grain size strengthening
10. Electrical conductivity in a metal alloy is expected to be higher if the metal is strengthened by a
 - a. coherent precipitate
 - b. noncoherent precipitate
11. We would expect a higher electrical conductivity in a metal that is
 - a. strain hardened
 - b. is given a recovery heat treatment
12. We expect to have a p-type semiconductor if
 - a. we dope silicon with phosphorous atoms
 - b. we dope germanium with indium atoms
 - c. we dope germanium with arsenic atoms
 - d. we dope GaAs with antimony atoms

13. We would expect to have a broader donor exhaustion plateau in a semiconductor based on
 a. silicon b. germanium c. GaP
14. Acceptor saturation provides an almost constant electrical conductivity over a range of temperatures in
 a. n-type semiconductors b. p-type semiconductors
15. When pressure is applied to a semiconductor, the energy gap
 a. increases b. decreases c. is unaffected
16. A current will flow when a p-n diode is connected to a power supply such that
 a. a recombination zone forms b. a depletion zone forms
17. In producing efficient transistors, it is particularly important to assure that the
 a. emitter is as thin as possible
 b. base is as thin as possible
 c. collector is as thin as possible
18. The dipoles in dielectric materials realign in an alternating electric field at the highest frequencies if the dipoles are
 a. electronic b. ionic c. molecular
19. The movement of the Ti ion in BaTiO₃ is an example of
 a. electronic polarization b. ionic polarization
 c. molecular polarization
20. Above the Curie temperature, BaTiO₃ loses much of its ability to be polarized because
 a. it melts above the Curie temperature
 b. it transforms from a tetragonal to a cubic crystal structure
 c. it becomes an electrical conductor rather than an insulator
21. The amount of charge Q that a capacitor can store
 a. increases as the dielectric constant (increases / decreases)
 b. increases as the surface area of the conducting plates (increases / decreases)
 c. increases as the distance between the conducting plates (increases / decreases)
 d. is higher when the conducting plates are (in a vacuum / separated by a dielectric material)
 e. increases when the dielectric strength (increases /decreases)
 f. is larger when the dielectric is (polyethylene / nylon)
22. The maximum electric field that can be imposed between two conductor plates without a current passing through the dielectric material is called the _____.
23. We would expect (thermoplastic / thermosetting) polymers to have a higher dielectric constant.

24. We would expect polyethylene to have the highest dielectric constant and highest degree of polarization in the (glassy / crystalline / leathery) condition.
25. We would expect that (crystalline / amorphous) polymers could serve as dielectrics at the higher frequencies.
26. You would expect that the least important property of a dielectric material to be used as an electrical insulator would be
- electrical conductivity
 - dielectric strength
 - dielectric constant
27. (Electrostriction / Piezoelectricity / Ferroelectricity) refers to the ability of a dielectric material to change dimensions when in an electric field or to create a voltage when a dimensional change is externally imposed by a stress.
28. Piezoelectricity can only occur when the material undergoes (electronic / ionic / molecular) polarization.
29. _____ refers to the polarization that occurs when all of the dipoles in the dielectric are aligned; _____ refers to the polarization that remains when an electric field is removed from a dielectric; _____ refers to the electric field required to eliminate polarization in a dielectric.

1. Relative permeability in a magnetic material is roughly equivalent to the (dielectric constant / dielectric strength / hysteresis loop) in a dielectric material.
2. Dipoles in magnetic materials such as iron, nickel, and cobalt are caused by
 - a. unfilled 3d energy levels
 - b. groups of atoms having a nonsymmetrical shape
 - c. movement of electrons or ions when a magnetic field is applied
3. _____ is a type of magnetic behaviour in which magnetic dipoles become aligned only when a very strong magnetic field is imposed. When the field is removed, magnetisation is completely lost.
4. _____ is a type of magnetic behaviour in which the dipoles associated with some ions reinforce the field, while dipoles associated with other ions oppose the field.
5. _____ refers to the type of magnetic behaviour displayed by Mn, MnO, and Cr.
6. The maximum BH product of a magnetic material refers to the _____ of the magnet.
7. Generally we expect the maximum permeability of a magnetic material to be much (greater / smaller) than the permittivity of a dielectric material.
8. The relative permeability of a magnetic material
 - a. is a constant
 - b. depends on the strength of the applied magnetic field
9. Magnetic materials used for electrical applications are said to be (soft / hard).
10. When dipoles in a magnetic material cannot be realigned quickly enough in an alternating field, heating occurs due to (dipole friction / eddy current losses).
11. It is more important to select a magnetic material that has a high electrical resistivity when the material is a (hard / soft) magnet.
12. We can get more useful work from an "electrical" magnet when the (saturation magnetisation / remanence / coercive field / power) is large.
13. An "electrical" magnet should possess all of the following characteristics except:
 - a. high relative permeability
 - b. large coercive field
 - c. small hysteresis loop
 - d. small remanence
14. In (hard / soft) magnets, it is essential that the magnetic domains be able to quickly realign when the field changes directions.

15. Fe-3% Si alloys are reasonable good "electrical" magnetic materials provided that they are processed in such a manner that the ($\langle 100 \rangle$ / $\langle 110 \rangle$ / $\langle 111 \rangle$) directions can be aligned with the magnetic field.
16. A material containing many lattice imperfections, such as dislocations or point defects, is expected to have a (larger / smaller) remanence.
17. Heat treatment of a group of metallic alloys called _____ allows the formation of tiny domain-sized precipitates that produce a high power.
18. Careful powder metallurgy processing of rare earth metal compounds, such as _____, allow each powder particle to serve as a domain and produce a high power.
19. A laminar composite of silicon iron and insulating sheets produce a soft magnetic material that
- has an unusually high power
 - has an unusually high remanence
 - has a high electrical resistivity
20. Magnetic powders embedded in an amorphous polymer matrix may be good soft magnets for each of the following reasons except:
- the domains can easily realign in an alternating field
 - the electrical resistivity is high
 - the coercive field is high
21. Magnetic ceramic materials are typically (ferromagnetic / ferrimagnetic).
22. In magnetite, or Fe_3O_4 ,
- the magnetic moments of all Fe^{3+} ions oppose the magnetic moments of all Fe^{2+} ions
 - the magnetic moments of Fe^{3+} ions oppose one another
 - the magnetic moments of both types of ions reinforce one another

1. The _____ is the wavelength of the lowest energy photon that is produced by a given stimulus.
2. A K_α photon is an example of the (continuous / characteristic) spectrum.
3. The visible spectrum contains photons having wavelengths of about _____ to _____ nm.
4. A (K_α / K_β / L_α) X-ray is produced if an electron drops from the $2s^22p^6$ energy level to the $1s^2$ energy level.
5. The wavelength of the photons emitted from GaAs is expected to be _____ nm. This (would / would not) be in the visible light spectrum.
6. _____ is a special form of luminescence in which photons are emitted only when the material is continually stimulated; however if photons continue to be emitted over some period of time after the stimulus is removed, _____ is said to have occurred.
7. (Phosphorescence / Fluorescence) is more likely to occur in extrinsic rather than intrinsic semiconductor compounds.
8. _____ are p-n junction devices that produce photons when a voltage is applied, while _____ are similar devices but produce a voltage when they are excited by photons.
9. As the energy of X-rays reaching a material increases, the linear absorption coefficient of the material typically (increases / decreases).
10. At X-ray energies that may excite an electron into a higher energy level, the linear absorption coefficient of the material will change dramatically; the wavelength corresponding to this energy is called the _____.
11. Fibre optic glass fibres may lose less of a beam of photons when the outer surface of the fibre has a (higher / lower) index of refraction than the centre of the fibre.
12. You would use a metal filter made of _____ in order to isolate the K_α peak of copper, producing an X-ray beam for diffraction experiments.
13. Transmission of a beam of photons may occur if their wavelength is (greater / less) than that corresponding to the energy gap of a semiconductor.
14. When YAG, or yttrium aluminium garnet, is doped with (As / Sb Nb), a laser beam having a wavelength determined by electronic reactions in the 4f energy level is produced.

1. Chemical corrosion of a polymer is more likely to occur if the chemical structure of the monomers is (similar to / different from) the chemical structure of the organic liquid in contact with the polymer.
2. Chemical corrosion (can / cannot) destroy ceramic components.
3. Chemical corrosion of a metal typically requires that the metal be (soluble / insoluble) in the attacking medium.
4. Electrochemical corrosion is a particular problem in (metals / ceramics / polymers).
5. Electrochemical corrosion occurs at the (anodic / cathodic) regions of a material.
6. The emf and galvanic series tell us
 - a. what metal will corrode
 - b. how fast corrosion will occur
7. Dissimilar metal corrosion is an example of a (concentration / composition / stress) cell.
8. Intergranular corrosion of a stainless steel is an example of a (concentration / composition / stress) cell.
9. Oxygen starvation is an example of a (concentration / composition / stress) cell.
10. Crevice corrosion is an example of a (concentration / composition / stress) cell.
11. Corrosion of a pure metal at grain boundaries is a result of a (concentration / composition / stress) cell.
12. "Pitting" corrosion is an example of a (concentration / composition / stress) cell.
13. Any by-product of electrochemical corrosion is expected to be found at the (anode / cathode).
14. The material that is higher in the emf or galvanic series is expected to (corrode / be protected).
15. The material exposed to highest stresses is expected to (corrode / be protected).
16. The material exposed to an electrolyte containing the highest oxygen concentration is expected to (corrode / be protected).

17. In a "sensitized" austenitic stainless steel, the anode is
- the high chromium austenite
 - the low chromium austenite
 - chromium carbide
18. Material such as Cu-Ni alloys may have a highly segregated structure following solidification. Corrosion may then occur as a consequence of a (concentration / composition / stress) cell. We can minimize this type of corrosion by doing a _____ heat treatment.
19. Materials that have experienced different levels of cold working may corrode as a consequence of a (concentration / composition / stress) cell. We can minimize the likelihood of corrosion without decreasing the mechanical properties if we do a _____ heat treatment.
20. (Tin-plated / Zinc-plated) steel is protected from corrosion due to the sacrificial anode effect.
21. "Terne" steel is made by coating a low carbon steel with lead. The lead will provide the same kind of protection that (tin / zinc) does.
22. When dissimilar metals are joined, we expect slower corrosion rates if the (anode / cathode) surface area is particularly large.
23. Inhibitors are introduced to the (metal / electrolyte) to reduce the rate of corrosion.
24. In providing cathodic protection using an impressed voltage, a corrosion engineer suggest using alternating current. Is this a good idea?
25. Producing a thick oxide layer on aluminium is an example of (anodic / cathodic) protection; this process in aluminium is usually called _____.
26. When the oxide that forms from the metal has a higher density than the metal,
- the metal will be protected from subsequent oxidation and corrosion
 - the oxide coating will be porous and oxidation will continue
 - the oxide coating will flake off and oxidation will continue
27. We expect that the rate of oxidation will
- increase with increasing temperature
 - decrease with increasing temperature
 - be unaffected by temperature

Failure - Origin, Detection and Prevention

1. A cup and cone fracture indicates brittle failure. T F
2. A dimpled fracture surface normally suggests ductile fracture. T F
3. A fracture surface with flat cleavage surfaces may indicate failure due to impact. T F
4. A cup and cone fracture appearance suggests failure due to simple tensile overload. T F
5. The presence of a chevron pattern indicates failure due to fatigue. T F
6. Beach marks on a fracture surface suggest that failure is due to fatigue. T F
7. Presence of voids in the structure near a fracture surface suggests creep failure rather than fatigue failure. T F
8. Presence of a fine network of numerous secondary cracks near a fracture surface supports failure by simple tensile overload rather than by stress-corrosion cracking. T F
9. We expect a ductile overload or brittle impact failure to occur in a very short time frame, while stress corrosion and fatigue failures occur over a longer time frame. T F
10. A proof test is an example of a destructive test. T F
11. A hardness test is often used to nondestructively determine whether a material has been properly heat treated. T F
12. Magnetic particle testing requires that a crack or other discontinuity actually penetrate to the surface of the part. T F
13. Dye penetrant inspection requires that a crack or other discontinuity actually penetrate to the surface of the part. T F
14. X-ray radiography easily detects cracks that are perpendicular to the X-ray beam. T F
15. Ultrasonic inspection easily detects cracks that are perpendicular to the X-ray beam. T F
16. Suppose there is a flat inclusion in a steel and that the inclusion has the same linear absorption coefficient as the steel. We would be more likely to detect this inclusion by ultrasonic inspection than by X-ray radiography. T F
17. If we measure the density and the velocity of sound in a material, we can calculate the modulus of elasticity. T F

18. We are more likely to accurately measure the thickness of aluminium foil if we use the through-transmission ultrasonic process rather than the reflection or resonance methods. T F

19. To detect a discontinuity in a steel part using magnetic particle inspection, the test setup must assure that the lines of magnetic flux be perpendicular to the discontinuity. T F

20. Magnetic particle inspection is particularly well suited for detecting cracks in aluminium and copper. T F

- _____ 11. Atoms move past one another, causing deformation, most easily when a force is applied to the material when the material possesses the
- a. metallic bond b. ionic bond c. covalent bond
- _____ 12. Bonding within the chains of a polymer is due primarily to
- a. metallic bonds b. ionic bonds
c. covalent bonds d. van der Waals bonds
- _____ 13. Bonding between the chains of a thermoplastic polymer is due primarily to
- a. metallic bonds b. ionic bonds
c. covalent bonds d. van der Waals bonds
- _____ 14. A very high bonding energy between the atoms is expected to
- a. provide a very stiff (high modulus of elasticity) material
b. provide a low stiffness (low modulus of elasticity) material
c. have no effect on the stiffness of a material
- _____ 15. A very high bonding energy between the atoms is expected to
- a. provide a material with a high coefficient of thermal expansion (that is, a large change in dimensions when the temperature changes)
b. provide a material with a low coefficient of thermal expansion (that is, a small change in dimensions when the temperature changes)
c. have no effect on the coefficient of thermal expansion
- _____ 16. Based on atomic bonding, we expect the electrical conductivity of a material to decrease when the temperature increases if the material possesses
- a. metallic bonds b. ionic bonds c. covalent bonds
- _____ 17. We would expect the typical metal to
- a. have good electrical and thermal conductivity but poor ductility
b. have good electrical and thermal conductivity and good ductility
c. have poor electrical and thermal conductivity and poor ductility
d. have poor electrical and thermal conductivity but good ductility
- _____ 18. We would expect the typical ceramic to
- a. have good electrical and thermal conductivity but poor ductility
b. have good electrical and thermal conductivity and good ductility
c. have poor electrical and thermal conductivity and poor ductility
d. have poor electrical and thermal conductivity but good ductility
- _____ 19. Most airplanes contain large amounts of aluminium and small amounts of iron and steel; the primary reason for this is:
- a. aluminium is stronger than steel
b. aluminium has a higher density than steel
c. aluminium has a higher strength-to-weight ratio than steel
- _____ 20. Ceramic materials such as SiO₂ typically contain anions and cations. We would generally expect that
- a. the anions have a larger radius than the cations
b. the cations have a larger radius than the anions

- _____ 1. The number of atoms or lattice points in a face centered cubic unit cell is: a. 1 b. 2 c. 3 d. 4 e. 14
- _____ 2. The atomic radius of an atom that is present in a BCC unit cell having a lattice parameter of 4.5 Å is:
a. 2.25 Å b. 1.95 Å c. 1.59 Å d. 1.12 Å
- _____ 3. In FCC unit cells, the close packed direction is:
a. along the edge of the unit cell
b. along the body diagonal of the unit cell
c. along the face diagonal of the unit cell
- _____ 4. The coordination number of the atoms in the BCC unit cell is:
a. 4 b. 6 c. 8 d. 12
- _____ 5. The packing factor of a FCC unit cell is about:
a. 0.34 b. 0.52 c. 0.68 d. 0.73
- _____ 6. The ability of a material to exist in more than one crystal structure, depending on temperature and pressure, is called:
a. anisotropy b. allotropy c. anthropology
- _____ 7. The fact that a material has different properties in different crystallographic directions means that the material is:
a. anisotropic b. allotropic c. antisocial
- _____ 8. The crystal structure of soft, ductile metals such as aluminium, copper, and gold is:
a. SC b. DC c. FCC d. HCP e. BCC
- _____ 9. The coordination number and packing factor of metals with the HCP crystal structure are the same as that of metals whose structure is the:
a. SC b. DC c. FCC d. BCC
- _____ 10. The stacking sequence of close packed planes is ABCABCABC in:
a. SC b. DC c. FCC d. HCP e. BCC
- _____ 11. The packing factor in diamond cubic silicon is low because:
a. silicon atoms come in different sizes
b. the directionality of the covalent bonds in silicon prevent close packing and high coordination numbers
c. the strong ionic bonding in silicon makes preserving electrical neutrality more important than high packing factors
- _____ 12. The repeat distance in the [111] direction in a BCC unit cell having a lattice parameter of 3.5×10^{-8} cm is:
a. 6.06×10^{-8} cm b. 4.95×10^{-8} cm
c. 3.03×10^{-8} cm d. 1.75×10^{-8} cm
- _____ 13. An atom located in an octahedral site has a coordination number of
a. 3 b. 4 c. 6 d. 8 e. 12
- _____ 14. Close packed directions in BCC unit cells have the form:
a. $\langle 100 \rangle$ b. $\langle 110 \rangle$ c. $\langle 111 \rangle$ d. $\langle 112 \rangle$
- _____ 15. Close packed planes in FCC unit cells have the form:
a. $\{100\}$ b. $\{110\}$ c. $\{111\}$ d. $\{123\}$

16. The interplanar spacing between two (114) planes in a BCC unit cell having a lattice parameter of 2.95 \AA is:
 a. 0.35 \AA b. 0.70 \AA c. 0.95 \AA d. 5.90 \AA

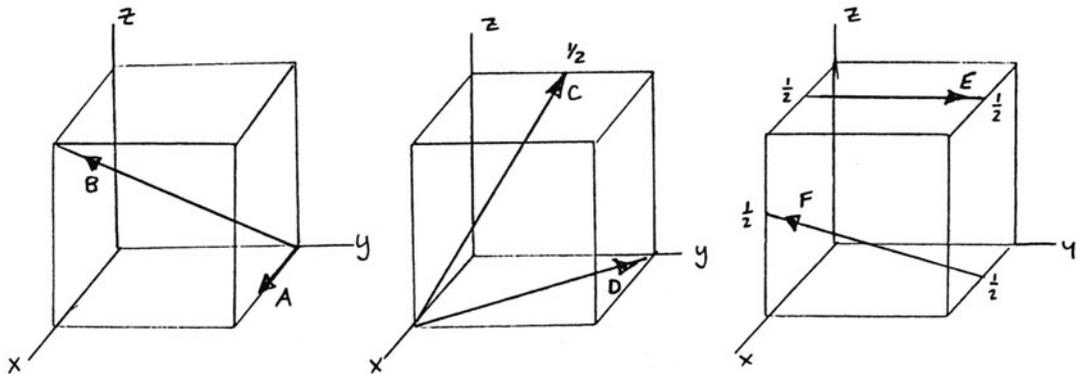
17. A material has a cubic-type of unit cell with one atom per lattice point. The lattice parameter is 0.40786 nm and the atomic radius is 0.1442 nm . What is the crystal structure?
 a. SC b. DC c. FCC d. BCC

18. We find that a metal having the FCC crystal structure has a lattice parameter of 3.84 \AA and a density of 22.65 g/cm^3 . Calculate the atomic weight of the metal.
 a. 21 g/mol b. 55.4 g/mol c. 193 g/mol d. $2 \times 10^{23} \text{ g/mol}$

19. TiAl is an important intermetallic compound that can be used at high temperatures. Ti atoms are located at the corners of a tetragonal unit cell and at two faces of the unit cell; Al atoms are located at four of the faces of the unit cell. The lattice parameters are $a_0 = 3.99 \text{ \AA}$ and $c_0 = 4.07 \text{ \AA}$. Calculate the density of TiAl.
 a. 1.65 g/cm^3 b. 2.65 g/cm^3 c. 3.84 g/cm^3 d. 4.5 g/cm^3

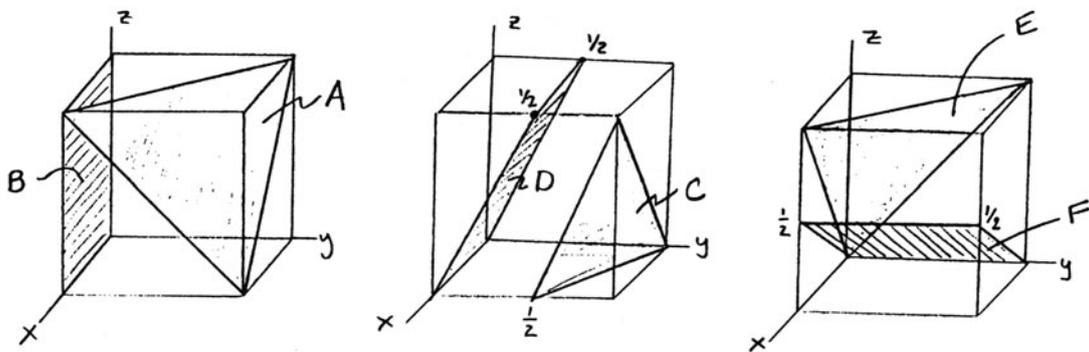
20. The following sketch shows six directions in unit cells. Determine the Miller indices for each of the directions:

A: _____ B: _____ C: _____
 D: _____ E: _____ F: _____



21. The following sketch shows six planes in unit cells. Determine the Miller indices for each of the planes:

A: _____ B: _____ C: _____
 D: _____ E: _____ F: _____



- _____ 1. The Burgers vector in FCC metals is a close packed direction of the form:
a. $\langle 100 \rangle$ b. $\langle 110 \rangle$ c. $\langle 111 \rangle$ d. $\langle 0001 \rangle$
- _____ 2. The Burgers vector in BCC metals is a close packed direction of the form:
a. $\langle 100 \rangle$ b. $\langle 110 \rangle$ c. $\langle 111 \rangle$ d. $\langle 0001 \rangle$
- _____ 3. The slip planes in FCC metals are close packed planes of the form:
a. $\{100\}$ b. $\{110\}$ c. $\{111\}$ d. $\{0001\}$
- _____ 4. The total number of slip systems in FCC metals is
a. 3 b. 12 c. 48 d. infinity
- _____ 5. The total length of dislocations, or dislocation density, in a metal is generally about
a. 10^5 to 10^7 m.m^{-3} b. 10^{10} to 10^{16} m.m^{-3}
c. 10 miles d. 100 kilometers
- _____ 6. When the angle between the slip direction and the direction of the applied stress is 90° , a dislocation
a. moves with very low applied stresses
b. requires applied stresses of about 0.7 MN.m^{-2} to move
c. requires applied stresses of about 70 MN.m^{-2} to move
d. will not move even if the applied stress is enormous
- _____ 7. In a nearly perfect crystal, the shear stress required to move a dislocation is
a. the tensile strength of the material
b. the critical resolved shear stress for the material
c. higher for FCC metals than for BCC metals
- _____ 8. Cross-slip of dislocations is NOT expected for
a. BCC metals b. FCC metals c. HCP metals
- _____ 9. The lattice imperfection produced when an extra half-plane of atoms is introduced into the lattice is called a
a. vacancy b. interstitial
c. edge dislocation d. screw dislocation
- _____ 10. Metals generally deform, particularly at room temperature, by
a. movement of vacancies
b. movement of interstitial atoms
c. movement of dislocations
d. movement of grains
- _____ 11. Ceramic materials are generally more brittle than metals because
a. they have higher strengths due to their ionic and covalent bonding
b. their crystal structures have a short Burgers vector and closely spaced close packed planes, making it difficult for dislocations to move
c. ionic bonding and the directionality of covalent bonds makes it difficult for atoms and ions to move relative to one another

_____ 12. Which of the following lattice imperfections will increase in number when we increase the temperature:

- a. dislocations
- b. grain boundaries
- c. vacancies
- d. interstitial atoms
- e. substitutional atoms

_____ 13. The Hall-Petch equation describes how the strength of a material depends on

- a. vacancies
- b. grain size
- c. dislocation density

_____ 14. Decreasing the grain size in a material usually

- a. increases the ductility of the material
- b. increases the strength of the material
- c. increases the melting temperature of the material

_____ 15. A stacking sequence of ABCABABABC means that

- a. the material has a HCP crystal structure everywhere
- b. a grain boundary must be present in the material
- c. a twin boundary must exist in the material
- d. a stacking fault is present in the material

_____ 16. Almost always, we find that increasing the number of lattice imperfections

- a. increases the strength of the material
- b. increases the ductility of the material
- c. decreases the strength of the material

_____ 17. FCC metals are expected to have good ductility because

- a. they have just the right number of intersecting slip systems
- b. they have only three slip systems, making it easy for dislocations to move without interfering with one another
- c. they have unusually high critical resolved shear stresses

_____ 18. When we increase the dislocation density, we expect

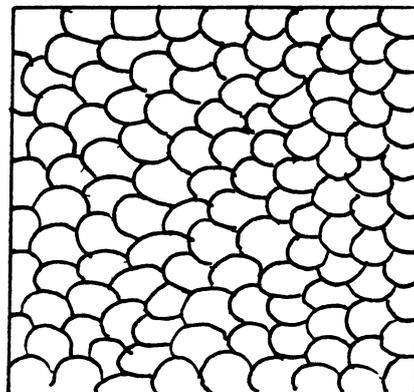
- a. the strength and ductility of the material to increase
- b. the strength and ductility of the material to decrease
- c. the strength to increase but the ductility to decrease
- d. the strength to decrease but the ductility to increase

_____ 19. Almost always, we strengthen a metal by

- a. making it easier for grain boundaries to move
- b. making it easier for dislocations to move
- c. making it harder for dislocations to move
- d. making it harder for vacancies to move

_____ 20. A sketch of the microstructure of a material is shown below; the sketch is drawn at a magnification of 100. The ASTM grain size for the material is about:

- a. 1
- b. 3
- c. 6
- d. 12



- _____ 1. An Arrhenius relationship means that, when the temperature changes, a characteristic or property of a material changes in a
a. linear manner b. parabolic manner c. exponential manner
- _____ 2. Most of the time, the activation energy Q for a particular reaction
a. increases as the temperature increases
b. decreases as the temperature increases
c. is a constant and is not affected by temperature
- _____ 3. The diffusion coefficient D for a particular system
a. increases as the temperature increases
b. decreases as the temperature increases
c. is a constant and is not affected by temperature
- _____ 4. Substitutional atoms, such as nickel atoms in a copper matrix, generally diffuse by
a. vacancy diffusion
b. interstitial diffusion
- _____ 5. We generally find that the activation energy is high, and therefore the rate of diffusion is comparatively slow, when atoms move by
a. vacancy diffusion
b. interstitial diffusion
- _____ 6. Diffusion is always occurring in materials, even when the material is absolutely pure. However a net flux is produced only if
a. the temperature is more than 0.4 times the absolute melting temperature of the material
b. a concentration gradient is present in the material
c. diffusion occurs by movement of interstitial atoms rather than substitutional atoms or vacancies
- _____ 7. In most cases involving material processing, such as heat treatments, the concentration gradient
a. increases with increasing heat treatment time
b. decreases with increasing heat treatment time
c. is always a constant
- _____ 8. We expect that carbon atoms will diffuse at a faster rate in
a. BCC iron b. FCC iron
- _____ 9. We would expect magnesium atoms or ions to diffuse at a faster rate in
a. magnesium metal b. magnesium oxide (MgO), a ceramic
- _____ 10. We would expect cerium atoms to diffuse at a higher rate in
a. aluminium, with a melting temperature of 660°C
b. nickel, with a melting temperature 1453°C
- _____ 11. We expect that copper atoms will diffuse more rapidly
a. through the bulk, or interior, of a copper grain
b. along grain boundaries in a polycrystalline block of copper
- _____ 12. Diffusion always occurs, even near absolute zero. However, generally the rate of diffusion is considered to be very slow and relatively unimportant until the temperature
a. is above room temperature
b. is above about 0.4 times the absolute melting temperature of the material
c. is above the melting temperature of the material

_____ 13. In a number of processes used to produce metal and ceramic parts, powders are compacted together and then heated, providing an opportunity for the powder particles to be joined together by diffusion. The step in the process during which the atoms are diffusing at the high temperature is most properly called:

- a. diffusion bonding b. grain growth c. sintering

_____ 14. The diffusion coefficient for carbon atoms in FCC iron at 1250°C is closest to:

- a. $3.546 \times 10^{-3} \text{ cm}^2/\text{s}$ b. $4.367 \times 10^{-6} \text{ cm}^2/\text{s}$ c. $8.521 \times 10^{12} \text{ cm}^2/\text{s}$

_____ 15. A Cu-Zn alloy is produced by a casting process. As a result of the process, a very nonuniform structure is produced. We find that if we perform a heat treatment at 800°C for 10 hours, the structure becomes uniform. About how long would it take to make the structure uniform if we performed the heat treatment at 300°C instead?

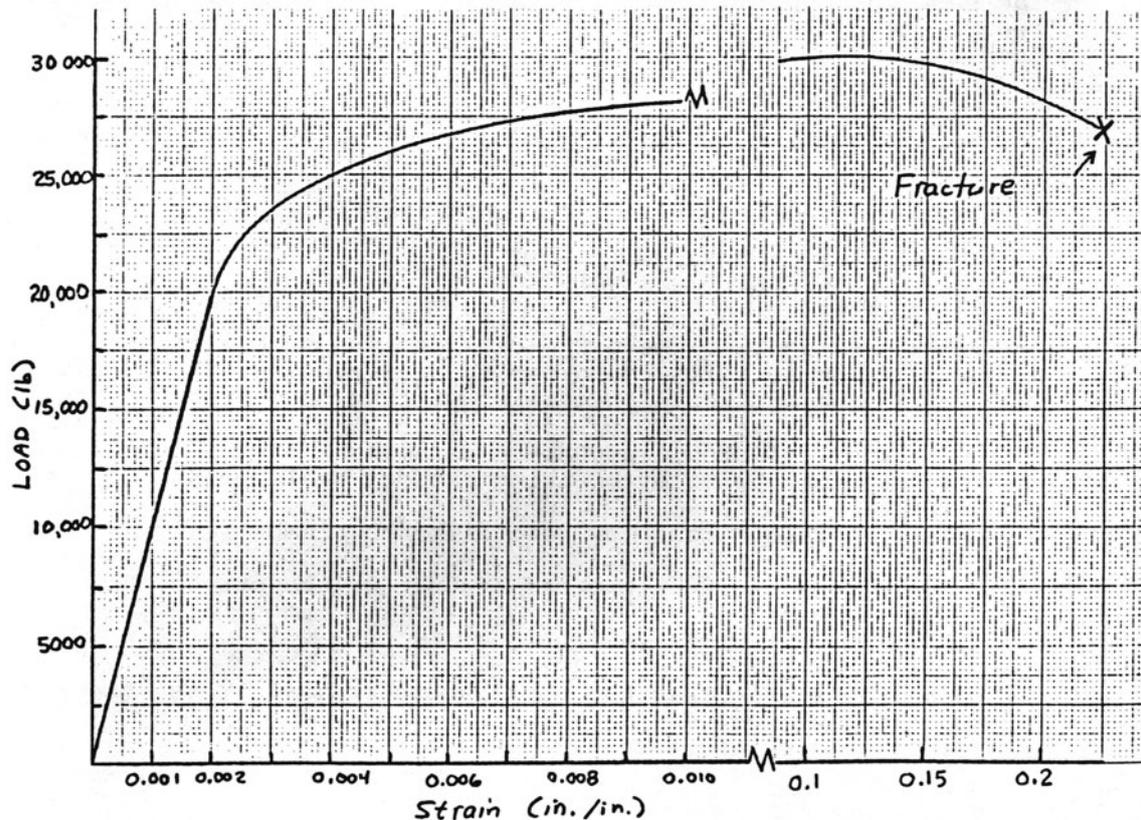
- a. 175 hours b. 3500 days c. 72,000 years

- _____ 1. The stress at which dislocations begin to slip and plastic deformation begins is called the
a. tensile strength b. modulus of elasticity
c. yield strength d. elongation
- _____ 2. The stress at which the maximum force that can be withstood by either a ductile or a brittle material is called the
a. tensile strength b. modulus of elasticity
c. yield strength d. breaking strength
- _____ 3. The slope of the stress-strain curve, which is determined primarily by the strength of the atomic bonds within the material, is called the
a. tensile strength b. modulus of elasticity
c. yield strength d. 0.2% offset yield strength
- _____ 4. When you design a part or select a material for a given application, you almost always want to be sure that the stress acting on the material during its service
a. is above the tensile strength
b. is below the yield strength
c. causes neither elastic nor plastic strain
- _____ 5. Necking of a ductile material occurs when the applied stress
a. reaches the yield strength
b. reaches the tensile strength
c. causes the material to fracture
d. causes at least some elastic strain
- _____ 6. The stiffness of a material is most closely related to
a. the yield strength
b. the tensile strength
c. the modulus of elasticity
d. the breaking strength
- _____ 7. Based on atomic bonding, we would expect that
a. the modulus of elasticity of ceramics is higher than that of metals
b. the modulus of elasticity of metals is higher than that of ceramics
- _____ 8. We would use a three-point bend test, rather than a standard tensile test, if we wished to determine the strength of a
a. metal b. ceramic
- _____ 9. For a given applied force, we expect that the true stress acting on a material will be
a. higher than the engineering stress
b. lower than the engineering stress
c. always equal to the engineering stress
- _____ 10. Almost always, as the temperature increases,
a. the ductility increases
b. the yield strength increases
c. the tensile strength increases
d. the modulus of elasticity increases
- _____ 11. Hooke's law only applies to
a. the plastic portion of a stress-strain curve
b. the elastic portion of a stress-strain curve

- _____ 12. We expect the modulus of elasticity of a metal to
- be higher for low melting point metals, such as aluminium
 - be higher for high melting point metals, such as iron or steel
- _____ 13. We would expect that the %Reduction in Area will be larger for a material that
- breaks with no necking
 - breaks after a large amount of necking has occurred
- _____ 14. If we want to shape a metal into a complex part by bending, forging, or some other metal-working process, we must
- always be sure that the stress applied during the metal-working process is below the yield strength
 - always be sure that the stress applied during the metal-working process is above the yield strength

15. A load-strain curve is shown below (the engineer was too lazy to convert the load to stress!). The sample that was tested had a diameter of 0.505 in. and a 2-in. long gage length was marked on the sample prior to testing. After the test was completed, the diameter of the broken sample was 0.465 in. and the distance between the gage marks was 2.46 in. Determine the following: (please indicate your work on the paper if at all possible)

- 0.2% offset yield strength = _____
- tensile strength = _____
- engineering stress at fracture = _____
- true stress at fracture = _____
- modulus of elasticity = _____
- %elongation = _____
- %reduction in area = _____



Mechanical Testing and Properties (Part II)

- _____ 1. Typically, we would expect to have a higher toughness in a metal if the crystal structure of the metal is:
a. HCP b. FCC c. BCC
- _____ 2. We always expect to find a ductile-brittle transition temperature in a metal if the crystal structure of the metal is:
a. FCC b. BCC
- _____ 3. We would prefer that a metal part operate at a temperature that is
a. above the transition temperature
b. below the transition temperature
- _____ 4. As a very general rule, we would expect that a metal will have the best toughness if:
a. the yield strength is 20,000 psi rather than 10,000 psi
b. the %elongation is 50% rather than 10%
- _____ 5. We are more likely to be able to identify an endurance limit in:
a. aluminium b. iron or steel
- _____ 6. The stress below which a metal is expected to be able to withstand a cyclical load for a given number of cycles, say 100,000, is called the
a. yield strength b. endurance limit
c. fatigue strength d. fatigue life
- _____ 7. Sometimes the surface of a part is shot peened (or blasted with steel shot in order to introduce compressive residual stresses at the metal surface). The shot peening is expected to
a. increase the applied cyclical stress that a metal can withstand
b. reduce the applied cyclical stress that a metal can withstand
c. have no effect on the fatigue properties of the metal part
- _____ 8. We never like to introduce sharp notches into a material, since the notches can help initiate cracks when a load is applied. We normally expect to find that the material is MOST sensitive to notches when failure is expected by
a. a simple overloading of the part in a tensile-type test
b. fatigue
c. creep
d. impact
- _____ 9. In ceramic materials, creep typically occurs by the sliding of grains past one another, while in polymers, creep typically occurs by sliding of polymer chains past one another. In metals, we find that creep is generally caused by
a. grain boundary sliding, just as in ceramics
b. sliding of atom chains by breaking van der Waals bonds, just as in polymers
c. movement of dislocations, by both slip and climb
- _____ 10. When we increase either the temperature or the stress, the creep rate in a material is expected to
a. increase b. decrease
- _____ 11. When we increase either the temperature or the stress, the rupture time caused by creep is expected to
a. increase b. decrease

- _____ 12. We make an impression and then measure the diameter of that impression when we determine the hardness of a material by the
 a. Rockwell C test b. Rockwell A test c. Brinell test
- _____ 13. We expect the fracture toughness of a particular metal to be highest if the metal is
 a. 0.125 in. thick b. 2.0 in. thick
- _____ 14. We usually find that, when we increase the yield strength of a metal, the critical fracture toughness K_{Ic}
 a. increases b. decreases
- _____ 15. The Charpy test is an example of a
 a. tensile test b. impact test c. fatigue test d. creep test
16. Suppose that we determine that the Brinell hardness number of a steel is 350. The tensile strength of the steel is about _____ psi and the endurance limit is about _____ psi.
17. The fracture toughness of a material is found to be 12,500 psi. Suppose a part made of this material contains a surface crack that is 0.5 in. deep and has a geometry factor "f" of 1.2. When we then apply a stress of 10,000 psi to the part, the crack will
 a. grow and the part will consequently fail
 b. not grow and the part will therefore not fail
- _____ 18. When we increase the temperature of a part, we would expect that the creep rate will
 a. increase linearly, or creep rate $\propto T$
 b. increase parabolically, or creep rate $\propto \sqrt{T}$
 c. increase exponentially, or creep rate $\propto \exp(-Q/RT)$
 d. decrease
- _____ 19. We would normally prefer that a cyclical stress applied to a material be
 a. above the endurance limit
 b. below the endurance limit
- _____ 20. Diffusion is most important in explaining
 a. tensile properties of a material
 b. impact properties of a material
 c. fatigue properties of a material
 d. creep properties of a material
 e. hardness of a material
 f. fracture toughness of a material

- _____ 1. When we bend a wire back and forth, each time causing at least some plastic deformation, the wire eventually breaks because
- fatigue cracks nucleate and grow until the wire fractures
 - the yield strength and breaking strength become virtually equal
 - we ate too many Wheaties for breakfast
- _____ 2. Plastic deformation of a metal at a low temperature is
- a mechanism for strengthening a metal
 - a manufacturing method for producing many products
 - both a strengthening mechanism and a manufacturing method
- _____ 3. We would expect to obtain a larger amount of strain hardening if we deform
- a FCC metal, such as aluminium
 - a HCP metal, such as magnesium
 - a BCC metal, such as iron
 - a ceramic, such as SiC
- _____ 4. A large strain hardening coefficient, say $n = 0.5$, means that
- a given amount of plastic strain produces only a small increase in strength
 - a given amount of plastic strain produces a large increase in strength
 - strain hardening is virtually impossible to achieve
- _____ 5. Suppose that we do a cold rolling process that reduces the thickness of a 57.5 mm thick plate to a final thickness of 17.5 mm. The amount of cold work is about
- 30%
 - 45%
 - 69%
 - 85%
 - 310%
- _____ 6. Suppose we could roll a 100 mm thick copper plate to 50 mm, then immediately roll it further to a 25 mm thickness, then immediately roll it still further to a 10 mm thickness. The total cold work that we have accomplished is
- 50%
 - 60%
 - 75%
 - 90%
 - 160%
- _____ 7. Suppose, in a wire drawing process, we produce a 0.125 mm diameter wire from a 0.25 mm diameter wire. The cold work involved in the process is
- 25%
 - 50%
 - 67%
 - 75%
 - 500%
- _____ 8. When we increase the amount of cold work done to a metal, we find that
- the yield strength and tensile strength decrease
 - the ductility decreases
 - the metal is less likely to fail by creep at high temperatures
- _____ 9. The strengthening that can be obtained in a metal by strain hardening is
- limited only by the size of the forming equipment that we use
 - usually less than 5% greater than the strength before cold working
 - limited by the reduced ductility of the cold worked metal
- _____ 10. Suppose you would like to produce a wire by a wire drawing process; this involves pulling the wire through an opening in a die. For this process to work
- it is essential that the process be done while the wire is very hot in order to minimize the forces required
 - it is essential that the process be done while the wire is cold in order that strain hardening occurs during the process
 - the drawing process can be done either hot or cold

- _____ 11. Which stage of the annealing of a cold worked metal causes the strength and grain size to decrease?
 a. recovery b. recrystallization c. grain growth
- _____ 12. A stress relief anneal is achieved as a result of
 a. recovery b. recrystallization c. grain growth
- _____ 13. Niobium melts at 2468°C; the recrystallization temperature of niobium is likely about
 a. 825°C b. 1100°C c. 1234°C d. 1375°C
- _____ 14. During recovery, we expect all of the following EXCEPT
 a. residual stresses to be reduced or eliminated
 b. electrical conductivity to increase
 c. the anisotropic behavior caused by cold working to be eliminated
 d. the strength to remain about the same as after cold working
- _____ 15. We would expect recrystallization to occur at a lower temperature for
 a. pure nickel cold worked 75%
 b. pure nickel cold worked 25%
 c. a nickel alloy cold worked 75%
 d. a nickel alloy cold worked 25%
- _____ 16. If we wish to produce a hot worked product, we
 a. deform the metal in the recovery temperature range
 b. deform the metal just above the recrystallization temperature
 c. deform the metal just above the grain growth temperature
- _____ 17. Generally a large shape or part is manufactured by
 a. hot working b. cold working
- _____ 18. We would expect to have better surface finish and dimensional accuracy in a part produced by
 a. hot working b. cold working
- _____ 19. Superplastic deformation requires all of the following EXCEPT
 a. the metal must be deformed at a low temperature to assure cold working
 b. the metal must have a very fine grain size
 c. the metal must deform at a slow rate
 d. grains should easily slide and rotate past one another
- _____ 20. Suppose you weld two cold worked plates together. The weld will most likely fail
 a. in the fusion zone where a liquid formed during welding
 b. in the heat affected area of the weld
 c. in the cold worked base metal
- _____ 21. During recovery
 a. the dislocation density increases
 b. the dislocation density decreases
 c. the dislocations are rearranged but their density remains constant
- _____ 22. If we want to produce a complex shape from magnesium, we should
 a. cold work the magnesium b. hot work the magnesium
- _____ 23. If we want to design a turbine blade to operate at 1000°C for long times, we should
 a. definitely consider cold working to help improve service life
 b. shot peen the surface to help improve service life
 c. NOT count on cold working to help improve service life

_____ and _____ 1. Two important commercial methods to produce a small grain size and therefore improved mechanical properties by controlling the solidification process are:

- a. achieving a slow cooling rate during solidification
- b. achieving a short solidification time
- c. producing a long solidification time
- d. encouraging homogeneous nucleation by producing large undercoolings
- e. encouraging heterogeneous nucleation by inoculation

_____ 2. Introducing tiny amounts of boron and titanium to liquid aluminum before pouring the liquid into a mold to produce a casting is called

- a. rapid solidification processing
- b. glass formation
- c. grain refinement
- d. homogeneous nucleation

_____ 3. The amount of heat that must be removed to eliminate the superheat in a liquid metal is called the

- a. specific heat
- b. latent heat of fusion

_____ 4. The heat contained in the metal that causes a thermal arrest, or plateau, in the cooling curve is the

- a. specific heat
- b. latent heat of fusion

_____ 5. If we can produce a casting without any type of nucleation occurring, we

- a. produce an exceptionally fine grained structure
- b. produce a glass, having no crystal structure
- c. are said to have caused heterogeneous nucleation

_____ 6. When poor inoculation leads to a large undercooling of the liquid metal prior to solidification, we expect to observe

- a. planar growth of the solid
- b. dendritic growth of the solid

_____ 7. By rapidly cooling the metal during solidification, we expect

- a. the secondary dendrite arm spacing (SDAS) to be large
- b. the secondary dendrite arm spacing (SDAS) to be small

_____ 8. According to Chvorinov's rule, a casting having a small volume but a large surface area should

- a. freeze rapidly and produce a high strength
- b. freeze slowly and produce a high strength
- c. freeze rapidly and produce a low strength
- d. freeze slowly and produce a low strength

_____ 9. We expect to have uniform, isotropic properties in a casting composed primarily of

- a. columnar grains
- b. equiaxed grains

_____ 10. In practice, we encourage a casting to have all equiaxed grains by

- a. encouraging rapid solidification
- b. encouraging slow solidification
- c. encouraging heterogeneous nucleation by inoculation

_____ 11. In order to improve the performance of turbine blades in jet engines, we encourage the formation of

- a. columnar grains
- b. equiaxed grains

_____ 12. The shrinkage that occurs during solidification can be controlled, producing a sound casting, if we

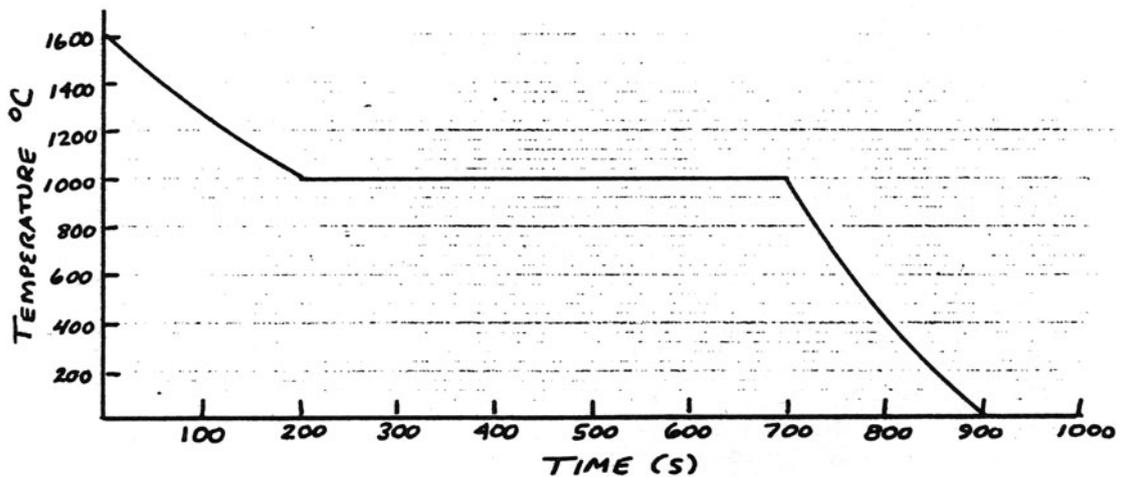
- a. use exceptionally high pouring temperatures or high superheats
- b. attach risers to the casting at key locations
- c. encourage dendritic growth

- _____ 13. The amount of gas porosity that forms in a casting is expected to increase, thus reducing the strength of the casting, if
- we melt and pour the metal while in a vacuum
 - we heat the metal to a very high superheat prior to pouring
 - we are sure to inoculate the metal prior to pouring
- _____ 14. The fusion zone in a weld is usually very strong because
- epitaxial growth encourages a fine grain size
 - rapid solidification of the fusion zone generally occurs
 - the fusion zone always contains some very strong type of alloy
- _____ 15. Two castings are sketched below. Which one of the castings is expected to have the highest strength?



16. A cooling curve is shown below.

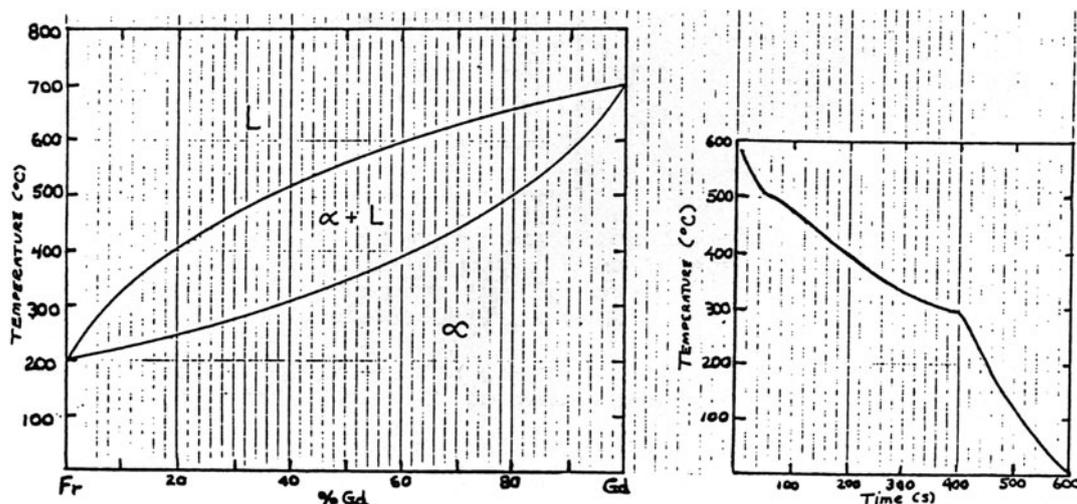
local solidification time = _____ s pouring temperature = _____ °C
 total solidification time = _____ s melting temperature = _____ °C
 superheat = _____ °C



17. In summary, we would like to control the solidification process so that
- the casting cools (rapidly / slowly)
 - the SDAS within each grain is (large / small)
 - inoculation produces a (large / small) grain size
 - usually, we want the grains to be (columnar / equiaxed)

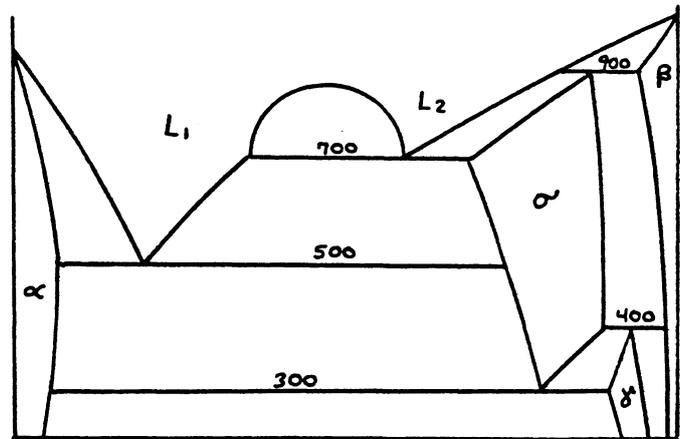
- _____ 1. Suppose the maximum solubility of Pb in Sn is 5% Pb. If 3% Pb is added to tin,
a. we expect to find a single-phase alloy
b. we expect to have a two-phase alloy
c. the question does not provide enough information to give an answer
- _____ 2. Which one of the following is NOT a condition for unlimited solid solubility?
a. only metals, and not ceramics, can display complete solubility
b. the two materials must have similar atomic radii
c. the two materials must have the same valence
d. the two materials must have the same crystal structure
- _____ 3. We expect to have a greater solid solution strengthening effect if the atomic radius of the alloying element is
a. much different from that of the "host" material
b. nearly the same as that of the "host" material
- _____ 4. If we increase the amount of alloying element,
a. we expect to obtain a greater increase in strength
b. we usually expect an increase in ductility
c. we expect to reduce the resistance of the material to creep
d. the electrical conductivity will not change
e. we expect all of the above
- _____ 5. The strengthening we obtain by solid solution strengthening is expected to be
a. a larger effect than grain size strengthening
b. a larger effect than strain hardening
- _____ 6. The temperature at which an alloy begins to melt during heating is called the
a. freezing range b. liquidus c. solidus
- _____ 7. The temperature at which an alloy is completely solid during cooling is called the
a. freezing range b. liquidus c. solidus
- _____ 8. The temperature difference between the liquidus and solidus temperatures is called the
a. superheat b. undercooling c. freezing range
- _____ 9. Between the liquidus and solidus temperatures, we expect to find
a. one phase containing a mixture of elements
b. two phases, each phase having the same composition
c. two phases, each phase having different compositions
- _____ 10. Suppose that you heat three alloys, each having a different composition, to the same temperature. In all three alloys, a mixture of liquid and solid form.
a. the composition of the solid phase in all three alloys is the same
b. the composition of the liquid phase in all three alloys is different
c. the amount of solid and liquid will be the same in all three alloys
- _____ 11. During nonequilibrium solidification, we expect that
a. the last solid to form contains a larger amount of the low melting point element
b. the first solid to form contains a larger amount of the low melting point element
c. the liquid phase always has the same composition as the solid phase

- _____ 12. In order to eliminate the effects of microsegregation, we might
 a. perform a recovery, or stress-relief, heat treatment
 b. perform a homogenization heat treatment
 c. use faster cooling rates during solidification
- _____ 13. The cooling rate of a casting is slowest
 a. just before the liquid cools to the liquidus
 b. just after the casting is completely solid
 c. between the liquidus and solidus temperatures
- _____ 14. If we plan to heat treat or hot work a casting that has been produced by nonequilibrium solidification, we should
 a. heat the casting between the liquidus and solidus
 b. heat the casting just a few degrees below the solidus
 c. be sure to keep the maximum temperature well below the solidus
- _____ 15. For a Fr-60% Gd alloy, the liquidus temperature is
 a. 700°C b. 600°C c. 500°C d. 400°C
- _____ 16. The freezing range for a Fr-20% Gd alloy is about
 a. 650°C b. 400°C c. 250°C d. 160°C
- _____ 17. A Fr-50% Gd alloy is heated to 400°C. The composition of the alpha phase is about
 a. 80% Gd b. 20% Gd c. 60% Gd d. 50% Gd
- _____ 18. A Fr-50% Gd alloy is heated to 400°C. The amount of the liquid phase is about
 a. 75% b. 25% c. 60% d. 50%
- _____ 19. A Fr-Gd alloy is heated to 500°C and a mixture of liquid and α is produced. The composition of the liquid is
 a. 40% Gd b. 70% Gd c. 82% Gd d. insufficient information
- _____ 20. A cooling curve is shown for a Fr-Gd alloy. The composition of the alloy from which the curve was obtained is about
 a. 17% Gd b. 40% Gd c. 64% Gd d. 92% Gd
- _____ 21. The local solidification time of the casting from which the cooling curve was obtained is about
 a. 50 s b. 350 s c. 400 s d. 600 s



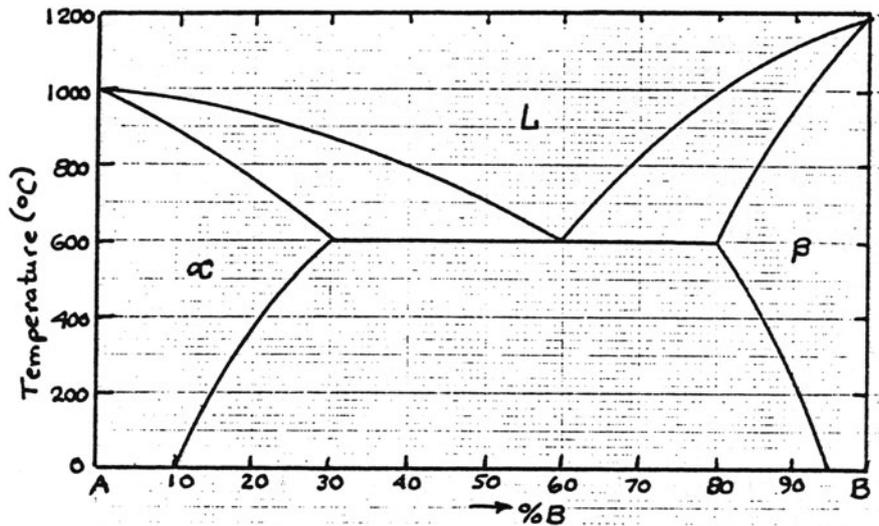
- _____ 1. For the best dispersion strengthening, the precipitate particles should be
 a. round, small, discontinuous, and present in a large volume percent
 b. angular, small, discontinuous, and present in a small volume percent
 c. round, large, continuous, and present in a large volume percent
 d. round, small, continuous, and present in a large volume percent
- _____ 2. A compound has a fixed, unalterable composition at all temperatures if it is
 a. stoichiometric b. nonstoichiometric
- _____ 3. Intermetallic compounds usually do NOT possess good
 a. resistance to oxidation and corrosion at high temperatures
 b. resistance to failure by creep
 c. high temperature strength
 d. ductility and formability at room temperature
- _____ 4. Which one of the following is NOT an intermetallic compound?
 a. Ni₃Al b. Ti₃Al c. MoSb₂ d. Al₂O₃
- _____ 5. Severe problems with segregation usually means that we would prefer to AVOID which one of the following three-phase reactions?
 a. eutectic b. eutectoid c. peritectoid d. monotectic
- _____ 6. Alloys such as Cu-Pb, in order to obtain better than usual machinability, may take advantage of which one of the following reactions?
 a. eutectic b. eutectoid c. peritectic d. monotectic
7. A complex phase diagram is shown below. Write in the name (eutectic, etc.) of the three phase reaction that occurs at each of the following temperatures:

- 900°C: _____
 700°C: _____
 500°C: _____
 400°C: _____
 300°C: _____



- _____ 8. In the phase diagram above, σ and γ are
 a. solid solutions of an alloying element in a metal
 b. stoichiometric intermetallic compounds
 c. nonstoichiometric intermetallic compounds
- _____ 9. Assuming that both solid solutions have good ductility, we would expect that the eutectic microconstituent that forms in the phase diagram above
 a. has reasonably good ductility b. is very brittle
- _____ 10. The line in a phase diagram that gives the solubility of one element in the other solid element as a function of temperature is called the
 a. solidus b. liquidus c. eutectic d. solvus

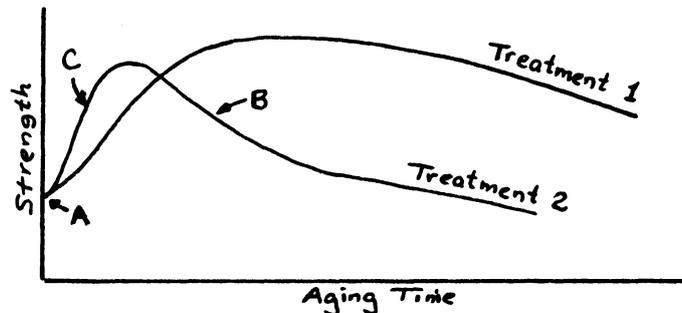
Consider the A-B phase diagram in the sketch for the remaining questions:



- _____ 11. An alloy that contains 70% B is called
a. a hypoeutectic alloy b. a hypereutectic alloy
- _____ 12. The solubility of B in element A at 0°C is
a. 10% B b. 30% B c. 60% B d. 80% B e. 95% B
- _____ 13. The maximum solubility of A in element B is
a. 10% A b. 20% A c. 60% A d. 80% A e. 95% A
- _____ 14. The composition of the eutectic microconstituent is
a. 10% B b. 30% B c. 60% B d. 80% B e. 95% B
- _____ 15. The microconstituent that forms first during the solidification of a 70% B alloy is
a. primary α b. primary β c. eutectic microconstituent
- _____ 16. The composition of the α phase in a 20% B alloy at 600°C is
a. 20% B b. 30% B c. 60% B d. 80% B
- _____ 17. The composition of the α phase in a 40% B alloy at 605°C is about
a. 30% B b. 40% B c. 60% B d. 80% B
- _____ 18. The amount of primary α in a 40% B alloy at 605°C is about
a. 33% b. 40% c. 50% d. 67% e. 80%
- _____ 19. The amount of eutectic microconstituent in a 75% B alloy is about
a. 25% b. 50% c. 75% d. 100%
- _____ 20. The total amount of α in a 50% B alloy at 0°C is about
a. 10% b. 53% c. 60% d. 95%
- _____ 21. The liquidus temperature for a 20% B alloy is
a. 400°C b. 600°C c. 660°C d. 920°C
- _____ 22. The amount of α in the eutectic microconstituent is about
a. 30% b. 40% c. 60% d. 80%
- _____ 23. In a 40% B alloy, the continuous microconstituent is expected to be
a. primary α b. primary β c. eutectic microconstituent

- _____ 1. When we cool an alloy very slowly from a single phase α region to a two phase $\alpha + \beta$ region, we expect
- a uniform dispersion of β particles and good strengthening
 - a uniform dispersion of β particles and embrittlement
 - the β to form as a grain boundary film and cause embrittlement
- _____ 2. When a precipitate β forms during the cooling process from a single phase to a two-phase region and grows as thin plates along specific crystallographic planes or directions in the α matrix, we are said to have produced a
- coherent precipitate
 - Widmanstatten precipitate
- _____ 3. The difference between precipitates in a "normal" dispersion strengthened alloy and an age hardened alloy is that the precipitate that forms during age hardening is
- coherent
 - noncoherent
 - Widmanstatten
- _____ 4. The greatest disruption of the matrix lattice is expected when we cause the formation of a
- coherent precipitate
 - noncoherent precipitate
- _____ 5. Normally we would solution treat an age hardenable alloy
- below the solvus temperature
 - above the liquidus temperature
 - above the solidus temperature
 - between the solvus and eutectic temperatures
- _____ 6. The purpose of the quenching step in an age hardenable alloy is to
- produce a round, uniformly distributed precipitate during quenching
 - produce a Widmanstatten precipitate during quenching
 - avoid the formation of any precipitate during quenching
- _____ 7. An age hardenable alloy should possess all of the following characteristics EXCEPT
- should be "quenchable"
 - should produce a coherent precipitate
 - should produce a strong, hard precipitate in a ductile matrix
 - should display increasing solid solubility with decreasing temperature
- _____ 8. The ageing temperature should be
- below the solvus temperature
 - between the solvus and eutectic temperatures
 - above the solidus temperature
- _____ 9. Certain alloys will undergo the ageing process at room temperature; these alloys are called
- artificial ageing alloys
 - natural ageing alloys
- _____ 10. Increasing the temperature at which ageing is done typically
- increases the maximum strength of the alloy
 - increases the time required for the alloy to age
 - reduces the time interval during which the strength of the alloy is near its maximum
- _____ 11. Overageing means
- we produced a structure that is too strong and too brittle
 - we aged for too long of a time and the precipitate lost its coherency
 - we aged the alloy two or three times

- _____ 12. As a general rule,
- the ageing process is an excellent way to achieve good creep resistance at high temperatures
 - the effects of age hardening are eliminated when the alloy is heated above some critical temperature
 - the ageing process is effective in producing high temperature strength only if the service temperature is above the solvus
- _____ 13. In age hardened aluminium alloys, the maximum service temperature should be approximately
- below about 150°C
 - somewhere between 200 and 400°C
 - any temperature up to 550°C
- _____ 14. The usual sequence in a age hardening process is
- quench, solution treat, then age
 - age, solution treat, then quench
 - solution treat, age, then quench
 - solution treat, quench, then age
- _____ 15. The coherent precipitate that usually forms in age hardening alloys is
- the equilibrium precipitate found in the phase diagram
 - a nonequilibrium precipitate that forms before the equilibrium precipitate is produced
- _____ 16. A supersaturated solid solution is present in the structure
- immediately after solution treating but before quenching
 - immediately after quenching
 - only after the alloy is completely aged
- _____ 17. The solution treating step is intended to
- produce large, rounded precipitate particles
 - produce a uniform, nonsegregated single phase structure
 - produce Widmanstätten precipitates
 - produce a coherent precipitate
- _____ 18. In aluminium alloys, we generally expect to obtain the greatest strengthening effect by
- controlling grain size strengthening
 - controlling solid solution strengthening
 - controlling the age hardening reaction
 - controlling strain hardening
- _____ 19. The sketch below shows how the strength of an alloy changes with increasing ageing time for two ageing treatments. The highest ageing temperature was used for
- treatment 1
 - treatment 2
- _____ 20. In the sketch below, point "A" refers to the
- strength of the final, aged structure
 - strength of the supersaturated solid solution
 - strength of the alloy before solution treatment
- _____ 21. A coherent precipitate is found in the portion of the sketch labeled
- A
 - B
 - C



- _____ 1. In the iron-carbon phase diagram, the eutectoid reaction is
a. austenite → ferrite + cementite
b. austenite → ferrite + δ -ferrite
c. ferrite → austenite + cementite
d. cementite → iron + carbon
- _____ 2. To obtain dispersion strengthening in iron-carbon alloys, we want
a. austenite to be the discontinuous precipitate
b. ferrite to be the discontinuous precipitate
c. cementite to be the discontinuous precipitate
- _____ 3. Austenite and ferrite are solid solutions of carbon in iron. The carbon in these solutions is present as
a. large substitutional atoms
b. small substitutional atoms
c. small interstitial atoms
- _____ 4. The solubility of carbon is HIGHEST in
a. ferrite (α) b. austenite (γ) c. δ -ferrite
- _____ 5. The lamellar, or plate-like, mixture of ferrite and Fe₃C is called
a. a primary microconstituent b. austenite
c. pearlite d. bainite
- _____ 6. The composition of pearlite in Fe-C alloys is about
a. 0.0218% C
b. 0.77% C
c. 6.67% C
d. always equal to the original composition of the steel
- _____ 7. The total amount of ferrite in a Fe-0.6% C steel is about
a. 22.7% b. 60% c. 77.3% d. 91.3%
- _____ 8. The amount of pearlite in a Fe-0.6% C steel is about
a. 22.7% b. 60% c. 77.3% d. 91.3%
- _____ 9. The amount of pearlite in a Fe-1.5% C steel is about
a. 12.4% b. 22.2% c. 77.8% d. 87.7%
- _____ 10. A steel containing 0.9% C is
a. a hypoeutectoid steel b. a hypereutectoid steel
- _____ 11. A steel containing a mixture of primary cementite and pearlite is
a. a hypoeutectoid steel b. a hypereutectoid steel
- _____ 12. A steel containing 30% pearlite is
a. a hypoeutectoid steel
b. a hypereutectoid steel
c. can't tell without additional information
- _____ 13. When we increase the carbon content from 0.1% C to 0.7% C,
a. the amount of primary ferrite increases
b. the amount of pearlite increases
c. the amount of cementite decreases
- _____ 14. We expect the highest strength in
a. ferrite b. austenite c. pearlite
- _____ 15. We expect pearlite to have a higher strength if
a. the interlamellar spacing is increased
b. the interlamellar spacing is decreased

- _____ 16. We expect the interlamellar spacing to be smaller if
- the cooling rate during the eutectoid reaction is faster
 - the cooling rate during the eutectoid reaction is slower
 - the temperature at which the eutectoid reaction is allowed to occur increases
- _____ 17. During cooling or during an isothermal transformation, we expect pearlite to form
- only at temperatures above 727°C
 - between about 550 and 727°C
 - between about 200 and 550°C
 - only below 200°C
- _____ 18. We can produce pearlite only if we start with
- ferrite
 - austenite
 - martensite
 - bainite
- _____ 19. A mixture of angular or slightly rounded Fe₃C particles in a continuous ferrite matrix, formed during a low temperature isothermal transformation, is called
- pearlite
 - martensite
 - bainite
 - tempered martensite
- _____ 20. If we austenitize a eutectoid steel at 800°C, quench the steel to 400°C and hold for 1000 s, then quench to 20°C, we expect the structure to be
- still all austenite
 - pearlite
 - bainite
 - martensite
- _____ 21. As a general rule, we would expect bainite to be
- weaker than pearlite
 - weaker than ferrite
 - stronger than pearlite
- _____ 22. Which one of the following is a "metastable" phase and does not appear in the phase diagram?
- ferrite
 - cementite
 - austenite
 - martensite
- _____ 23. Which one of the following forms by an "athermal" transformation, or one that depends on temperature rather than on time?
- pearlite
 - bainite
 - martensite
 - ferrite
- _____ 24. The hardness of martensite
- increases as the carbon content increases
 - decreases as the carbon content increases
 - is independent of carbon content
- _____ 25. Martensite in steels is considered to be
- a soft ductile material that is desirable as a continuous matrix in a dispersion strengthened steel
 - a hard, very brittle material that must be softened before it is actually used
 - a material that should be avoided during any heat treatment
- _____ 26. When we heat martensite to some temperature below the eutectoid temperature,
- the martensite transforms to pearlite
 - the martensite decomposes to form cementite in a ferrite matrix
 - the martensite transforms back to austenite
- _____ 27. When we increase the tempering temperature of martensite, we expect
- the strength and hardness of the steel to increase
 - the strength and hardness of the steel to decrease
- _____ 28. Which one of the following is not a mixture of ferrite and Fe₃C?
- pearlite
 - bainite
 - martensite
 - tempered martensite

- _____ 1. The steel that contains 94% pearlite and 6% primary cementite is
a. 1020 b. 1070 c. 10110 d. 10180
- _____ 2. A cold worked 0.2%C steel is found to be too brittle to be formed into a complicated shape. A low cost heat treatment to restore the ductility might be
a. process annealing b. full annealing
c. normalising d. quench and temper
- _____ 3. In order to produce a 0.60%C steel containing fine pearlite, in addition to the primary ferrite, your heat treatment would likely be
a. process annealing b. annealing
c. normalising d. spheroidising
- _____ 4. In order to improve the machinability of a 52100 steel, your heat treatment would likely be
a. process annealing b. annealing
c. normalising d. spheroidising
- _____ 5. The first step for each of the following heat treatments is austenitising EXCEPT for
a. annealing b. normalising
c. spheroidising d. quench and tempering
- _____ 6. When we anneal a hypereutectoid steel, we should
a. austenitise above the A_{cm} line to assure 100% austenite
b. austenitise between the A_1 and A_{cm} lines to prevent the formation of a grain boundary Fe_3C film
c. austenitise below the A_1 temperature
- _____ 7. In a 4340 steel, we expect
a. the carbon content to be 40%
b. to find no alloying elements
c. the carbon content to be 0.40%
- _____ 8. The Fe_3C is generally round and very coarse in
a. bainite b. pearlite
c. spheroidite d. tempered martensite
- _____ 9. We would NOT expect to perform which one of the following treatments on a 10110 steel?
a. annealing b. normalising
c. spheroidising d. process annealing
- _____ 10. In order to minimise quench cracks during a quench and temper heat treatment, we might
a. temper the steel at a higher temperature than normal
b. quench the steel in agitated water instead of still oil
c. perform a marquenching heat treatment
d. increase the carbon content in the steel
- _____ 11. A low-alloy, 0.2% C steel is cooled under conditions which produce a cooling rate of 7.5 °C/s. The expected microstructure is
a. all martensite b. ferrite, bainite, and martensite
c. all pearlite d. ferrite, pearlite, and bainite
- _____ 12. The major effect of adding alloying elements to a steel is to
a. increase the hardness of any martensite that forms
b. increase the amount of martensite that forms during quenching
c. increase the cooling rate required to produce martensite

- _____ 13. The hardness of a steel quenched under conditions that give a zero Jominy distance is determined primarily by
- the amount of alloying element in the steel
 - the amount of carbon in the steel
 - the type of quenching medium that is used
- _____ 14. Both a 1030 and a 4130 steel are quenched to produce a hardness of HRC 40. Both are then tempered for 30 minutes at 350°C. After tempering
- the 1030 steel will be harder than the 4130 steel
 - the 1030 steel will be softer than the 4130 steel
 - both steels will still have the same hardness
- _____ 15. A 50 mm diameter 1040 steel is quenched to produce a surface hardness of HRC 30. A 50 mm diameter 9310 steel quenched under the same conditions will likely have a surface hardness of about
- HRC 35
 - HRC 41
 - HRC 48
 - HRC 60
- _____ 16. A 12 mm diameter 4320 steel is quenched to produce a hardness of HRC 45 at the centre of the bar. The hardness at the same location in a 25 mm diameter 1040 steel quenched in the same medium should be closest to
- HRC 20
 - HRC 30
 - HRC 45
 - HRC 55
- _____ 17. A 25 mm diameter 1080 steel bar is quenched to produce a hardness of HRC 40 at the centre of the bar. A 50 mm diameter 1080 steel bar is quenched in the same medium; what is the microstructure expected in the centre of this bar?
- coarse pearlite
 - fine pearlite
 - pearlite and martensite
 - martensite
- _____ 18. Steel A has a flat hardenability curve and a maximum hardness of HRC 35, while steel B has a steeply sloping hardenability curve and a maximum hardness of HRC 65. The steel containing the highest amount of alloying element is
- Steel A
 - Steel B
- _____ 19. When a steel is austenitised and furnace cooled, the heat treatment is
- normalising
 - spheroidising
 - annealing
 - process annealing
- _____ 20. We would expect the least problems with residual stresses and quench cracking if we quench a steel in:
- agitated water
 - still oil
 - agitated brine
 - still water
- _____ 21. When we increase the amount of alloying element in a steel,
- the pearlite start and finish temperatures become longer
 - the bainite start and finish temperatures become shorter
 - the carbon content at which the eutectoid reaction occurs is larger
- _____ 22. We would like to produce a dual phase steel by heating a 0.3%C steel to 750°C. The amount of martensite in the steel after quenching is expected to be about
- 35%
 - 49%
 - 65%
 - 100%
- _____ 23. The composition of the martensite in the dual phase steel described in question 22 is about
- 0.02% C
 - 0.3% C
 - 0.6% C
 - 0.77% C
- _____ 24. We expect to obtain the greatest percentage of martensite in a steel when we quench the steel in
- unagitated oil
 - agitated brine
 - agitated water

- _____ 1. We generally would expect to find a larger case depth, perhaps as much as 6 mm deep, in a steel that is
- surface hardened using a gas flame or induction coil
 - carburised
 - nitrided
- _____ 2. We would expect to obtain a better combination of high toughness coupled with good fatigue and wear resistance if we
- surface harden a 1040 steel
 - carburise a 1010 steel
 - completely quench and temper a 1080 steel
- _____ 3. In order to obtain a high carbon content in the martensite that forms in the case during carburising
- we perform the carburising treatment below the A_1 temperature
 - we perform the carburising treatment between the A_1 and the A_3
 - we perform the carburising treatment above the A_3
- _____ 4. We expect to have better weldability, i.e. less chance of martensite formation in the heat affected zone, when we weld
- a 1020 steel
 - a 1050 steel
 - a 4320 steel
- _____ 5. When we add chromium to a steel, we expect that the austenite portion of the phase diagram will
- increase in size
 - decrease in size
- _____ 6. In order to produce a "stainless" steel, we must
- be sure to quench and temper the steel
 - add at least 12% chromium to the steel
 - include a significant amount of nickel in the steel
 - have enough carbon present to assure the formation of pearlite
- _____ 7. For stainless steels, the best combination of corrosion resistance, formability, strain hardening, and ductility is expected in
- martensitic stainless
 - austenitic stainless
 - ferritic stainless
 - PH stainless
- _____ 8. A stainless steel containing large amounts of nickel is most likely
- ferritic
 - martensitic
 - austenitic
- _____ 9. A stainless steel containing a mixture of ferrite and austenite is called
- PH stainless steel
 - duplex stainless steel
- _____ 10. If we find that a stainless steel is not attracted to a magnet, the steel is most likely
- ferritic
 - austenitic
 - martensitic
 - duplex
- _____ 11. If a eutectic reaction forms during the solidification of an iron-carbon alloy, we have a
- tool steel
 - dual phase steel
 - stainless steel
 - cast iron
- _____ 12. The $L \rightarrow \gamma + Fe_3C$ eutectic reaction
- is the stable reaction and produces gray cast iron
 - is the metastable reaction and produces ductile cast iron
 - is the metastable reaction and produces white cast iron
 - is the stable reaction and produces malleable cast iron

- _____ 13. When we produce a cast iron containing 3.5% C and 1.8% Si, we expect that the iron is
- hypoeutectic, with primary austenite
 - hypereutectic, with primary austenite
 - hypoeutectic, with primary graphite
 - hypereutectic, with primary cementite
- _____ 14. In steels, annealing produces coarse pearlite. In cast irons, annealing produces a matrix of
- coarse pearlite
 - fine pearlite
 - ferrite
 - bainite
- _____ 15. We expect to have good vibration damping and good resistance to thermal fatigue, properties important in a cast iron automotive block, for
- ductile cast iron
 - malleable cast iron
 - gray cast iron
 - white cast iron
- _____ 16. We typically expect the strength of a cast iron to
- increase when the carbon equivalent increases
 - decrease when the carbon equivalent increases
 - be unaffected by the carbon equivalent
- _____ 17. When we introduce at least 0.03% Mg to liquid cast iron, then to do an effective job of inoculation, we expect to produce
- malleable cast iron
 - white cast iron
 - ductile cast iron
 - grey cast iron
- _____ 18. In order to produce malleable cast iron, we must first produce
- grey cast iron
 - white cast iron
 - ductile cast iron
 - compacted graphite cast iron
- _____ 19. We generally expect the strength of a cast iron to
- increase when the amount of ferrite in the matrix increases
 - increase when the amount of pearlite in the matrix increases
 - be unaffected by the amount of ferrite and pearlite present
- _____ 20. One of the differences between ferritic malleable iron and pearlitic malleable iron is
- only pearlitic malleable iron goes through first stage graphitisation
 - only ferritic malleable iron goes through second stage graphitisation
 - only ferritic malleable iron goes through the drawing process
- _____ 21. Due to the solid solution strengthening effect, we expect
- a ferritic malleable iron to be stronger than a ductile iron with a ferrite matrix
 - a ferritic malleable iron to be weaker than a ductile iron with a ferrite matrix
 - a ferritic ductile iron to be tougher than a ferritic malleable iron
- _____ 22. All cast irons include, besides carbon, substantial amounts of
- chromium
 - nickel
 - silicon
- _____ 23. Suppose you treat liquid iron with magnesium, inoculate, then wait 10 minutes before pouring. If you produce a white cast iron microstructure instead of ductile iron, then you expect
- the nodulising element faded
 - the inoculating agent faded
- _____ 24. Vermicular graphite is an undesirable microstructural feature in ductile iron; however it is the desired type of graphite in
- white cast iron
 - malleable cast iron
 - gray cast iron
 - compacted graphite cast iron

- _____ 1. The most powerful strengthening method in aluminium is
a. strain hardening b. solid solution strengthening
c. dispersion strengthening d. age hardening
e. grain size strengthening
- _____ 2. An aluminium alloy with the designation 3003-H18 is
a. an age hardenable casting alloy
b. a cold worked casting alloy
c. an age hardenable wrought alloy
d. a cold worked wrought alloy
- _____ 3. Aluminium and its alloys tend to have all of the following properties EXCEPT for
a. low density and high strength-to-weight ratio
b. good resistance to creep, fatigue, and wear
c. high electrical and thermal conductivity
d. good resistance to oxidation and corrosion
- _____ 4. The addition of lithium as an alloying element in aluminium
a. increases the density compared to pure aluminium
b. increases the modulus of elasticity compared to pure aluminium
c. does not permit an age hardening heat treatment to be done
- _____ 5. An aluminium alloy with the designation 7075-T6 is
a. a naturally aged wrought alloy strengthened by CuAl_2
b. an artificially aged wrought alloy strengthened by a Mg-Zn precipitate
c. a cold worked wrought alloy that has been recrystallized
d. a heat treated casting alloy
- _____ 6. Compared to aluminium alloys, the magnesium alloys
a. have both a lower density and lower strength, leading to similar strength-to weight ratios
b. have much better resistance to creep, fatigue, and wear
c. have much better resistance to oxidation and corrosion
d. are stiffer, or have a higher modulus of elasticity
- _____ 7. The amount by which magnesium and its alloys can be strengthened by cold working is
a. limited due to the HCP crystal structure
b. comparable to that obtained in aluminium alloys
c. limited by the unusually high strength of magnesium
d. is usually large, taking advantage of the good dimensional accuracy caused by magnesium's high modulus of elasticity
- _____ 8. Beryllium and its alloys have the following characteristics EXCEPT for
a. the modulus of elasticity is greater than that of steel
b. the strength-to-weight ratio is as high or higher than that of most other metals and alloys
c. beryllium is inexpensive to obtain and fabricate
d. Be maintains its strength and stiffness at high temperatures
- _____ 9. Copper and its alloys generally do NOT have
a. good electrical and thermal conductivity
b. good corrosion resistance
c. high strength-to-weight ratios
d. good ductility and formability

- _____ 10. If we wanted to produce a copper-base alloy that has the high strength and large amount of elastic deformation needed for use as a spring, we might select
- brass containing 30% Zn
 - quenched and tempered aluminium bronze
 - age hardened beryllium copper
 - cold worked cupronickel containing 30% Ni
- _____ 11. Nickel-base superalloys do NOT rely for their high temperature properties on
- strain hardening
 - age hardening
 - dispersion strengthening by carbides
 - solid solution strengthening
- _____ 12. A martensitic reaction can help provide strength in
- aluminium alloys and magnesium alloys
 - copper alloys and aluminium alloys
 - nickel alloys and copper alloys
 - copper alloys and titanium alloys
- _____ 13. A titanium alloy is to be used in an application where good resistance to growth of fatigue cracks is desired. We might select
- an alpha titanium alloy that has been rapidly cooled to produce a Widmanstätten microstructure
 - an alpha titanium alloy that has been slowly cooled to produce a more equiaxed or plate-like structure
 - commercially pure titanium
- _____ 14. We generally find that the largest amounts of alloying elements are present in
- alpha titanium alloys
 - beta titanium alloys
 - alpha-beta titanium alloys
- _____ 15. Martensite can form in titanium alloys. In titanium alloys,
- martensite is hard, strong, and brittle, just as in steels
 - the alloy becomes stronger, rather than weaker, during tempering
 - overtempering cannot occur as it does in steels
- _____ 16. If we wanted to select an alloy for use in a corrosive environment at 600°C, we might first look at
- aluminium alloys
 - magnesium alloys
 - titanium alloys
 - nickel-base superalloys
- _____ 17. Refractory metals may be coated to reduce problems with oxidation. The coatings must have all of the following characteristics EXCEPT
- should have poor bonding with the refractory metal to keep the coating from cracking when the refractory metal expands
 - must prevent diffusion of gases through the coating in order to prevent contamination of the refractory metal
 - must have a high melting temperature
 - must expand or contract about the same amount as the metal when the temperature changes

- _____ 1. When ceramic parts produced from silica are heated, cracks may form. This is primarily due to
- an unusually high coefficient of thermal expansion
 - the large volume change that accompanies an allotropic transformation
 - the very low melting temperature of silica
- _____ 2. Slip does not occur easily in most ceramic materials. This may be due to all of the following EXCEPT
- the Burger's vector in ceramics is large
 - the strong bonding and directional nature of any covalent bonds make it difficult for dislocations to move
 - repulsive forces due to ionic bonding may make it difficult for any dislocations to move
 - there are no dislocations in ceramic materials
- _____ 3. When modifying agents, such as Na_2O , are added to silica,
- the melting temperature of any glass that forms will decrease
 - the viscosity of a glass at any particular temperature increases
 - it is more difficult to form the glassy material into a complex shape
- _____ 4. It is less likely that a flaw in a ceramic material will grow if
- the radius of curvature of the flaw is made smaller
 - the length of the flaw is reduced
 - the fracture toughness of the ceramic is made smaller
 - the modulus of elasticity of the ceramic is reduced
- _____ 5. An example of embedding hard ceramic particles into a softer, more ductile matrix is
- transformation toughened zirconia
 - a cermet
 - stabilised zirconia
 - tempered glass
- _____ 6. Some improvement in fracture toughness of zirconia can be obtained by adding CaO or MgO . The improvement occurs because
- stabilised zirconia does not go through a phase transformation on cooling
 - the CaO or MgO make it easier for dislocations to move in the ceramic
 - a glass-ceramic is produced, in which crystalline precipitates interfere with the movement of dislocations
- _____ 7. When we produce a crystalline ceramic part by using very small powder particles, thus giving a very fine grain structure, we find that
- the fracture toughness often is improved
 - the creep resistance is often improved
 - both fracture toughness and creep resistance are improved
- _____ 8. We would expect to have the best resistance to creep for
- ceramics that are entirely glassy
 - ceramics that are completely crystalline
 - ceramics that contain a mixed glassy and crystalline structure
- _____ 9. When a glass is heated and a crystalline phase precipitates,
- the material has vitrified
 - the material has devitrified
- _____ 10. Glass fibres are generally made at a temperature that is
- below the strain point
 - near the softening temperature
 - between the annealing and softening temperatures
 - above the melting temperature
 - near the working temperature

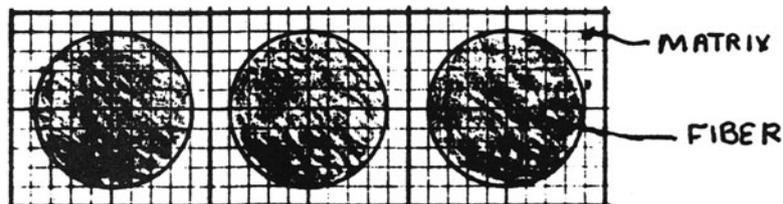
- _____ 11. The most common glass is a soda-lime silicate glass. The lime, or CaO, is added to the glass primarily to
- significantly reduce the liquidus temperature of the glass
 - make it easier for a crystalline phase to precipitate
 - reduce the solubility of the glass in water
 - improve the strength of the glass at high temperatures
- _____ 12. In producing a glass-ceramic, we must first
- cool the glass slowly to assure that the crystalline phase has sufficient time to precipitate
 - cool the glass rapidly to assure that no crystalline phase precipitates
- _____ 13. In producing glass-ceramics
- the temperature at which nucleation of the precipitate occurs is lower than the temperature at which the precipitates are allowed to grow
 - the ceramic is first heated to a liquid, quenched to some intermediate temperature, and then held at that temperature until the crystalline phase forms
- _____ 14. The greatest dimensional changes that occur when a clay body is heated happen when
- the interparticle water is evaporated
 - the pore water is evaporated
- _____ 15. As the sintering temperature of a clay body increases,
- the amount of glass that forms in the structure increases
 - a greater amount of crystalline phase precipitates
- _____ 16. We typically expect to obtain improved mechanical properties in an advanced ceramic when it is produced by
- hot pressing or hot isostatic compaction
 - compaction of the powders at room temperature, followed by sintering
 - slip casting
 - reaction bonding
- _____ 17. By appropriate selection of raw materials and processing techniques,
- we can produce ceramic materials with good ductility
 - we can produce ceramic materials with good fracture toughness
- _____ 18. Generally we expect the corrosion resistance of ceramics to be
- good
 - poor
- _____ 19. Generally we expect the melting temperature of ceramics to be
- high
 - low
- _____ 20. Generally we expect the electrical and thermal conductivity of ceramics to be
- high
 - low
- _____ 21. We do NOT expect any of the ceramics to be bonded by
- covalent bonds
 - van der Waals bonds
 - ionic bonds
 - metallic bonds
- _____ 22. We expect ceramics to usually have a
- high modulus of elasticity
 - low modulus of elasticity

- _____ 1. The polymerization mechanism that produces a by-product is called
a. addition b. condensation
- _____ 2. If a monomer has a functionality of two, we expect to form
a. only linear chains b. three-dimensional networks
- _____ 3. An unsaturated bond is
a. a bond that has been fired to drive off absorbed water
b. required for the addition polymerization mechanism
c. required to provide the by-product that forms during condensation reactions
d. necessary to bond the carbon atoms within the polymer chains
- _____ 4. Cross-linking does not generally occur between chains in
a. thermoplastics b. elastomers c. thermosets
- _____ 5. Which one of the following is NOT true for the degree of polymerization?
a. refers to the average number of repeat units, or mers, in a chain
b. increasing the degree of polymerization increases the melting point
c. increases if the amount of initiator increases
d. increasing the degree of polymerization increases polymer strength
- _____ 6. Thermoplastic polymers can have usable mechanical properties even above the glass transition temperature if they
a. are in the rubbery range
b. are in the leathery range
c. are above the degradation temperature
- _____ 7. Crystallinity in polymers will be encouraged if
a. the degree of polymerization is large
b. the liquid polymer is rapidly cooled to room temperature
c. the amorphous polymer is plastically deformed between the glass transition temperature and the melting temperature
d. the monomers that produce the polymer chains are complex
- _____ 8. We can increase the effectiveness of bonding between chains by all of the following EXCEPT
a. promoting crystallization of the polymer
b. producing strong bonds between chains by replacing some hydrogen atoms with chlorine atoms in the monomers
c. encouraging branching
d. using isotactic rather than atactic monomers
- _____ 9. We would expect PEEK to be stronger than polyacrylonitrile because
a. the bonding between chains will be much stronger in PEEK
b. the bonding within the chains will be much stronger in PEEK
c. the chains will be much longer in PEEK than in PAN
- _____ 10. Kevlar, a tradename of DuPont, is all of the following EXCEPT
a. an aromatic polyamide b. an aramid
c. a liquid-crystalline polymer d. a vinylidene polymer
- _____ 11. On average, how many monomers of polymethyl methacrylate are present in each chain of PMMA if the molecular weight of the polymer is 235,000 g/mol?
a. 2350 b. 235,000 c. 2.35×10^7 d. 1849

- _____ 12. Polymers that are capable of very large elastic deformations at room temperature but can be easily manufactured, formed, or even recycled at higher temperatures are
- thermoplastics
 - thermosets
 - elastomers
 - thermoplastic elastomers
- _____ 13. A thermoplastic polymer is expected to be most resistant to creep when the polymer is in the
- leathery condition
 - rubbery condition
 - glassy condition
- _____ 14. Polymers having chains so strong that they behave as rigid rods even above their melting temperatures are called
- thermoplastic elastomers
 - liquid-crystalline polymers
 - interpenetrating polymer networks
 - vulcanized elastomers
- _____ 15. When repeat units become more complex, or atoms other than hydrogen are attached to the chains,
- the melting temperature is reduced
 - the glass transition temperature is increased
 - the strength is reduced
 - crystallization occurs more easily
- _____ 16. Adhesives that rely on the solidification of a relatively low melting point polymer to provide bonding are
- pressure-sensitive adhesives
 - hot-melt adhesives
 - chemical (thermosetting) adhesives
 - evaporation adhesives
- _____ 17. The density of polymers is typically
- higher than that of light metals such as aluminium
 - between about 0.9 and 1.5 g/cm³
 - about the same as that of ceramic materials
- _____ 18. The modulus of elasticity of polymers is typically
- lower than that found for metals or ceramics
 - higher than metals but lower than ceramics
 - higher than ceramics but lower than metals
 - higher than both ceramics and metals
- _____ 19. Polymers are generally considered to be
- electrical and thermal conductors
 - electrical and thermal insulators
- _____ 20. Polymers are generally considered to have
- good resistance to corrosion
 - poor resistance to corrosion
- _____ 21. The most temperature-resistant polymers typically can operated
- only at room temperature
 - up to a maximum of about 350°C
 - well above the melting temperature of aluminium
 - well above the melting temperature of steel
- _____ 22. The usual cross-linking agent for elastomers is
- oxygen
 - sulfur
 - nitrogen
 - formaldehyde
- _____ 23. The strength-to-weight ratio of typical polymers is
- very high compared to metals and ceramics due to the low density
 - very low compared to metals and ceramics due to the low strength
- _____ 24. Superglues, which rely on moisture for curing, are typically
- epoxies
 - acrylics
 - cyanoacrylates
 - silicones

- _____ 1. Tungsten carbide particles embedded in a matrix of cobalt would be a typical example of a
- dispersion strengthened composite
 - laminar composite
 - ceramic-ceramic composite
 - cermet
- _____ 2. We would expect to have the best retention of strength at temperatures near the melting point of aluminium if we select alloy
- Borsic-reinforced aluminium
 - age hardenable aluminium
 - SAP aluminium
 - strain hardened aluminium alloy
- _____ 3. Which one of the following composites is expected to become stronger as the temperature is increased?
- Si_3N_4 fibres in a SiC matrix
 - carbon fibres in a carbon matrix
 - tungsten fibres in a nickel alloy matrix
 - Ni_3Al whiskers in a polyester matrix
- _____ 4. We would have the LEAST isotropic behaviour in
- three-dimensional fibre-reinforced composites
 - chopped fibre-reinforced composites
 - laminar composites
 - internally oxidized dispersion strengthened materials
- _____ 5. One of the most important limiting factors concerning Kevlar fibre-reinforced composites is
- their poor fracture toughness
 - their poor performance at elevated temperatures
 - their low specific modulus
 - their low specific strength
- _____ 6. A major advantage of dispersion strengthened composites compared to typical metals is
- their high room temperature strength
 - their superior resistance to creep
 - their high room temperature strength-to-weight ratio
 - their exceptionally light weight
- _____ 7. When sheets only one fibre thick are prepared by surrounding the fibres with only partly polymerized resin, we have produced a
- roving
 - prepreg
 - tow
 - yarn
 - whisker
- _____ 8. Delamination is NOT likely to be a problem in
- particulate composites
 - fibre-reinforced composites
 - laminar composites
- _____ 9. Increasing the temperature at which carbon fibres are carbonized
- increases the modulus of elasticity of the fibre
 - increases the modulus of rupture of the fibre
 - increases the strength of the fibre
 - increases the ductility of the fibre to at least 10% elongation
- _____ 10. Increasing the aspect ratio by making fibres with a smaller diameter will increase the strength of a composite primarily by
- increasing the bond strength between the fibre and the matrix
 - reducing the probability that critically sized flaws are present on the fibre surface
 - increasing the melting point of the fibre
 - reducing the density of the fibre

- _____ 11. The most important combination of properties that Alclad is intended to obtain is
- high strength and high modulus of elasticity
 - high strength and excellent wear resistance
 - good corrosion resistance and excellent electrical conductivity
 - good corrosion resistance and high strength
- _____ 12. Nearly perfect single crystal fibres containing no mobile dislocations are called
- prepregs
 - tows
 - whiskers
 - rovings
- _____ 13. We do not expect to manufacture fibre-reinforced composites of
- carbon fibres in a polymer matrix
 - ceramic fibres in a metal matrix
 - Kevlar fibres in a metal matrix
 - metal fibres in a polymer matrix
- _____ 14. When fibres separate from the matrix due to poor bonding,
- delamination has occurred
 - pull-out has occurred
 - a Griffith crack has grown
 - sizing has occurred
- _____ 15. Improving strength and stiffness is most likely to be the major reason for producing a
- true particulate composite
 - fibre-reinforced composite
 - laminar composite
- _____ 16. We expect the lowest specific strength and specific modulus for
- polyethylene fibres
 - nylon fibres
 - carbon fibres
 - Kevlar fibres
- _____ 17. The sketch below shows the typical cross-section of a carbon fibre-reinforced epoxy matrix composite. The expected modulus of elasticity parallel to the fibres is expected to be: (the modulus for carbon is 77×10^6 psi and the modulus for epoxy is 400,000 psi)
- 77.4×10^6 psi
 - 38.7×10^6 psi
 - 800,000 psi
 - 23.6×10^6 psi



- _____ 18. The laminar composite composed of aluminium sheets sandwiched between Kevlar fibre-reinforced polymer sheets is called
- Alclad
 - Arall
 - Glare
 - honeycomb
- _____ 19. Borsic fibres contain tungsten, boron, and silicon carbide.
- the tungsten is present primarily to provide high strength at high temperatures
 - the boron is intended to provide good ductility to the fibre
 - the silicon carbide is intended to provide good bonding between the fibre and an aluminium matrix
- _____ 20. Good bonding between the fibres and the matrix is NOT always desired in
- ceramic fibre-polymer matrix composites
 - glass fibre-polymer matrix composites
 - ceramic fibre-ceramic matrix composites
 - ceramic fibre-metal matrix composites

- _____ 1. We normally expect softwoods to
a. have a higher density than hardwoods
b. have a lower density than hardwoods
c. have a higher compressive strength than hardwoods
- _____ 2. The strength of a wood increases significantly when the moisture content falls below about 30% because
a. the water in pores and vessels begins to evaporate
b. the water in the cellulose fibres begins to evaporate
- _____ 3. In general, we expect the tensile strength to be greater than the compressive strength in
a. woods b. concrete c. ceramics
- _____ 4. We expect the greatest changes in dimensions of a part due to changes in the humidity for
a. woods b. concrete c. ceramics d. metals
- _____ 5. We normally expect a wood to have the highest strength in the
a. longitudinal direction
b. radial direction
c. tangential direction
- _____ 6. We expect a slow rate of curing when cement contains a large percentage of
a. $3\text{CaO Al}_2\text{O}_3$ b. 3CaO SiO_2 c. 2CaO SiO_2
- _____ 7. Increasing the amount of air entrained in concrete
a. increases the strength of the concrete
b. increases the bulk density of the concrete
c. increases the workability of the concrete
- _____ 8. We normally find that the strength of a concrete
a. increases as the amount of entrained air increases
b. increases as the water/cement ratio increases
c. increases as the curing time increases
- _____ 9. To produce good strength in a concrete,
a. the aggregate particles should be as large as possible, depending on the size of the structure that is to be built
b. the aggregate particles should be round, rather than angular
c. we should adjust the concrete mix to provide a large slump
- _____ 10. An external load on reinforcing bars in concrete is permanently applied for
a. reinforced concrete
b. prestressed concrete
c. poststressed concrete

- _____ 1. Copper-base alloys are expected to have the lowest electrical conductivity when they are strengthened by
a. grain size strengthening b. dispersion strengthening
c. age hardening d. solid solution strengthening
e. cold working
- _____ 2. When the temperature increases, the electrical conductivity of a metal almost always
a. increases b. decreases c. is unchanged
- _____ 3. When the temperature increases, the electrical conductivity of a ceramic material almost always
a. increases b. decreases c. is unchanged
- _____ 4. When the temperature increases, the electrical conductivity of an intrinsic semiconductor almost always
a. increases b. decreases c. is unchanged
- _____ 5. In electrical conduction, which one of the following is analogous to the activation energy in diffusion?
a. the flux b. the energy gap
c. the conductivity d. the electric field
- _____ 6. Changes in temperature and number of lattice imperfections affect the electrical conductivity of metals primarily by
a. changing the energy gap
b. changing the mobility
c. changing the number of charge carriers
d. changing the amount of charge carried by each charge carrier
- _____ 7. Introducing lattice imperfections, such as antimony atoms, to a semiconductor such as silicon affects the electrical conductivity primarily by
a. changing the mobility of the charge carriers
b. changing the number of charge carriers
c. changing the amount of charge carried by each charge carrier
- _____ 8. We do not expect to have an energy gap between the conduction and valence bands for
a. metals b. polymers c. ceramics d. semiconductors
- _____ 9. When we add a dopant such as arsenic to germanium, we expect the graph of conductivity versus temperature to show a
a. donor exhaustion plateau
b. acceptor saturation plateau
c. continuously increasing conductivity with increasing temperature
d. continuously decreasing conductivity with increasing temperature
- _____ 10. No current will flow in the circuit when we connect a power supply to a p-n diode such that
a. a recombination zone is produced
b. a depletion zone is produced
- _____ 11. The donor exhaustion plateau in GaP should extend over a wider temperature range than that in silicon. The reason for this is
a. electrical charge is carried in GaP by diffusion of ions
b. the mobility of the charge carriers in GaP is several orders of magnitude greater than that in silicon
c. the energy gap in GaP is larger than that in silicon

- _____ 1. The power of a magnetic material refers to the
a. coercive field required to randomise the magnetic moments
b. the maximum product of inductance B and magnetic field H
c. the sum of the magnetic moments of all of the atoms in the material
d. the maximum temperature to which a soft magnet will heat
- _____ 2. Magnetic materials such as Alnico that are to be used as permanent magnets are said to be magnetically
a. hard b. soft
- _____ 3. The spin of each electron in nickel produces a magnetic moment called a Bohr magneton. Nickel has 8 electrons in its 3d energy level and 2 electrons in its 4s energy level. The net magnetic moment for each nickel atom is
a. 2 Bohr magnetons b. 3 Bohr magnetons
c. 8 Bohr magnetons d. 10 Bohr magnetons
- _____ 4. Dipole friction causes heating of a magnet used in an electrical application because
a. it causes eddy currents which produce resistance heating
b. it represents energy lost when dipoles cannot be realigned quickly enough in an alternating field
- _____ 5. Iron-silicon alloys oriented in such a way that the <100> directions are aligned with the magnetic field are generally considered for
a. electrical applications b. permanent magnets
- _____ 6. Ceramic magnets used in electrical applications are particularly attractive compared to most metal-based magnetic materials because
a. the ceramics have a high electrical resistivity
b. the ceramics develop a much higher remanence
c. the coercive field is particularly high in ceramics
- _____ 7. A permanent magnet material should possess all of the following characteristics EXCEPT:
a. high relative permeability b. large coercive field
c. small hysteresis loop d. large remanence
- _____ 8. A reinforcement of an applied magnetic field is achieved only while the field is applied for materials that display
a. ferromagnetic behaviour
b. ferrimagnetic behaviour
c. paramagnetic behaviour
d. diamagnetic behaviour
- _____ 9. Domain-size precipitates of a ferromagnetic material are produced during the heat treatment of magnetic materials such as
a. Co₅Sm b. Alnico c. ceramic ferrites d. silicon iron
- _____ 10. We can increase the overall magnetic moment in a ceramic ferrite by introducing ions such as
a. Zn²⁺ b. Ni²⁺ c. Mn²⁺ d. Cu²⁺

- _____ 1. Generally X-rays are produced by electrons jumping from one energy level to another
- in the valence band of the atoms
 - in the conduction band of the atoms
 - in the "inner" energy levels of the atoms
- _____ 2. Electron traps that delay the transition of an electron from the conduction band to the valence band are responsible for
- phosphorescence
 - fluorescence
 - lasers
- _____ 3. Photons are NOT emitted in
- photoconductors
 - luminescent materials
 - lasers
 - light emitting diodes
- _____ 4. We normally expect that the linear absorption coefficient of a material for photons will increase when the energy of the photons
- increases
 - decreases
- _____ 5. The energy of an X-ray produced when an electron jumps from the M shell to the K shell
- is the same for all elements, such as Ca, Zn, or Al
 - depends primarily on whether the element is in the pure form, such as pure Al, or in an alloy, such as Al-Cu
 - is different for each element
- _____ 6. The wavelength of the photons produced in a light emitting diode
- depends primarily on the mobility of electrons in the LED
 - depends primarily on the energy gap of the LED material
 - will change significantly as the voltage applied to the LED changes
- _____ 7. We would expect a photon to be transmitted through a pure semiconductor if the energy of the photon is
- greater than the energy gap of the semiconductor
 - less than the energy gap of the semiconductor
- _____ 8. When the energy of a photon increases, the wavelength of the photon
- increases
 - decreases
- _____ 9. In order to identify a material based on its X-ray spectrum, we must use the
- continuous spectrum of X-rays
 - characteristic spectrum of X-rays
- _____ 10. A material with a high reflectivity is expected to have a
- high dielectric constant
 - low dielectric constant

- _____ 1. We would expect the microstructure of a material to have the greatest effect on
- specific heat
 - thermal conductivity
 - linear coefficient of thermal expansion
- _____ 2. Higher specific heats are generally found for
- metals and alloys
 - ceramics
 - polymers
 - semiconductors
- _____ 3. Above room temperature, the specific heat of metals
- generally increases dramatically as temperature increases
 - generally decreases dramatically as temperature increases
 - is relatively constant as the temperature increases
- _____ 4. We expect to have a higher coefficient of thermal expansion when
- the strength of the atomic bonds decreases
 - the melting temperature of the material increases
 - the material has a high modulus of elasticity
- _____ 5. As the temperature increases, the coefficient of thermal expansion
- remains constant for a given material
 - increases for a given material
 - decreases for a given material
- _____ 6. We expect to produce higher thermal stresses in a material for each of the following EXCEPT
- increasing the coefficient of thermal expansion
 - increasing the specific heat
 - increasing the temperature change
 - increasing the modulus of elasticity
- _____ 7. Generally the thermal conductivity of ceramics increases as the temperature increases because
- the mobility of electrons is much higher at high temperatures
 - the number of electrons that are able to move is much higher at high temperatures
 - the lattice vibrations are much more pronounced at high temperatures
- _____ 8. When the temperature of a metal increases, the thermal conductivity may
- increase because more valence electrons are available
 - increase because the mobility of the electrons is higher
 - increase due to larger lattice vibrations
- _____ 9. The resistance of a material to thermal shock is improved if
- the modulus of elasticity is high
 - the thermal conductivity is high
 - the coefficient of thermal expansion is high
- _____ 10. The resistance of a metal to thermal shock is generally very good, primarily because
- the modulus of elasticity is very low
 - the metal can deform rather than fracture
 - the coefficient of thermal expansion is very low

- _____ 1. A by-product of an electrochemical cell occurs as a result of various reactions at the
a. anode b. cathode
- _____ 2. Actual removal of metal during corrosion occurs at the
a. anode b. cathode
- _____ 3. Which one of the following is NOT an example of a composition cell?
a. intergranular corrosion b. a scratched tin-plated steel
c. crevice corrosion d. corrosion of a two-phase alloy
- _____ 4. Generally electrochemical corrosion is limited to metals and does not typically occur in ceramics or polymers because
a. only metals have good electrical conductivity
b. only metals are likely to dissolve in an electrolyte or solvent
c. only metals can be joined together tightly enough to produce effective physical contact
d. ceramics and polymers typically behave as anodes
- _____ 5. Corrosion occurs in a concentration cell wherever the oxygen content of the electrolyte is
a. highest b. lowest
- _____ 6. We can protect a steel pipeline by
a. connecting a large piece of tin to the pipeline to produce an electrochemical cell
b. connecting the steel pipeline to a source of an alternating current
c. connecting the steel pipeline to a direct current source such that electrons continually flow from the pipeline
d. connecting the steel pipeline to a direct current source such that electrons continually flow to the pipeline
- _____ 7. When an electrochemical cell cannot be avoided, we would find that the rate of corrosion will be most RAPID if
a. the anode area is larger than the cathode area
b. the cathode area is larger than the anode area
- _____ 8. A heat treatment called a quench anneal is used to
a. deliberately cause a stainless steel to corrode so that grain-sized particles are produced for powder metallurgy processing
b. eliminate sensitization and possible intergranular corrosion
c. produce martensite in stainless steels for good wear resistance
d. produce a thick protective oxide layer on aluminium
- _____ 9. Intergranular corrosion that occurs in a sensitized austenitic stainless steel is an example of a
a. composition cell b. stress cell c. concentration cell
- _____ 10. Anodizing is
a. introducing chemicals into the electrolyte that concentrate at the anode and prevent an electrical circuit from developing
b. coating steel with a metal such as copper, tin, or gold
c. producing a thick protective oxide layer on aluminium
- _____ 11. Corrosion of alumina or silica is most likely to occur by
a. electrochemical corrosion when placed in contact with other oxides
b. chemical attack when placed in contact with liquid ceramics
c. oxidation when exposed at elevated temperatures

- _____ 12. Suppose that a metal oxidizes and the density of the oxide is greater than the density of the metal that is removed:
- a. the oxide will form a protective, nonporous, adherent film
 - b. the oxide will be porous and oxidation will continue
 - c. the oxide will flake off and oxidation will continue
- _____ 13. When we place a piece of copper in contact with a piece of gold, we expect that the
- a. copper will corrode
 - b. gold will corrode
- _____ 14. Due to a stress cell, we would expect faster corrosion for a
- a. metal with a large grain size
 - b. metal with a small grain size
- _____ 15. Oxidation generally is most severe at
- a. high temperatures
 - b. low temperatures
- _____ 16. Suppose a steel post is partly under water and partly above water. The anode in this concentration cell is expected to be
- a. the steel that is under water
 - b. the steel that is above the water line
- _____ 17. We would like to protect nickel by using a sacrificial anode. The sacrificial anode might be
- a. lead (Pb)
 - b. chromium (Cr)
- _____ 18. Impurities in a metal can concentrate at the grain boundaries of the metal. Typically this will cause corrosion due to a
- a. concentration cell
 - b. stress cell
 - c. composition cell

- _____ 1. The fracture of a metal part by an impact load is often characterized by the presence of
- a chevron pattern
 - a cup-and-cone appearance
 - beach marks
 - extensive crack branching
- _____ 2. A ductile fracture of a metal part might be recognized by the presence of
- dimples on the fracture surface
 - striations on the surface
 - a corrosion product
 - beach marks on the surface
- _____ 3. A cup-and-cone fracture appearance is typical of
- ductile fracture
 - brittle fracture
- _____ 4. When a material fails by fatigue, we expect to observe all of the following features EXCEPT
- beach marks, usually with only a magnifying glass
 - striations, usually requiring a scanning electron microscope
 - extensive plastic deformation and bending of the overall part
 - a smooth fracture surface where the fatigue crack slowly grew
- _____ 5. When we observe a fracture surface that contains multiple cracks, typically filled with a nonmetallic material, we suspect failure occurred by
- ductile overload
 - stress corrosion cracking
 - creep failure
 - impact
- _____ 6. The presence of voids at triple points, or where three grain boundaries meet, is a common indication that failure occurred by
- ductile overload
 - stress corrosion
 - creep failure
 - impact
- _____ 7. We never expect to see any evidence of plastic deformation when failure occurs due to
- ductile overload
 - fatigue failure
 - creep failure
- _____ 8. We are able to detect imperfections beneath the surface of any type of material, although we will not know how far beneath the surface the imperfection is, when we use
- X-ray radiography
 - ultrasonic inspection
 - magnetic particle inspection
 - dye penetrant inspection
- _____ 9. We would expect to observe a crack perpendicular to the surface of a flat piece of steel if we use
- X-ray radiography
 - ultrasonic inspection
- _____ 10. We would NOT be able to use magnetic particle inspection to locate imperfections in
- nickel
 - steel
 - silica
 - cobalt
- _____ 11. Longer exposure times for X-ray radiography would be expected if the linear absorption coefficient of the material being examined
- is increased
 - is decreased
- _____ 12. We typically use a dielectric, piezoelectric material such as BaTiO₃ when we use
- X-ray radiography
 - ultrasonic inspection
 - dye penetrant inspection
 - magnetic particle inspection

_____ 13. By applying a load below the yield strength of the material, we can nondestructively determine whether there is a subcritical crack in the part by the

- a. ultrasonic inspection test
- b. hardness test
- c. acoustic emission test
- d. magnetic particle inspection test

_____ 14. We generally cannot detect whether a crack is present in a material when we use the

- a. ultrasonic inspection test
- b. hardness test
- c. acoustic emission test
- d. magnetic particle inspection test

_____ 15. A crack must actual be present at the surface in order for it to be detected by

- a. ultrasonic inspection
- b. acoustic emission
- c. dye penetrant inspection
- d. magnetic particle inspection